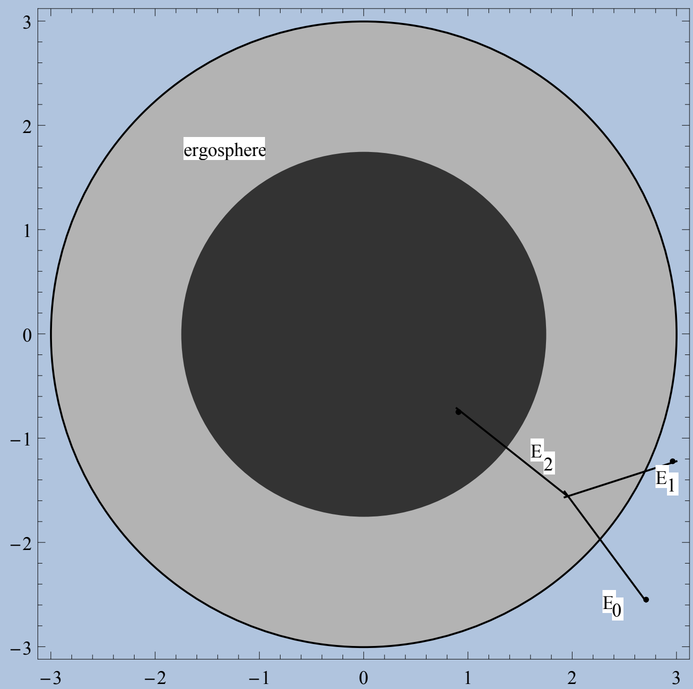
Ergosphere of the Born-Infeld black hole

Nora Bretón
Depto. de Física, Cinvestav-IPN, México.
GR21/NY2016

OUTLOOK

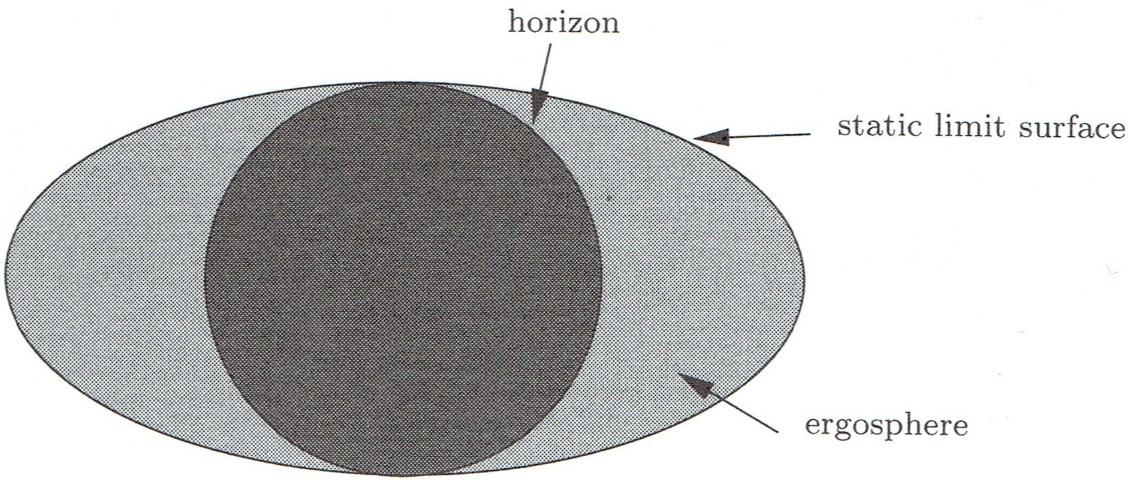
- Energy extraction from a black hole
- NLEM generalization of the Reissner-Nordstrom b.h.
- Finiteness of the e.m. field at $r = 0$
- Effective potential and the effective ergosphere
- Irreducible mass of the BI b.h.

Penrose process for energy extraction



The energy of the swallowed particle is negative as measured at ∞, but positive in the local Lorentz frame: $E_2 = E_0 - E_1$.

Energy can be extracted from a rotating black hole



Ergosphere of a Kerr black hole.

The ergosphere is the region between the horizon and the surface of infinite redshift (static limit).

Energy can be extracted from a charged black hole

[Christodoulou & Ruffini, 1971]
For a STATIC AXISYMMETRIC spacetime,
$ds^2 = \psi dt^2 - \psi^{-1} dr^2 - r^2 d\Omega^2$
the conserved quantities of a test particle μ, q are:
$p_t = E = -\mu\psi\dot{t} - qA_t, \quad p_\varphi = L_z = \mu g_{\varphi\varphi}\dot{\varphi} + qA_\varphi$
$\psi\dot{t}^2 - \psi^{-1}\dot{r}^2 - r^2\dot{\theta}^2 - \sin^2\theta\dot{\varphi}^2 = 1$
on the equatorial plane $\theta = \pi/2$ and at the turning point, substituting the conserved quantities,

$$\left(\frac{E}{\mu} - \frac{q}{\mu}A_t\right)^2 = \psi\left(1 + \frac{p_\varphi^2}{\mu^2 r^2}\right),$$

the eq. for E has the form $\alpha E^2 - 2\beta E + \gamma = 0$ with roots

$$E_\pm = qA_t \pm \mu\left[\psi\left(1 + \frac{p_\varphi^2}{\mu^2 r^2}\right)\right]^{1/2},$$

For $q < 0$ negative energy E_+ states exist and then energy can be extracted. E_+ corresponds to 4-momentum pointing toward future.
Note that no extraction is possible in the limit $p_\varphi \to \infty$

THE REISSNER-NORDSTROM (RN) BLACK HOLE

IT IS THE UNIQUE SSS SOLUTION OF THE EINSTEIN-MAXWELL EQUATIONS

It is characterized by M, Q independent parameters, and the line element:

$$ds^2 = \Psi(r)dt^2 - \Psi^{-1}(r)dr^2 - r^2 d\Omega^2, \quad \Psi(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2},$$

the horizon radius is solution of

$r^2 - 2Mr + Q^2 = 0, \quad r_\pm = M \pm \sqrt{M^2 - Q^2}$

The extreme RN black hole is when $M^2 = Q^2$, in this case the two horizons coalesce. The extreme RN b.h. is of great importance in string theory as a BPS state.

The electric field diverges at the charge position $r = 0$,

$$\vec{E}(r) = \frac{Q}{r^2},$$

To mend this situation Born and Infeld proposed (1934) the finiteness of the e.m. field at the charge position, b would be the maximum allowed field; the Lagrangian is

$$\mathcal{L}_{\rm BI} = 4b^2 \left(-1 + \sqrt{1 + \frac{F}{2b^2} + \frac{G^2}{16b^4}} \right)$$

$F = F_{\mu\nu}F^{\mu\nu} = 2(B^2 - E^2)$ and $G = \tilde{F}_{\mu\nu}F^{\mu\nu} = \vec{E} \cdot \vec{B}$
b is the maximum allowed field, $b = 10^{20}$Volt/m

- Maxwell's Lagrangian, $\mathcal{L}_{\rm Max} = F$.
- No bi-refringence,

- Euler, Heisenberg and Schwinger showed that BI Lagrangian may be an effective Lagrangian of QED vacuum polarization.

  Expanding $\mathcal{L}_{\rm BI}$ for "small"b,

$$\mathcal{L}_{\rm BI} = 4b^2\left(-1+\sqrt{1+\frac{F}{2b^2}+\frac{G^2}{16b^4}}\right) \approx F - \frac{F^2}{8b^2} + \frac{G^2}{8b^2} + \cdots$$

- Comparing with the Heisenberg-Euler Lagrangian,

$$\mathcal{L}_{\rm HE} = \mathcal{L}_{\rm Max} + \frac{e^4}{360m^4\pi^2}\left\{\frac{F^2}{4} + 7G^2\right\},$$

  that describes the low energy limit of the light-light scattering.

- The BI action is the effective action for low-energy degrees of freedom of the D-brane

  $\mathcal{L}_{\rm ST} = -T\sqrt{-det(\eta_{\mu\nu} - 2\pi\alpha F_{\mu\nu})}$

  α is the string tension and is related to b, On a D3-brane, open strings attached to the brane may couple to a $U(1)$ field at the end of the string so that the lowest order contribution to the partition function is given by $\mathcal{L}_{\rm ST}$

- The Born-Infeld (BI) coupled to gravity action :
  $S = \frac{1}{16\pi}\int d^4x\sqrt{g}\,(R - \mathcal{L}_{\rm BI}),$
- The BI black hole line element (static),

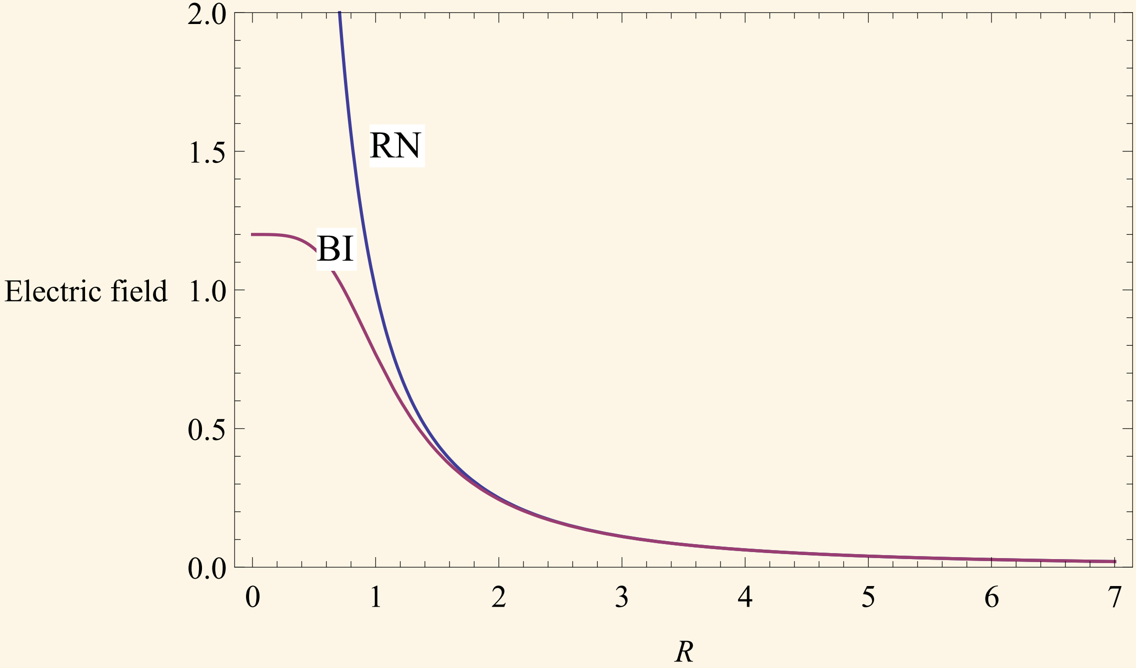
$$ds^2 = \psi(r)dt^2 - \psi^{-1}(r)dr^2 - r^2 d\Omega^2,$$

$$\psi_{\rm BI} = 1 - \frac{2M}{r} + \frac{2}{3}r^2b^2\left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}}\right) + \frac{4Q^2}{3r}\int_r^\infty \frac{dz}{\sqrt{z^4 + Q^2/b^2}}$$

- For the BI b.h. $E(r=0)$ is finite, although there are curvature singularities at $r = 0$
- The SSS solution of the coupled Einstein-Maxwell eqs.

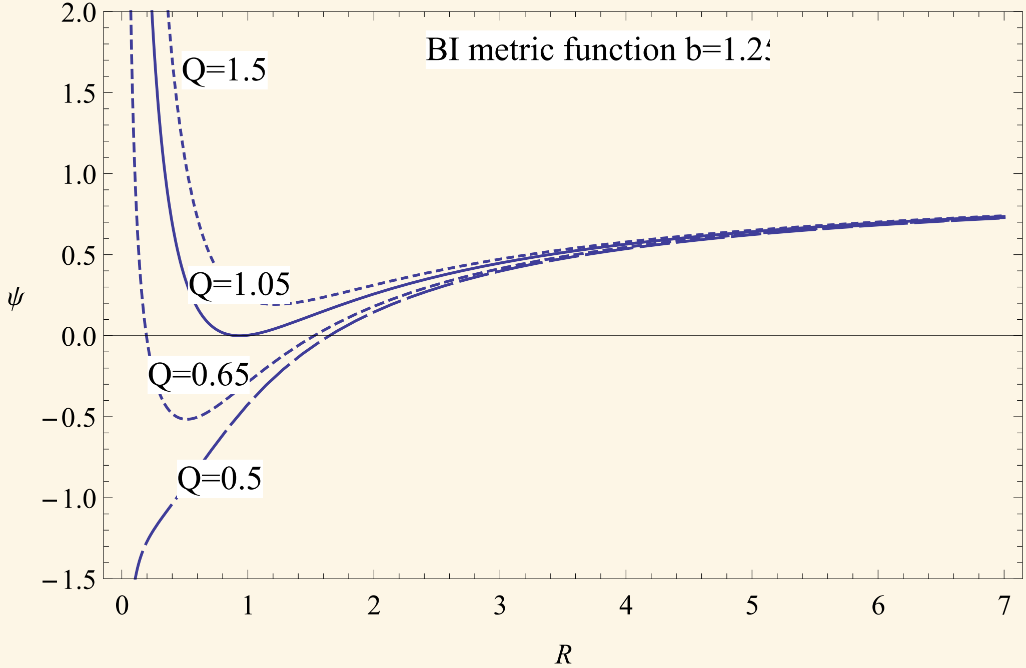
$$\psi_{\rm RN}(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2},$$

ELECTRIC FIELDS OF REISSNER-NORDSTROM AND BORN-INFELD BLACK HOLES



The electric field of the BI b.h. is finite at $r = 0$ as was the aim of Born-Infeld theory; RN electric field goes to infinity at $r = 0$

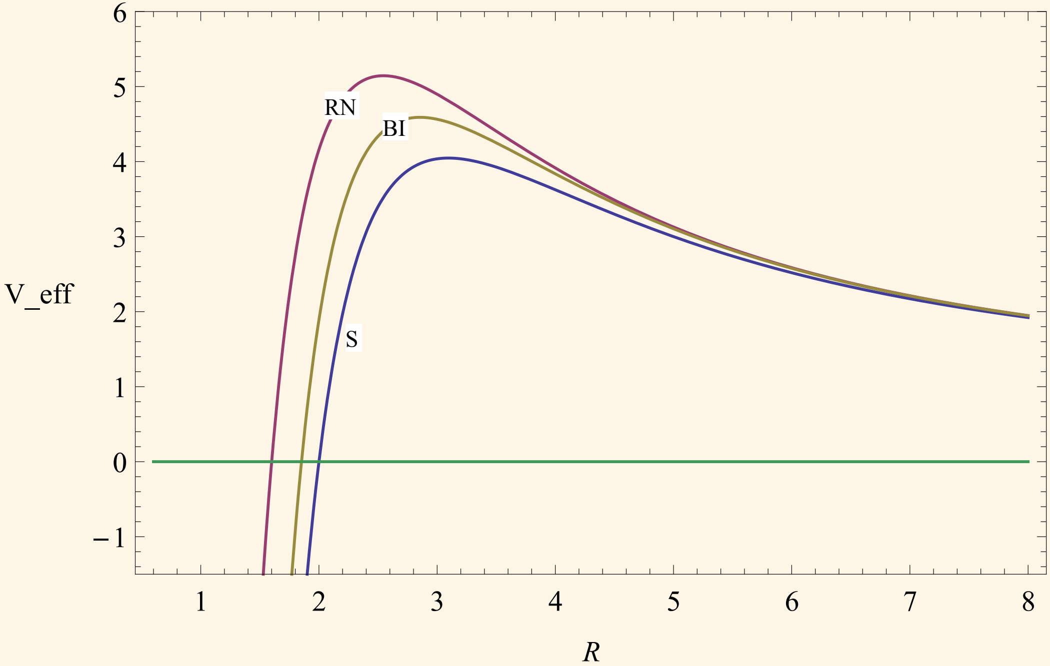
The metric function of BI black hole for fixed b and different values of Q



$$\psi_{\rm BI} = 1 + \tfrac{2}{3}b^2r^2\left(1-\sqrt{1+\frac{Q^2}{b^2r^4}}\right) + \frac{2}{r}\left\{-M + \tfrac{2}{3}Q^2\int_r^\infty \frac{dz}{\sqrt{z^4+Q^2/b^2}}\right\}$$

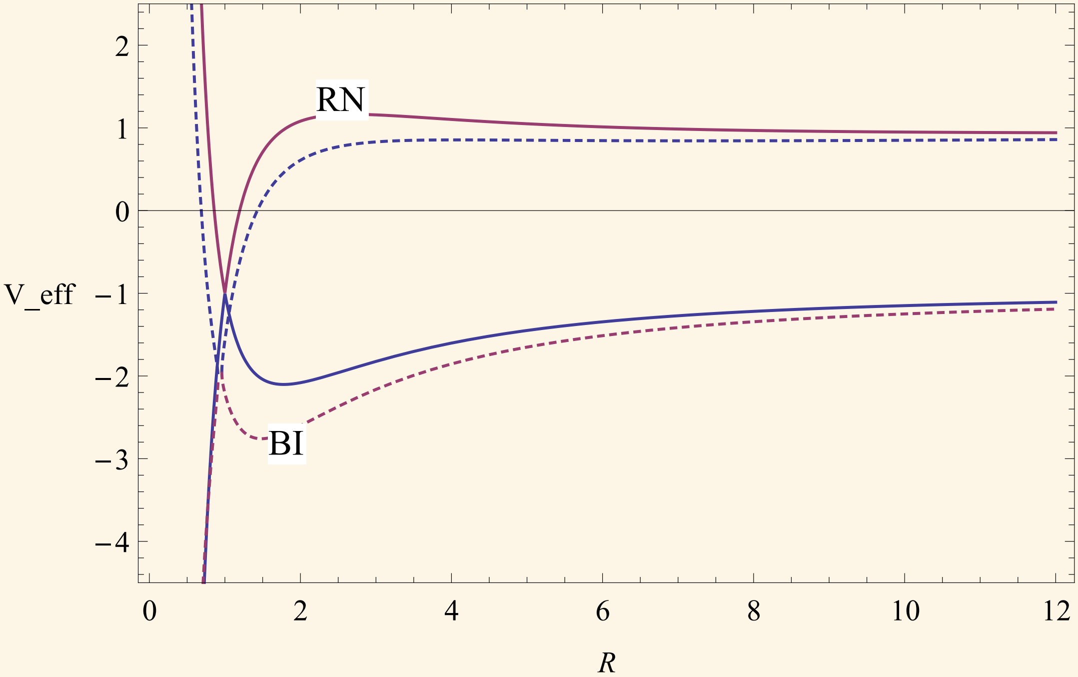
The horizons may be one, two or none, depending on the values of the parameters; for low charges the behavior is Schwarzschild-like; while for large values of Q the RN behavior dominates.

The effective potentials of Schwarzschild, RN and BI black holes



The e.m. nonlinear effect is of shielding the charge, as if the b.h. had less charge; RN is a more compact object, the maximum of the V_{eff} of BI b.h. is lower than the one of RN. $R = r/M$

The effective potential shows negative regions



Effective potential can be identified from $\dot{r}^2 + V_{\rm eff} = E^2$. The regions with negative energy may be thought as an effective ergosphere from which energy can be extracted. The dotted curves are the BI ones and they are more negative than the ones for the RN b.h.

$$E_{\pm} = qA_t \pm \mu \left[\psi \left(1 + \frac{p_\varphi^2}{\mu^2 r^2} \right) \right]^{1/2} ,$$

The most larger negative E_+ occurs for $p_\varphi = 0$,

$$E_{\pm} = qA_t \pm \mu \left(\psi \right)^{1/2} ,$$

The largest root $r_{\rm es}$ of $E_+ = 0$ defines the outer limit of the ergosphere, being the inner radious the horizon,

So, for RN,

$A_t = \frac{Q}{r}, \quad \psi_{\rm RN} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2},$

the ergosphere extends from the horizon up to $r_{\rm es}$

$r_+ = M + \sqrt{M^2 - Q^2} \leq r \leq M + \sqrt{M^2 - Q^2 \left(1 - \frac{q^2}{\mu^2} \right)} = r_{\rm es}$

for BI the equation is more complicated, as $A_t(r) = Q \int_r^\infty \frac{dx}{\sqrt{x^4 + Q^2/b^2}}$

and

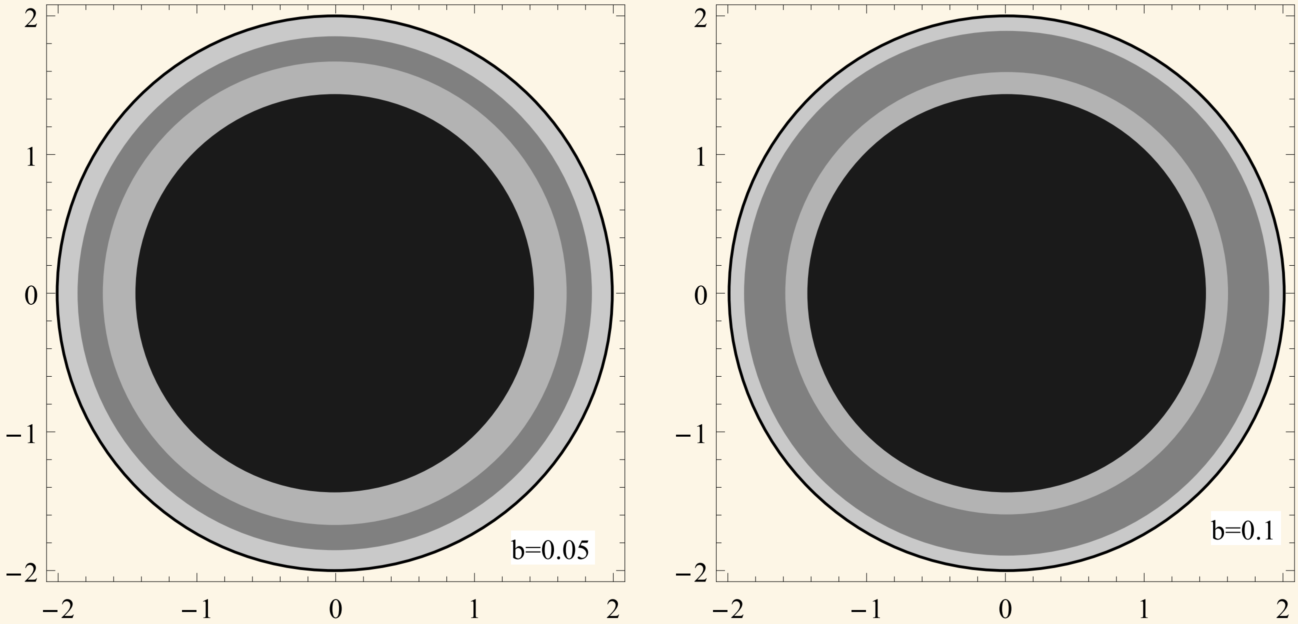
$\psi_{\rm BI} = 1 - \frac{2M}{r} + \frac{2}{3} r^2 b^2 \left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}} \right) + \frac{4Q^2}{3r} \int_r^\infty \frac{dz}{\sqrt{z^4 + Q^2/b^2}}$

then $r_{\rm es}$ is given by the largest root of

$r - 2M + \frac{2}{3} b^2 r (r^2 - \sqrt{r^4 + Q^2/b^2}) + Q^2 I(r) \left[\frac{4}{3} - r \frac{q^2}{\mu^2} I(r) \right] = 0$

$I(r) = \int_r^\infty \frac{dx}{\sqrt{x^4 + Q^2/b^2}} = \frac{1}{2} \sqrt{\frac{b}{Q}} \mathbf{F} \left[\arccos \left(\frac{r^2 - Q/b}{r^2 + Q/b} \right), \frac{1}{\sqrt{2}} \right].$

BI and RN ergospheres



Dark grey ring (spherical shell) is the BI ergoregion that is smaller than the one of RN (light grey); as b grows, it approaches the RN case. The black disk represents the RN horizon. In the plot $M = 1, Q = 1, b = 0{,}05$ (left); $b = 0{,}1$ (right), $\mu = 1, q = -1$.

How much energy can be extracted?

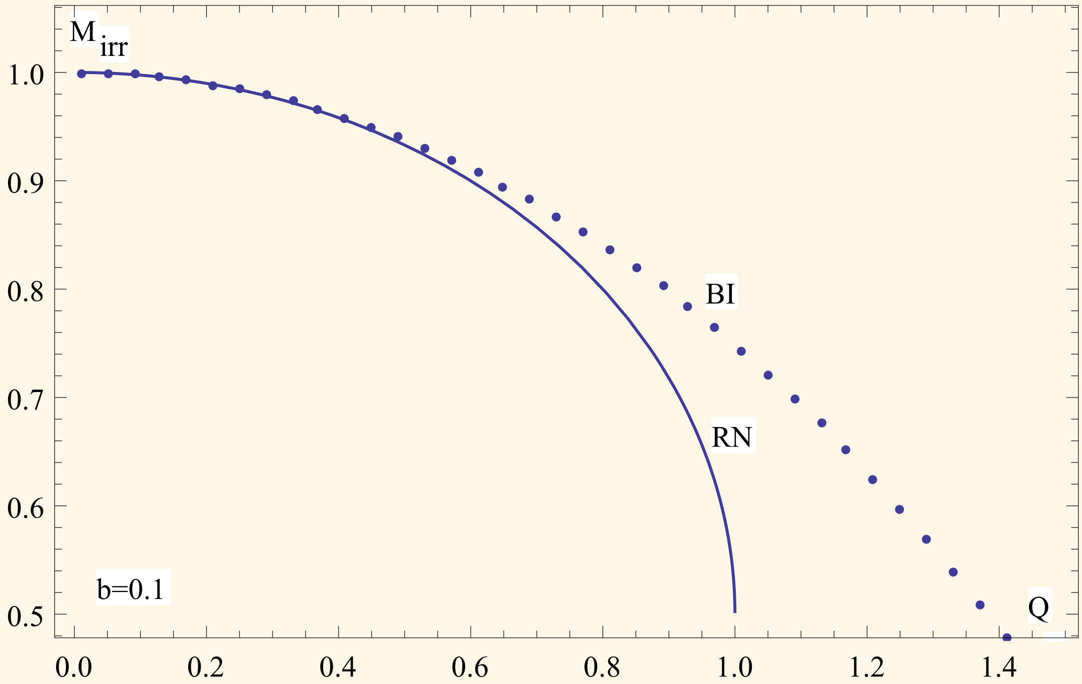
The irreducible mass, $M_{\rm ir}$

By injecting a test particle with E, q, L_z, changes occur to the black hole:

$$\delta M = E; \quad \delta Q = q; \quad \delta J = L_z$$

δM has a lower limit, the minimum E for given q, L_z,
From the eq. for E, $\alpha E^2 - 2\beta E + \gamma = 0$, the orbit of minimum E crosses the horizon, and corresponds to
$E = \frac{\beta}{\alpha}$ evaluated at $r_+ \Rightarrow M_{\rm ir} = \frac{r_+}{2}$
$M - M_{\rm ir}$ is the maximum energy that can be extracted from the black hole.
There exists no IRREVERSIBLE PROCESS which will decrease the irreducible mass.
(In the process grav. radiation should be negligible.)

More energy can be extracted from RN than from BI



$M_{\rm ir} = \frac{r_+}{2}$

Since $r_{+RN} < r_{+BI}$ then $M_{\rm ir,RN} < M_{\rm ir,BI}$, consequently more energy can be extracted from RN b.h.

$M_{\rm ir,RN} = \frac{1}{2}(M + \sqrt{M^2 - Q^2})$

while $M_{\rm ir,BI} = \frac{1}{2} r_{+,BI}$ with $r_{+,BI}$ is the largest root of

$r - 2M + \frac{2}{3} b^2 r (r^2 - \sqrt{r^4 + Q^2/b^2}) + \frac{4}{3} Q^2 I(r) = 0$

CONCLUSIONS

- The BI ergoregion is smaller than the RN one; as b increases the BI ergoregion approaches the RN one
- The BI irreducible mass is greater than the RN one: less energy can be extracted from BI BH than from the RN one.
- Irreducible mass of the RN BH has a lower limit that corresponds to the extreme BH ($M = Q$), $M_{\rm ir} = M/2$
- NLED (at least BI) decreases the extractable energy from black holes respect to Einstein-Maxwell