# Nonaxisymmetric Horizon Instability of Extremal Black Holes


Peter Zimmerman

w/ Marc Casals and Sam Gralla


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University of Arizona

GR 21 NYC

# Extremal black holes in a nutshell

- Parameters are saturated (a=M in Kerr)

- Enhanced symmetries (Kerr/CFT)
  - SL(2,R) x U(1)

- Vanishing surface gravity (zero Hawking temperature)
  - redshift effect $e^{-\kappa v} \to 1$

- Conserved quantities
  - infinite number of charges (Aretakis constants)

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    - Mode stability (Whiting, 1989) forbids exponentially growing modes, but doesn’t prove boundedness of generic perturbations.
    - Aretakis (2010) shows linear perturbations decay but transverse derivatives blow up polynomial on the event horizon. Lucietti and Reall extend to gravity (2012).

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- No. The derivative instability at the horizon is recovered as a branch point in the complex frequency plane.

- The mode technique allows us to predict the growth of *nonaxisymmetric* modes.

    - We find enhanced growth rates for these modes.

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# The Laplace transform and linear stability

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- The late-time dynamics of a linear system is dictated by singular points of its transfer (Green) function in the complex frequency plane.
  - *Mode stability* - lack of singular points in the upper half plane
    - No exponentially growing modes.

We compute the Laplace transform of the Green function for *nonaxisymmetric* massless scalar perturbations.

spheroidal harmonics (SL problem)

$$G = \frac{1}{2\pi}\sum_{\ell,m} e^{im\psi}\int_{-\infty+ic}^{\infty+ic} S_{\ell m\omega}(\theta)S^*_{\ell m\omega}(\theta')\tilde{g}_{\ell m\omega}(x,x')e^{-i\omega v}d\omega$$

radial Green function

inverse **Laplace Transform**

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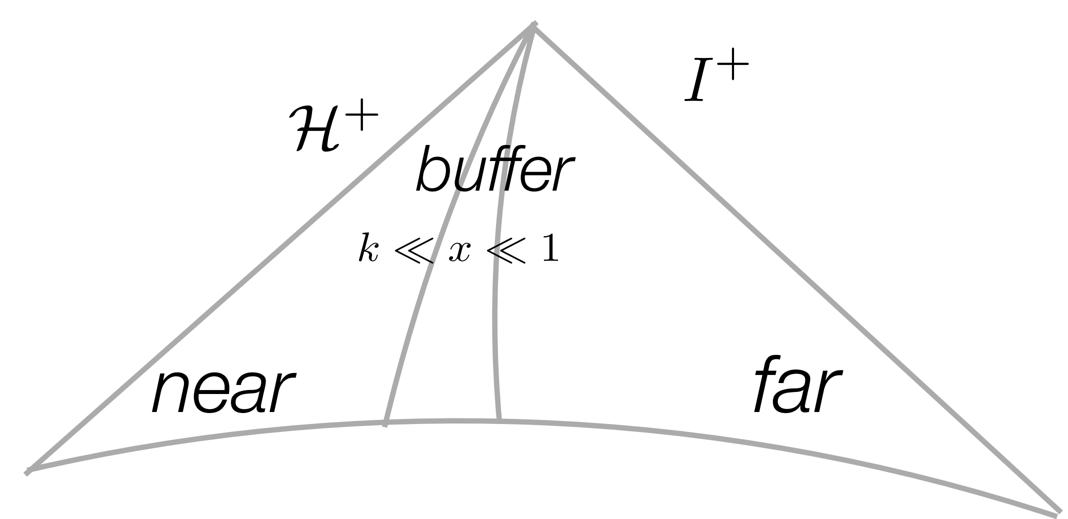
# Matched asymptotic expansion

- Construct $\tilde{g}_{\ell m \omega}$ from homogeneous solutions via the method of *matched asymptotic expansions*.
  - valid for $k \equiv \omega - m/2 \ll 1$
- Near zone $x \ll 1$
- Far zone $x \gg k$

Horizon at $x = 0$



# Extremal Kerr Green function

$$\tilde{g}_{\ell m\omega}(x,x') \sim f(x')\left[\frac{(-ik)^{-H+2i\omega}}{k(-ik)^{-2H}+\mathcal{N}}\right] e^{i\omega x}\sum_{j=0}^{\infty} A_j(\omega)\left(\frac{x}{ik}\right)^j, \quad x\to 0.$$

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- The branch point at the superradiant bound frequency k=0 determines the late-time solution.
- The character of the singularity is determined by the *conformal weight*.

# Conformal Weight

- The conformal weight labels representations of the near-horizon symmetry group, SL(2,R).

- At the superradiant bound, the highest weight solution has conformal weight given by

$$H \text{ is } \begin{cases} = 1/2 + ib, & |m| \gtrsim .74\ell \qquad\qquad \text{(dominant)}, \\ > 1, \ \& \notin \mathbb{Z}, & 0 < |m| \lesssim .74\ell \quad \text{(subdominant)}, \\ = \ell + 1, & m = 0 \qquad\qquad\qquad \text{(axisymmetry)}. \end{cases}$$

# Growth Rates on the event horizon

- Axisymmetric perturbations (compact data off H)

$\Phi^{(n)}_{\mathcal{H}} \equiv (\partial^n_x \Phi)|_{\mathcal{H}}$

when
$n = \ell + 1$

$$\Phi^{(n)}_{\mathcal{H}} \simeq v^{-2}, \qquad v \to \infty$$

otherwise

$$\Phi^{(n)}_{\mathcal{H}} \simeq v^{n-\ell-2}, \quad v \to \infty$$

Consistent with Aretakis' estimates and numerical simulations (Lucietti et al 2013)

# Growth rates on the event horizon

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  - Complex H $\quad \Phi^{(n)}_{\mathcal{H}} \simeq v^{n-1/2}, \qquad v \to \infty$

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The nonaxisymmetric modes yield the dominate derivative growth.

One derivative grows for compact “far” initial data.

$$\Longrightarrow \quad \text{local energy density} \simeq v$$

# Interpretation

- Off the horizon, the field decays slower (1/v). The jump in the decay *at* the horizon explains the derivative growth.

simple example $f(v,x) = e^{-vx}$

# Relation to QNMs

- Extended ringing of nearly extremal Kerr (Detweiler, 1980)

$$\omega_n = m/2 + O(\kappa)$$

  - The collective excitation of weakly damped overtones slows decay (transiently) from exponential to power law. (Yang et al 2013)

  - The modes vanish in the extremal limit (no real, non-superradiant QNMs) and the *branch point* "emerges" at the superradiant bound.

# Looking ahead - current work

- We have generalized to spin weighted fields

$$\psi_4^{(n)} \simeq v^{3/2+n}, \quad v \to \infty$$

Curvature blowup!

- We're relating our analysis to the MST solution
    - MAE is recovered as the first term in a convergent series representing the full linearized solution.

# Open questions

- What are the rates when we consider initial data that penetrates the horizon?
- Can we find nonaxisymmetric conserved quantities on the horizon?
- Is there a CFT analog of the instability?