

Black Hole Entropy Perspective on Neutron Star Mass

Contributed Talk : Joint Session C4-D4 on Black Holes,
GR21, Columbia University, New York

Parthasarathi Majumdar

Department of Physics
Ramakrishna Mission Vivekananda University
Belur, West Bengal, India

July 13, 2016

Neutron Star Facts

- Highest Neutron Star (NS) mass (most accurate measurement so far) : $2.01 \pm 0.04 M_{\odot}$ (Antoniadis 2013)

$$M_{NS} \leq M_{max} \simeq \xi M_{\odot} , \quad \xi = 2$$

unstable under gravitational collapse ?

- Schwarzschild radius to size (radius) ratio is unusually high for the heaviest NS

$$\frac{R_S}{R_{NS}} = O(10^{-1}) ,$$

Neutron Star Facts

- Highest Neutron Star (NS) mass (most accurate measurement so far) : $2.01 \pm 0.04 M_{\odot}$ (Antoniadis 2013)

$$M_{NS} \leq M_{max} \simeq \xi M_{\odot} , \quad \xi = 2$$

unstable under gravitational collapse ?

- Schwarzschild radius to size (radius) ratio is unusually high for the heaviest NS

$$\frac{R_S}{R_{NS}} = O(10^{-1}) , \quad \text{cf.} \quad \frac{R_S}{R_{WD}} = O(10^{-3})$$
$$\frac{R_S}{R_{\odot}} = O(10^{-5}) , \quad \frac{R_S}{R_E} = O(10^{-8})$$

Stable against gravitational collapse ?

$$M_* = \xi \left(\frac{M_P}{\Lambda_{QCD}} \right)^2 M_P, \quad \xi \sim 20 - 30$$

'...the combination of natural constants (above), providing a mass of proper magnitude for the measurement of stellar masses, is at the base of a physical theory of stellar structure.'

Intricate interplay between QCD and Quantum Gravity !?

M_* usually determined by Hydrostatic equilibrium : $P_{grav} = P_{Fermi}$

Spherical GR star Tolman, 1930; Oppenheimer, Volkov 1930

$$P_{grav} = P_{core} = \rho_0 \left[\frac{\left(1 - \frac{R_S}{R}\right)^{1/2} - 1}{1 - 3\left(1 - \frac{R_S}{R}\right)^{1/2}} \right]$$

NS: P_{Fermi} strongly dependent on NS EoS (Glendenning 2004) ← LE eff model of strong int → construction ambiguities !

'Model Independent' Approaches

- $dP/d\rho \in [0, 1]$ (Rhoades, Ruffini, 1974)

$$P_{deg} < \frac{1}{8} \left(\frac{\rho_0}{m_n} \right)^{4/3}, \quad m_n \rightarrow \text{neutron mass}$$

Hydrostat equil : $M_{max} \leq \xi \frac{M_P^3}{m_n^2}$, where ξ determined by matching to Harrison-Wheeler EoS as fiducial

- I-Love-Q (Yagi, Yunes, 2013)
 - Most EoSs show convergence vis-a-vis I-Love-Q
 - Proximity to collapse to Black Hole

Std assumption : GR effects small for determining NS EoS. Is Hydrostatic Equilibrium consistent with this ?

Alternative : GRQFT \Rightarrow additional technical complications (vacuum, spin-statistics thm, ...)

A Different Take

Reexpress maximum mass as

$$\left(\frac{M_{max}}{M_P}\right) = \xi \left(\frac{\lambda_{Cn}}{l_P}\right)^2 = \xi \frac{A_{Cn}}{A_P}$$

**Planck scale l_P appears nonperturbatively : rhs \nearrow
as $l_P \searrow$**

Contrast with perturbative QG effects $\rightarrow \sim O(l_P)$!

Reminiscent of black hole entropy :

$$S_{bh} = \frac{A_{hor}}{4A_P} + \text{quantum corr. for } k_B = 1$$

Motivation for Entropic Origin of Maximum Mass

Maximum mass for stability of NS \rightarrow Minimum Mass for Formation of Black Hole horizon

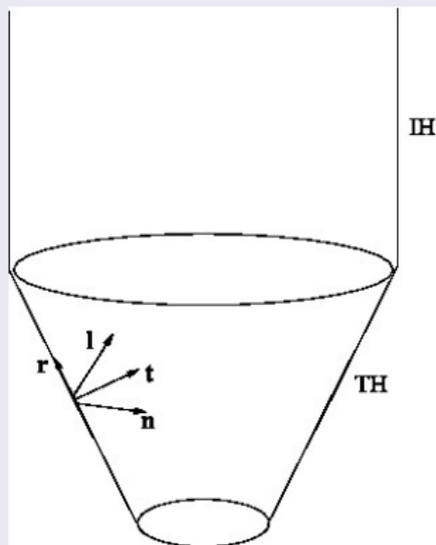
Critical Mass given in terms of ratio of areas : akin to black hole entropy

Speculate : possible entropic origin of critical mass not strongly dep on details of EoS of dense neutron matter

Speculate : Horizon formation not sudden, but akin to nucleation in 1st order phase transition

Assume : Trapping (dynamical) horizons evolving to Isolated Horizons as quasi-equil configurations

Trapping and Isolated Horizon (Ashtekar, Krishnan, 2005)



TH foliated by splk 2-surface : null normals l , n have $\Theta_l = 0$, $\Theta_n < 0$ (marginally trapped)

Splk TH : accreting energy and growing : $L_n \Theta_l < 0$

New Perspective : evolution of NS – EITHER collapse to black hole

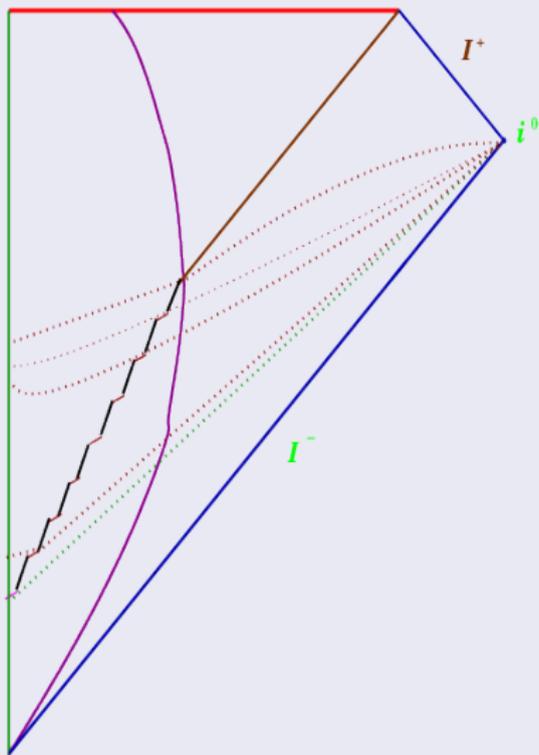
New Perspective : evolution of NS – EITHER collapse to black hole

New Perspective : evolution of NS – OR Stabilization

New Perspective : evolution of NS – OR Stabilization

New Perspective : collapse and stabilization in Eddington-Finkelstein frame

New Perspective : conformal frame - quasi-equilibrium



'Quantum General Relativity' : indep qu fluct on bdy :

$$\mathcal{H} = \mathcal{H}_b \otimes \mathcal{H}_h \otimes \mathcal{H}_m$$

$$|\Psi\rangle = \sum_{b,h,m} C_{bhm} |\psi\rangle_b \otimes |\chi\rangle_h \otimes |\phi\rangle_m$$

$$\hat{H} = \hat{H}_b \otimes \mathbf{1} \otimes \mathbf{1} + \mathbf{1} \otimes \hat{H}_h \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{1} \otimes \hat{H}_m$$

Hamiltonian constraint : bulk

$$\left(\hat{H}_b \otimes \mathbf{1} + \mathbf{1} \otimes \hat{H}_m \right) |\psi\rangle_b \otimes |\phi\rangle_m = 0$$

$|\phi\rangle_m$ might involve conserved charges $\Rightarrow \hat{H}_m \rightarrow \hat{H}'_m$ including chemical potentials (electric potential, angular frequency)

Bulk vs Boundary

Partition Function

$$\begin{aligned} Z(\beta) &= \text{Tr}_h \text{Tr}_{b,m} \exp -\beta \hat{H} \\ &= \sum_{b,m,h} |C_{b,m,h}|^2 \langle \chi |_m \langle \phi |_b \langle \psi | \exp -\beta \hat{H} | \psi \rangle_b | \phi \rangle_m | \chi \rangle_h \\ &= \text{Tr}_h \exp -\beta \hat{H}_h \cdot \sum_{b,m} |C_{b,m,h}|^2 || | \psi \rangle_b | \phi \rangle_m ||^2 \\ &= \text{Tr}'_h \exp \beta \hat{H}_h \equiv Z_h(\beta) \end{aligned}$$

Bulk states decouple! Boundary states determine bh thermodynamics \rightarrow Thermal holography !

Weaker than Holographic Hypothesis 't Hooft 1992; Susskind 1993; Bousso 2002
... Given any closed surface, we can represent all that happens (gravitationally) inside it by degrees of freedom on this surface itself. This ... suggests that quantum gravity should be described by a **topological** quantum field theory in which all (gravitational) degrees of freedom are projected onto the boundary.

Canonical ensemble of Trapping Horizons

- Assume $Z_h(\beta)$ is determined by dynamics of A_h , A_{NS} (kinem)
- Assume macroscopic areas
 $A_h \simeq n \cdot A_P$, $n \gg \gg 1$; $A_m \equiv A_m(A_{NS}, A_h) = A_{NS} - A_h$
- Rescale areas $A_h \rightarrow A_h/A_P$, $A_m \rightarrow A_m/A_{QCD}$, $A_{QCD} \sim \Lambda_{QCD}^{-2}$
- Assume time-scale such that on every $\Sigma_t \rightarrow$ quasi-equil with IH of fixed \bar{A}_h and $\bar{M}_h = M(\bar{A}_h)$; $\bar{M}_{mat} = M(\bar{A}_m)$, $\bar{A}_m = A_{NS} - \bar{A}_h$
- Keep Gaussian fluct. (Das, PM, Bhaduri 2001; Chatterjee, PM 2004; PM 2007)

$$Z_h(\beta) \simeq \int dA_h \exp [S_h(A_h) + S_m(A_{NS} - A_h) - \beta(M_h + M_m)]$$

for large area eigenvalues $n \gg \gg 1$.

**Evaluate Z_h by saddle-pt expansion around $\bar{A}_{IH}, \mathcal{A}_m \Rightarrow$:
Gaussian approx**

Thermal Stability Criterion

Extremal pt condition at \bar{A}_h , $\bar{A}_m = A_{NS} - \bar{A}_h \Rightarrow$

$$\beta(\bar{A}_h) = \frac{S_{h,A_h}(\bar{A}_h) + S_{m,A_h}(A_{NS} - \bar{A}_h)}{M_{h,A_h}(\bar{A}_h) + M_{m,A_h}(A_{NS} - \bar{A}_h)}$$

Saddle pt condition (at \bar{A}_h , \bar{A}_m)

$$\beta [M_{h,A_h A_h} + M_{m,A_h A_h}] |_{\bar{A}_h} > [S_{h,A_h A_h} + S_{m,A_h A_h}] |_{\bar{A}_h}$$

This can be further expressed as (with slight change notation)

$$(\log [\beta(A_h)_{A_h}])_{A_h} (\bar{A}_h) < 0$$

Stability of TH \Rightarrow the local temperature must increase with area

- **Thermal Stability Criterion for horizon (IH) : if satisfied, NS collapses into BH; if not, NS stabilizes**
- **How does this lead to a maximum mass for a stable NS ? (in progress)**

BH (IH) Limit of Thermal Stability Criterion

- Absence of matter : $(\beta M_h - S_{grav})_{A_h A_h} |_{\bar{A}_h} > 0 \rightarrow$ Thermal Stability Criterion for radiant black hole (Chatterjee, PM 2005; PM 2007; Majhi, PM 2011; Sinha, PM 2015)
- Equil $\beta \Rightarrow$ th stab crit :

$$\frac{M_{hA_h A_h}}{M_{hA_h}} > \frac{S_{hA_h A_h}}{S_{hA_h}}.$$

- Generalizes to charged, rotating horizons (Majhi, PM 2011; Sinha, PM 2015)
- **No classical metric used in derivation**
- **Corrections to area law for S_{IH} (characteristic of LQG) plays crucial role** (Kaul, PM 1998, 2000; Majhi, PM 2014)

$$S_{IH} = S_{BH} - \frac{3}{2} \log S_{BH} + O(S_{BH}^{-1}), \quad S_{BH} \equiv \frac{1}{4} \mathcal{A}_{IH}$$

Kerr-Newman black hole

$$M^2 = \frac{A}{16\pi} + \frac{\pi}{A}(4J^2 + Q^4) + \frac{Q^2}{2}$$

Violates stability bound \rightarrow thermally unstable

Anti-de Sitter Kerr-Newman :

$$M^2 = \frac{A}{16\pi} + \frac{\pi}{A}(4J^2 + Q^4) + \frac{Q^2}{2} + \frac{J^2}{l^2} + \frac{A}{8\pi l^2} \left(Q^2 + \frac{A}{4\pi} + \frac{A^2}{32\pi^2 l^2} \right)$$

Satisfies stability bound for

$$A \gg 4\pi(-\Lambda)^{-1/2}, \quad A \gg \sqrt{4J^2 + Q^4}$$

Implications of Thermal Stability Criterion

- Thermal Stability Criterion valid at every Σ_t where quasi-equilibrium is assumed with $\bar{A}_h(t)$, $\bar{A}_m(t) = A_{NS}(t) - \bar{A}_h(t)$ as quasi-equil area config
- Define NS mass on each Σ_t

$$M_{NS}(t) \equiv M_h(\bar{A}_h(t)) + M_m(A_{NS}(t) - \bar{A}_h(t))$$

- Stability of NS implies instability by decay of trapping hor; alternatively, stability of hor implies collapse of NS
- **Determine maximum $M_{NS}(t = t_i)$ for which $\bar{A}_h(t_f) = 0 \Rightarrow$ NS stabilizes**
- **Alternatively, determine minimum $M_{NS}(t = t_i)$ for which $\bar{A}_h(t_f) = A_{NS}(t_f) \Rightarrow$ NS collapses to a bh**
- To what extent can S_{mat} be computed without details of LE eff strong ineractions ?

Aspects of S_{mat}

- Gauge-gravity correspondence : in large N limit use semiclassical gravity dual to strong int for S_{mat} ?
- Holographic Entanglement Entropy (Ryu, Takayanagi 2006; Calabrese, Cardy 2004, 2009; Hubeny et. al. 2008; Banerjee 2014; ...)
- 3 dim spm submanifold \mathcal{M} to be defined, s.t. $\partial\mathcal{M}$ has area occurring in HEE. For S_{mat} , explore possibility of A_h , A_{NS} being submanifold area
- Holographic Entanglement Entropy $S_m \equiv -Tr_{\mathcal{M}}\rho_{m\mathcal{M}} \log \rho_{m\mathcal{M}}$ where $\rho_{m\mathcal{M}} \equiv Tr_{blk}\rho_{m,full}$
- General form : $S(A) = \xi_0(A/A_{UV}) + \xi_1 \log(A/A_{UV}) + \dots$
- LQG : $S_{grav} = -Tr_h\rho_{gh} \log \rho_{gh}$, $\rho_{gh} \equiv Tr_{blk}\rho_{g,full} \rightarrow$ interpret as entanglement entropy. Holography is automatic in LQG

Summary and Pending Issues

- Max mass for stable NS assumed to be close to min mass for collapse. Need to justify assumption, cf gravitational waves, neutrinos
- Conventional : stiffer EoS \Rightarrow higher max mass; softer (lower mass hadrons/quarks) EoS \Rightarrow lower max mass of NS. Here, postulate existence of partially trapping horizon, whose stability/growth or decay governs evolution of NS
- TH traps lighter and lighter matter as it grows with NS collapse, until it traps light \rightarrow birth of IH.
- TH turns tmlk from splk through a null phase, if decay dominates when TH turns momentarily into an IH
- How reliable is the 'thermal stability criterion' ? Matches IH (black hole) limit. Permits rather precise formulation of the problem : **What is the max $M_{NS}(t = t_i)$ such that $\bar{A}_h(t_f) \ll A_{NS}(t_f)$? Need to explore further**