Influence of a plasma on the shadow of a spherically symmetric black hole

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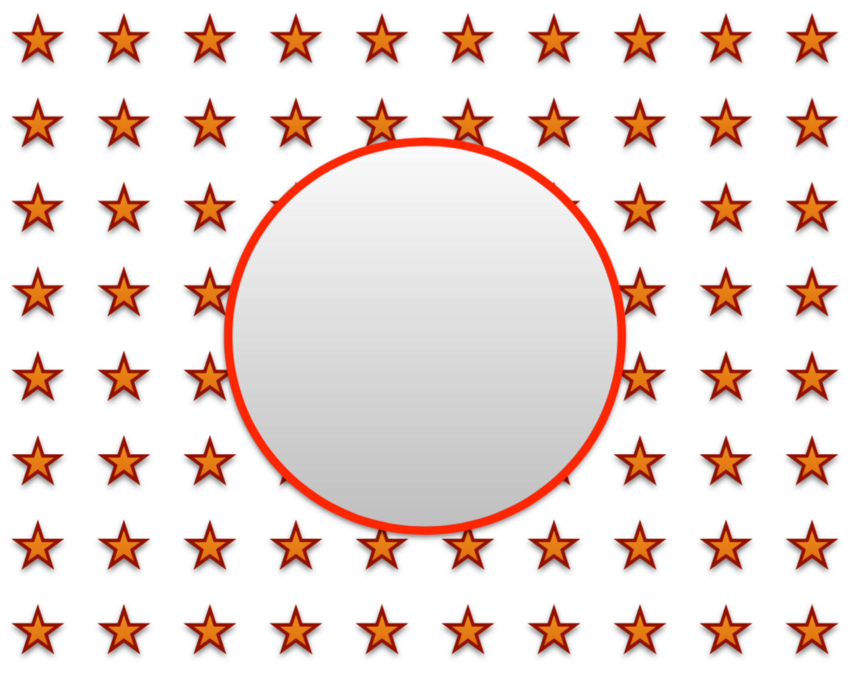
What is a black hole shadow?

There are SMBH in the center of galaxies, for example, in the center of our galaxy

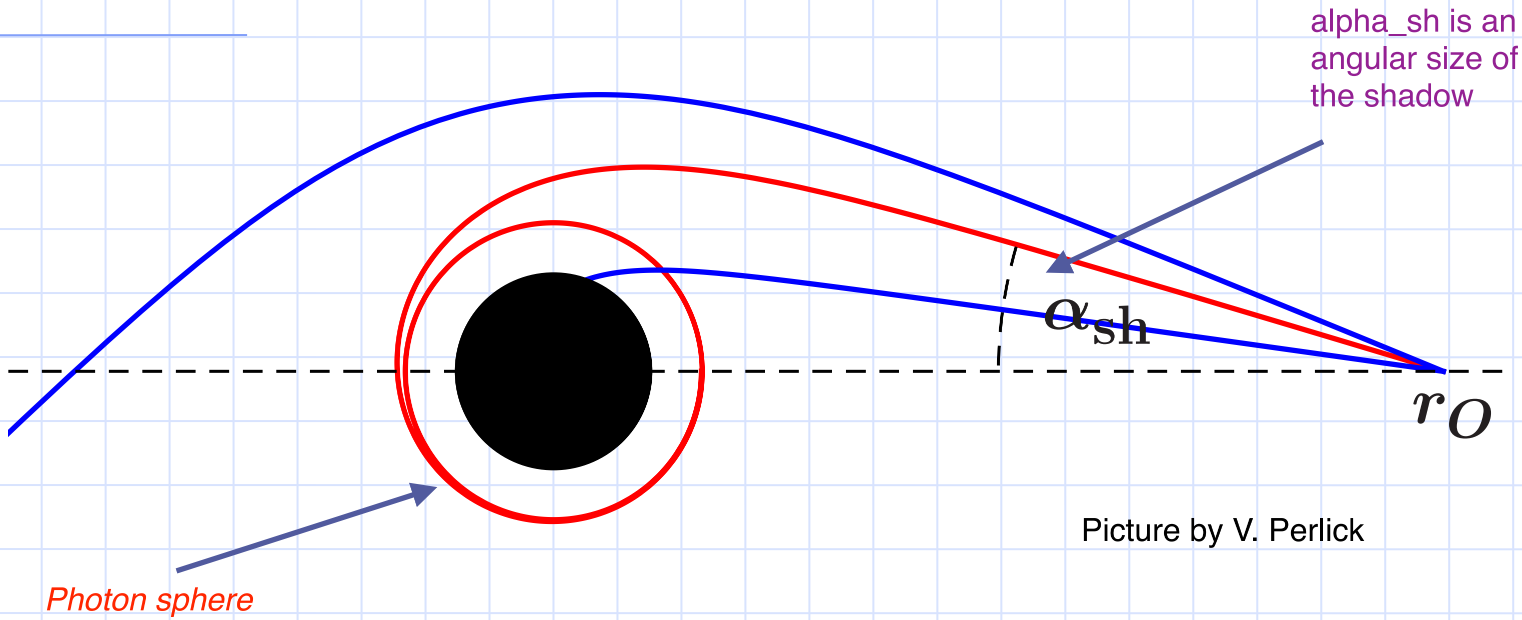
A distant observer should “see” this black hole as a dark disk in the sky which is known as the “shadow”



For the black hole at the center of our galaxy, size of the shadow is about 53 μas (size of grapefruit on the Moon).

Two projects are under way to observe this shadow with radio telescopes:
Event Horizon Telescope and the BlackHoleCam.

On the theoretical side, the shadow is defined as the region of the observer's sky that is left dark if there are light sources distributed everywhere but not between the observer and the black hole.



Picture by V. Perlick

Note: The shadow is *not an image of the event horizon*. The boundary of the shadow corresponds to light rays that asymptotically approach the photon sphere (at r = 3M in the Schwarzschild case) and not the horizon (at r = 2M in the Schwarzschild case). Moreover, light rays are bent due to gravity. So the shadow is the image of region inside the photon sphere increased by light bending.

Angular size of the shadow of Schwarzschild BH in vacuum

J.L. Synge, MNRAS 131, 463 (1966)

$$\sin^2\alpha_{\rm sh} = \frac{27M^2(1-2M/r_{\rm O})}{r_{\rm O}^2}$$

Schwarzschild radius

Radius of the photon sphere

$$R_S = 2M, \quad r_{ph} = 3M$$

For distant observer, $r_{\rm O} >> M$:

$$\alpha_{\rm sh} \simeq 3\sqrt{3}\,\frac{M}{r_{\rm O}}$$

Analytical calculation of the shadow size and shape in vacuum, other metrics:

Kerr:

J.M. Bardeen, in Black Holes, edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1973), p. 215.

Class of Plebański-Demiański spacetimes:

A. Grenzebach, V. Perlick, and C. Lämmerzahl, Phys. Rev. D 89, 124004 (2014).

A. Grenzebach, V. Perlick, and C. Lämmerzahl, Int. J. Mod. Phys. D 24, 1542024 (2015).

Here we work only with spherically symmetric BH

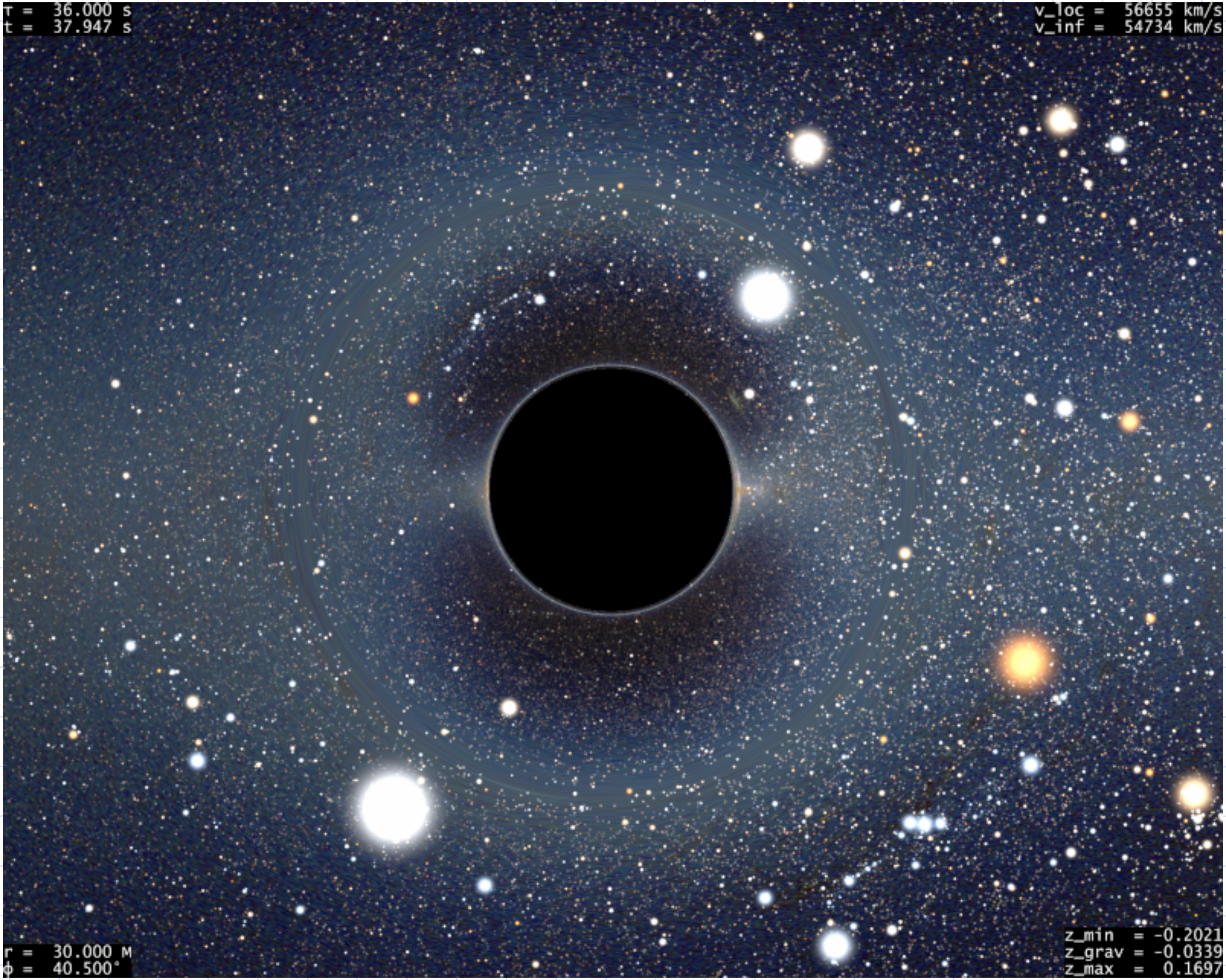
Influence of matter around BH on the observed size of the shadow

We would like to investigate the influence of matter around of BH on the observed size of the shadow.

Modeling is made by different groups.

For example, H. Falcke, F. Melia, and E. Agol, Astrophys. J. 528, L13 (2000).

Also, ray tracing programs have been written for producing realistic images of a black hole surrounded by an accretion disk, e.g., for the movie *Interstellar*.
The numerical techniques used for this movie are described in detail in O. James, E. Tunzelmann, P. Franklin, and K. Thorne, Classical Quantum Gravity 32, 065001 (2015).



Riazuelo A 2014 Simulation of starlight lensed by a camera orbiting a Schwarzschild black hole
(www2.iap.fr/users/riazuelo/interstellar)

BH shadow in presence of plasma, analytical approach

We perform the first attempt of **analytical** investigation of plasma influence on the shadow size, in frame of **geometrical optics**, taking into account effects of general relativity and plasma presence.

In this approximation, the presence of the plasma leads only to a **change of the geometrical size** of the shadow via a change of the light ray trajectories in this medium.

We consider general spherically symmetric space-time, and spherically symmetric distribution of plasma. Plasma is considered as a **medium with a given index of refraction**. Masses of plasma particles are not taken into account.

Geometrical optics in presence both gravity and plasma

To calculate, we need **general theory for geometrical optics in arbitrary medium** (dispersive or not) **in curved space-time** (in presence of gravity).

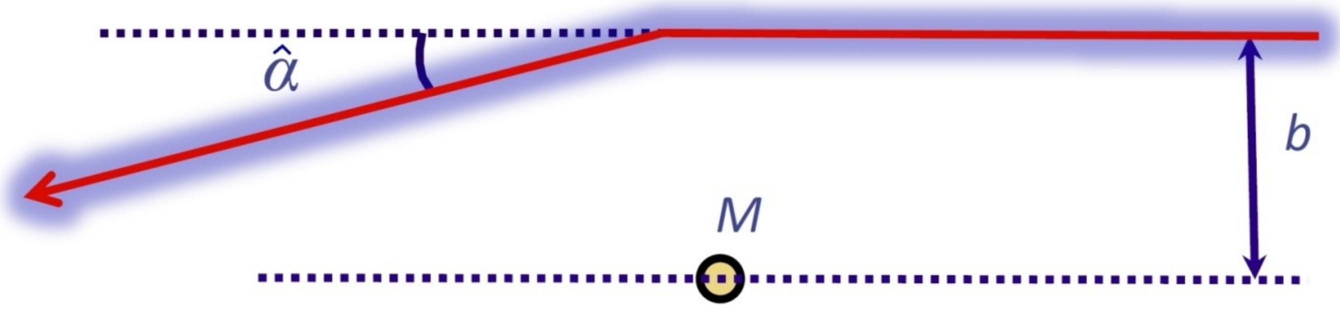
J.L. Synge, Relativity: The General Theory (1960)
The first self-consistent approach for geometrical optics in medium in presence of gravity
See also: J. Bicˇa´k and P. Hadrava, Astron. Astrophys. 44, 389 (1975)

The first monograph completely devoted to review of general-relativistic ray optics in medium:
V. Perlick, Ray Optics, Fermat's Principle, and Applications to General Relativity (Springer-Verlag, Berlin, 2000)

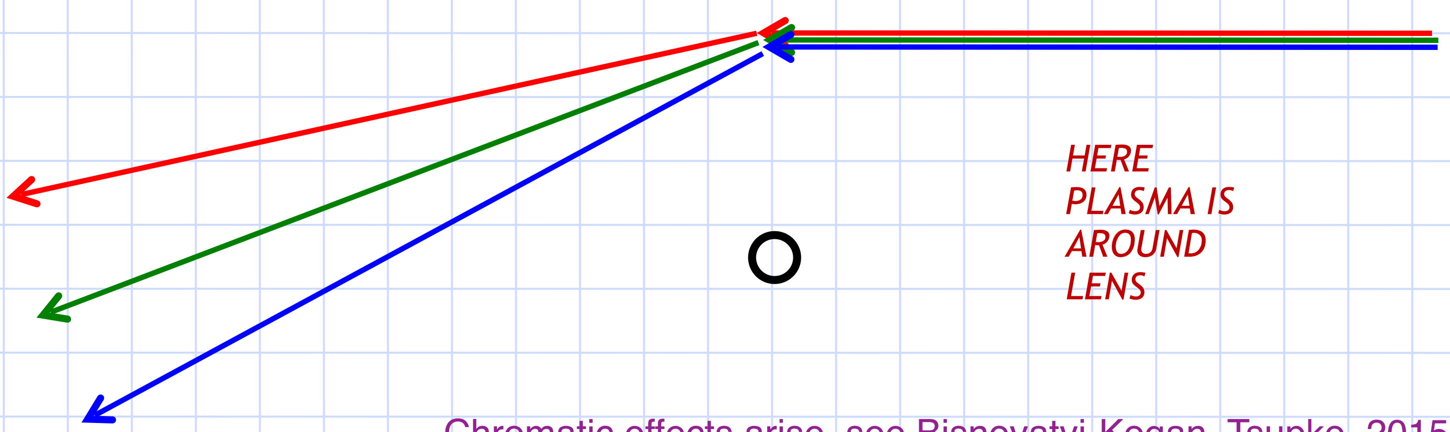
On language of gravitational lensing the problem of plasma influence is considered for the first time:
P.V. Bliokh and A.A. Minakov, Gravitational Lenses [Russian] (Naukova Dumka, Kiev, 1989).

Detailed description of different effects of gravitational lensing in presence of plasma:
G.S. Bisnovatyi-Kogan, O.Yu. Tsupko (2009, 2010, 2013, 2015)

In vacuum the deflection angle of the photon does not depend on the photon frequency:



In presence of both plasma and gravity the deflection angle do depend on the photon frequency:



Chromatic effects arise, see Bisnovatyi-Kogan, Tsupko, 2015 for review

BH shadow in presence of plasma, analytical approach

We consider general spherically symmetric space-time,

$$g_{ik} dx^i dx^k = -A(r) dt^2 + B(r) dr^2 + D(r)(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2)$$

and spherically symmetric distribution of plasma:

$$n(r,\omega)^2 = 1 - \frac{\omega_p(r)^2}{\omega^2} \qquad \omega_p(r)^2 = \frac{4\pi e^2}{m} N(r)$$

Photon frequency depends on space coordinates in presence of gravity (gravitational redshift)

$$\omega = \omega(r) \, !$$

electron plasma frequency

number density of the electrons in the plasma

We have derived analytical formula for angular size of the shadow of BH surrounded by spherically symmetric plasma distribution and succeeded to rewrite it in very compact way:

Angular radius $\alpha_{\rm sh}$ of shadow:

$$\boxed{\sin^2 \alpha_{\rm sh} = \frac{h(r_{\rm ph})^2}{h(r_{\rm O})^2}}$$

$r_{\rm ph}$ is the radius of photon sphere

$r_{\rm O}$ is the observer position

function $h(r)$ contains all information about metric, plasma distribution and photon frequency:

$$h(r)^2 = \frac{D(r)}{A(r)} \left(1 - A(r) \frac{\omega_p(r)^2}{\omega_0^2} \right)$$

$$g_{ik} dx^i dx^k = -A(r) dt^2 + B(r) dr^2 + D(r) (d\vartheta^2 + \sin^2\vartheta \, d\varphi^2)$$

Condition for photon sphere:

$$\boxed{\frac{d}{dr} h(r)^2 \Big|_{r=r_{\rm ph}} = 0 \quad \Rightarrow \quad r_{\rm ph}}$$

Dependence on the photon frequency, significant for radio waves

V. Perlick, O.Yu. Tsupko, G.S. Bisnovatyi-Kogan, Phys. Rev. D 92, 104031 (2015)

Using this formula, it is possible to calculate an angular radius of the BH shadow:

for any spherically symmetric metric, for example Schwarzschild BH, without approximation of weak field

for any position of observer, in particular very close to BH and very far from BH

for any spherically symmetric distribution of plasma

for any photon frequency

Note that plasma is a dispersive medium therefore the radius of the shadow depends on the photon frequency (more rigorously, on the ratio of the plasma frequency and the photon frequency), effect is significant for very long radio waves

RAINBOW SHADOW!

Example: accretion of free falling plasma onto Schwarzschild BH

$$A(r) = B(r)^{-1} = 1 - \frac{2GM}{c^2 r}, \quad D(r) = r^2, \quad \frac{\omega_p(r)^2}{\omega_0^2} = \beta_0 \left(\frac{GM}{c^2 r}\right)^{3/2}$$

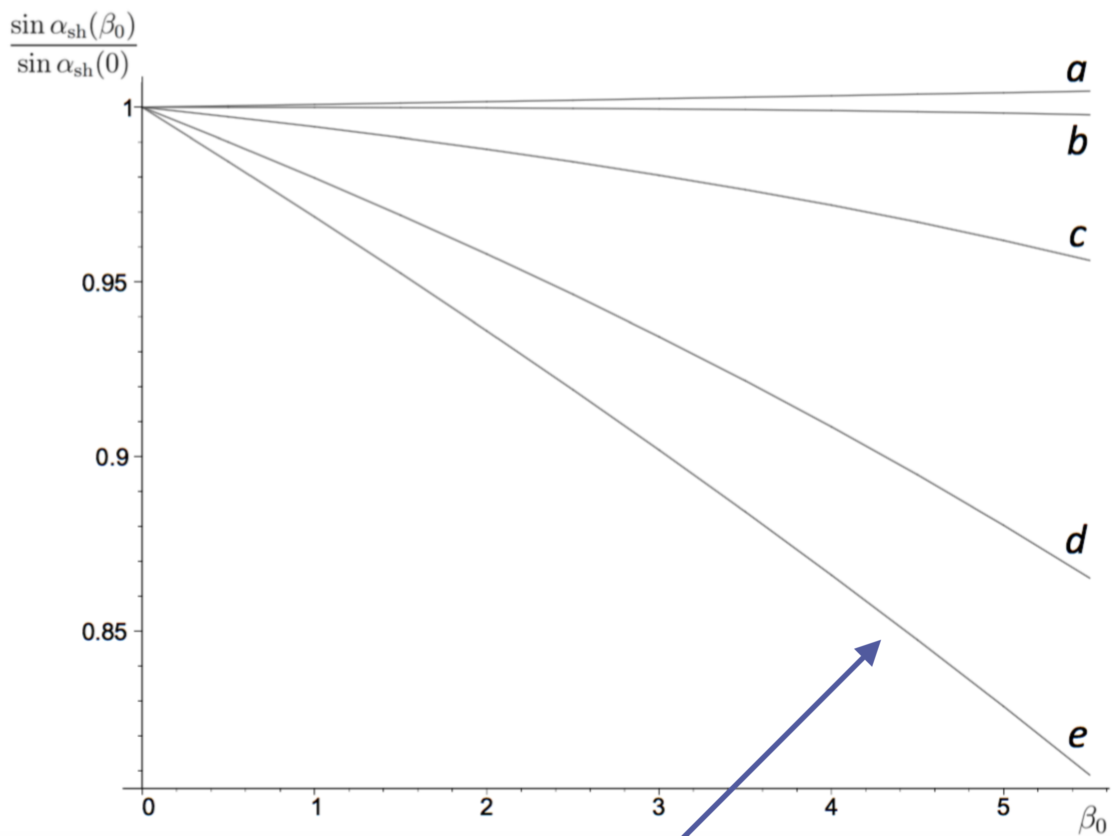
$a : r_O = 3.3\, GM/c^2$

$b : r_O = 3.8\, GM/c^2$

$c : r_O = 5\, GM/c^2$

$d : r_O = 10\, GM/c^2$

$e : r_O = 50\, GM/c^2$



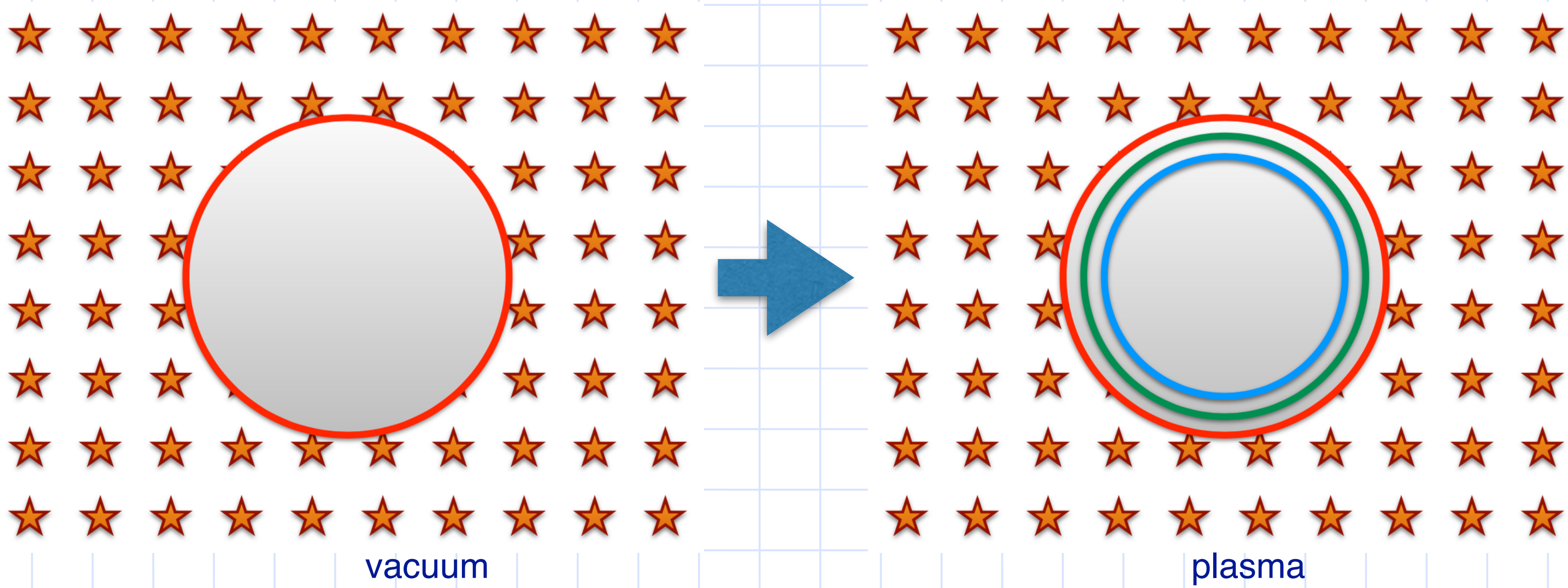
For distant observer the shadow becomes smaller due to plasma if we tend to smaller photon frequencies

Main results for the shadow:

In the presence of a plasma the size of the shadow **depends on the wavelength** at which the observation is made, in contrast to the vacuum case where it is the same for all wavelengths.

For an observer far away from a Schwarzschild black hole the plasma has a **decreasing effect** on the size of the shadow.

The effect of the plasma is significant only in the **radio** regime.



V. Perlick, O.Yu. Tsupko, G.S. Bisnovatyi-Kogan, Phys. Rev. D 92, 104031 (2015)