Modeling Dynamical Scalarization of Neutron-Star Binaries

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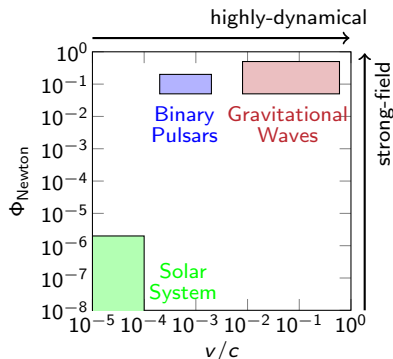
Testing general relativity with gravitational waves

A century of experiments indicate gravity closely resembles GR [Will 1993] [Turyshev 2008], [Will 2014],...

Gravitational waves provide the first window into the *highly-dynamical, strong-field* regime of gravity

Extracting a signal from the noise in a GW detector requires *accurate waveform models*.

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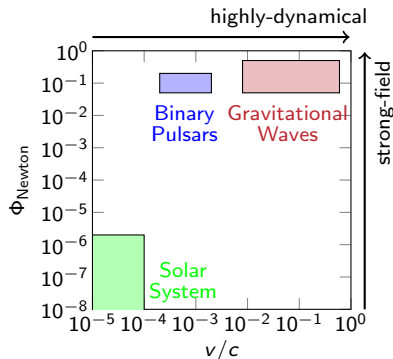
Given the tight constraints on GR, what deviations could emerge in this new regime?

Extracting a signal from the noise in a GW detector requires *accurate waveform models*.

How can we accurately model the GW signal from such deviations?

Scalar-tensor theories of gravity

Scalar-tensor theories are amongst the most natural and well-studied alternatives to GR. We consider theories with one massless scalar [Damour, Esposito-Farèse 1992]

$$S = \int d^4x \frac{c^3\sqrt{-g}}{16\pi G}\left[\phi R - \frac{\omega(\phi)}{\phi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right] + c^2\sum_A \int d\tau_A m_A(\phi)$$

Violation of the SEP$\Rightarrow$*variable mass* & *dipole radiation*.

Certain couplings $\omega(\phi)$ allow for novel behavior in the strong-field regime while satisfying weak-field constraints [Damour, Esposito-Farèse 1993]. We consider theories whose scalar-to-matter coupling is characterized by
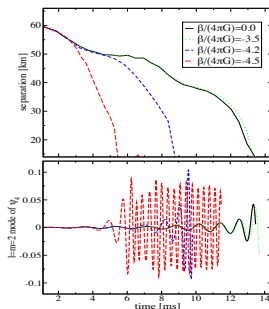
$$a(\phi) = \sqrt{\frac{1}{3+2\omega(\phi)}} = \sqrt{\frac{B\log\phi}{2}}$$

or equivalently, $\qquad a(\varphi_{\text{DEF}}) = \frac{B\varphi_{\text{DEF}}}{2} \qquad$ where $\phi = e^{B\varphi_{\text{DEF}}^2/2}$

Dynamical scalarization

Numerical relativity simulations uncovered *dynamical scalarization* in neutron-star binaries [Barausse+ 2013], [Shibata+ 2014].
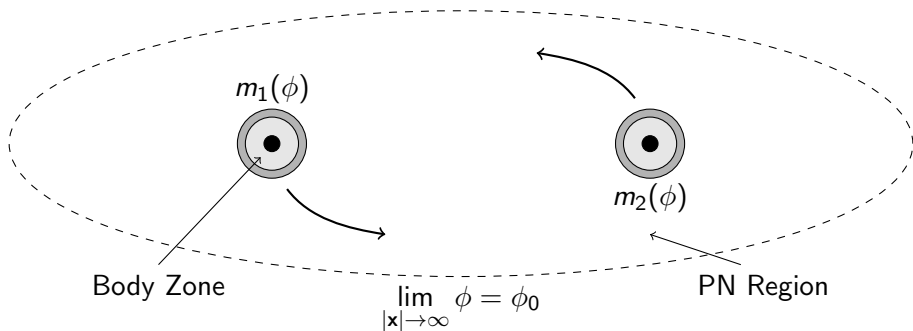
- Binary system evolves as in GR until late inspiral or merger
- Scalar field rapidly grows by orders of magnitude
- Inspiral shortened by up to 30-60 GW cycles (out of last 250) [Taniguchi+ 2015]

Taken from [Barausse+ 2013]

Goal: Improve on previous models [Sampson+ 2014], [Palenzuela+ 2014] with a *self-consistent* framework that incorporates dynamical scalarization from *first principles*.

Post-Newtonian approach

$m_1(\phi)$

$m_2(\phi)$

Body Zone

$$\lim_{|\mathbf{x}| \to \infty} \phi = \phi_0$$

PN Region
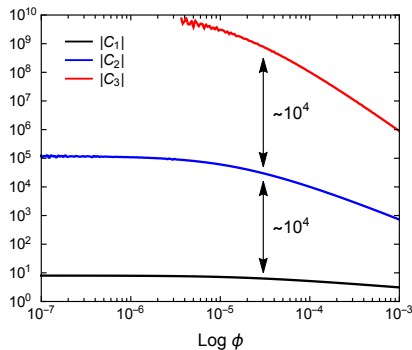
Post-Newtonian prescription

Expand ϕ about ϕ_0 and $g_{\mu\nu}$ about $\eta_{\mu\nu}$ in powers of c^{-2}.

Dynamical scalarization falls outside of the PN framework

The mass of each body is expanded about the background ϕ_0

$$m_A(\phi) = m_A(\phi_0) \left(1 + C_1(\phi - \phi_0) + C_2(\phi - \phi_0)^2 + \cdots\right)$$

The coefficients in this expansion increase dramatically at each order.

[**NS**, Buonanno 2016]

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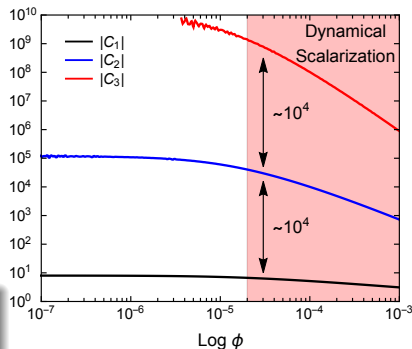$$m_A(\phi) = m_A(\phi_0) \left(1 + C_1(\phi - \phi_0) + C_2(\phi - \phi_0)^2 + \cdots\right)$$

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Dynamical scalarization occurs when

$$C_n(\phi - \phi_0)^n \sim C_{n+1}(\phi - \phi_0)^{n+1}$$

where this expansion should break down.

Instead, one should *resum* the expansion of $m_A(\phi)$ and its derivatives.

[**NS**, Buonanno 2016]

Resumming the post-Newtonian expansion

The PN region is determined by

$$S = \int d^4x \frac{c^3 \sqrt{-g}}{16\pi G} \left[\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right]$$
$$+ c^2 \sum_A \int d^4x \int d\tau_A \, m_A(\phi) \, \delta^{(4)}\Big(x - \gamma_A(\tau_A)\Big)$$
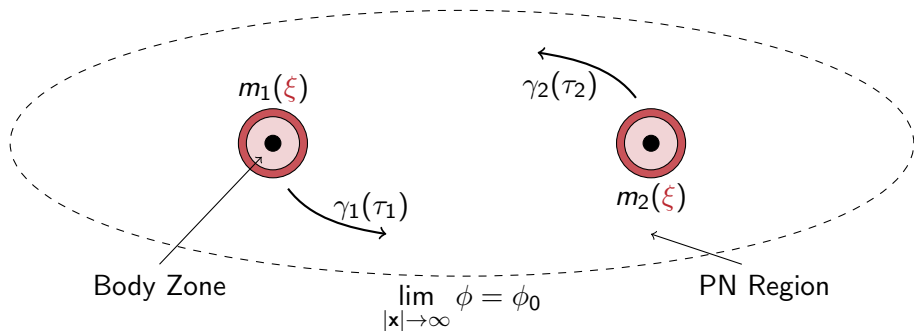
Resumming the post-Newtonian expansion

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$$+ c^2 \sum_A \int d^4x \int d\tau_A \left[m_A(\xi) + \lambda_A(\tau_A)(\phi - \xi) \right] \delta^{(4)}\left(x - \gamma_A(\tau_A) \right)$$

Resumming the post-Newtonian expansion

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We solve the field equations by expanding ϕ and $g_{\mu\nu}$ about the background *but leave ξ unexpanded* (and thus, also $m(\xi)$ and its derivatives).

Earlier PN calculations can be used with slightly modified source terms.

The Lagrange multipliers λ_A yield a system of algebraic equations for ξ that must be *solved exactly* (numerically).

[**NS**, Buonanno 2016]

Equations of motion

Modifying the source terms and repeating the calculation of [Mirshekari, Will 2013], we compute the equations of motion through next-to-leading order *(full expressions up to $\mathcal{O}\left(c^{-4}\right)$ given in [**NS**, Buonanno 2016])*

$$a_1^i = -\frac{Gm_2(\xi_2)\left(1 + \alpha_1(\xi_1)\alpha_2(\xi_2)\right)}{\phi_0 r^2} n^i + [\text{NLO}] + \mathcal{O}\left(c^{-4}\right)$$

$$a_2^i = (1 \rightleftharpoons 2)$$

$$\xi_1 = \phi_0 + \frac{2G\mu_0 m_2(\xi_2)\alpha_2(\xi_2)}{\phi_0 r c^2} + [\text{NLO}] + \mathcal{O}\left(c^{-4}\right)$$

$$\xi_2 = (1 \rightleftharpoons 2)$$

where α_A is the scalar charge, related to the derivative of m_A
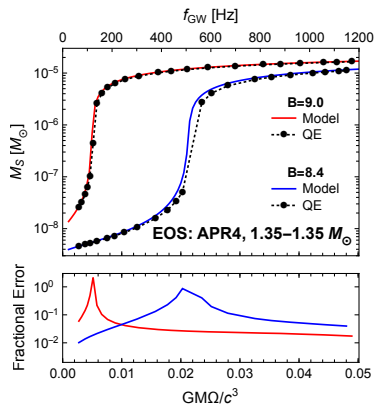
$$\alpha_A \sim \frac{d\log m_A}{d\log \xi}$$

Testing the model against numerical relativity

We compute the scalar mass M_S, a gauge invariant measure of scalarization,
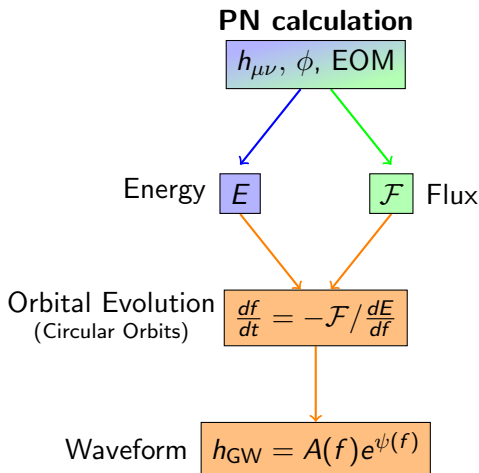
$$M_S \equiv -\frac{c^2}{8\pi G} \oint_{|\mathbf{x}|\to\infty} \delta^{ij}\partial_i\phi \, dS_j$$

and compare against quasi-equilibrium (QE) configuration calculations from [Taniguchi+ 2015].

[**NS**, Buonanno 2016]

Progress towards a full waveform model

PN calculation

$h_{\mu\nu}, \phi,$ EOM

Energy E $\mathcal{F}$ Flux

Orbital Evolution (Circular Orbits) $\frac{df}{dt} = -\mathcal{F}/\frac{dE}{df}$

Waveform $h_{\rm GW} = A(f)e^{\psi(f)}$

Resummed PN calculation

$h_{\mu\nu}, \phi,$ EOM

M_S $M_{\rm ADM}$ $\mathcal{F}$

$\frac{df}{dt} = -\mathcal{F}/\frac{dE}{df}$

$h_{\rm GW} = A(f)e^{\psi(f)}$

[Damour, Esposito-Farèse 1992], [Damour, Esposito-Farèse 1996], [Mirshekari, Will 2013]
[Damour, Esposito-Farèse 1992], [Lang 2014], [Lang 2015]
[Will 1994], [**NS**, Marsat, Buonanno 2016]
[**NS**, Buonanno 2016]

Conclusions

- Gravitational wave detectors allow us to probe the *highly-dynamical, strong-field regime* of gravity.
- Detecting deviations from GR requires *accurate waveform models* in alternative theories of gravity.
- Dynamical scalarization is a promising feature for which to search, but occurs as the *PN approximation breaks down*.
- We construct a perturbative model that incorporates dynamical scalarization *from first principles* by straightforwardly *resumming the PN expansion*.

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- Detecting deviations from GR requires *accurate waveform models* in alternative theories of gravity.
- Dynamical scalarization is a promising feature for which to search, but occurs as the *PN approximation breaks down*.
- We construct a perturbative model that incorporates dynamical scalarization *from first principles* by straightforwardly *resumming the PN expansion*.
- Our model reproduces numerical relativity prediction of *location and magnitude of scalarization to* $\lesssim 10\%$; the ultimate goal is to produce inspiral waveforms within this framework.

Backup Slides

Transformation between Jordan and Einstein frames

The class of scalar-tensor theories we consider can be rewritten in either the Jordan frame or Einstein frame:

Jordan Frame

$$S = \int d^4x \frac{c^3\sqrt{-g}}{16\pi G} \left(\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right) + S_m[g_{\mu\nu}, \Xi]$$

Einstein Frame

$$S = \int d^4x \frac{c^3\sqrt{-\tilde{g}}}{16\pi G} \left(\tilde{R} - 2\tilde{g}^{\mu\nu} \nabla_\mu \tilde{\varphi} \nabla_\nu \tilde{\varphi} \right) + S_m \left[e^{-\int 2d\tilde{\varphi}/\sqrt{3+2\omega(\tilde{\varphi})}} \tilde{g}_{\mu\nu}, \Xi \right]$$

with the transformation

$$\tilde{g}_{\mu\nu} \equiv \phi g_{\mu\nu}, \qquad\qquad \tilde{\varphi} \equiv \int d\phi \frac{\sqrt{3+2\omega(\phi)}}{2\phi}$$

Coupling in the Einstein frame

$$S = \int d^4 x \frac{c^3 \sqrt{-\tilde{g}}}{16 \pi G} \left[\tilde{R} - 2 \tilde{g}^{\mu\nu} \nabla_\mu \tilde{\varphi} \nabla_\nu \tilde{\varphi} \right] + S_m \left[e^{-\int 2 d\tilde{\varphi} / \sqrt{3 + 2\omega(\tilde{\varphi})}} \tilde{g}_{\mu\nu}, \Xi \right]$$

The scalar field is coupled to matter (Ξ) only through the metric, so as to avoid introducing a "fifth force." The coupling is characterized by

$$a = (3 + 2\omega)^{-1/2}$$

The most commonly considered couplings include

$a(\tilde{\varphi})$	$\phi(\tilde{\varphi})$	$\omega(\tilde{\varphi})$	Parameters	GR Limit
$\dfrac{1}{\sqrt{3 + 2\omega_{\mathrm{BD}}}}$	$\exp\left(\dfrac{2\tilde{\varphi}}{\sqrt{3 + 2\omega_{\mathrm{BD}}}}\right)$	ω_{BD}	ω_{BD}	$\omega_{\mathrm{BD}} \to \infty$
$\dfrac{B\tilde{\varphi}}{2}$	$\exp\left(\dfrac{B\tilde{\varphi}^2}{2}\right)$	$\dfrac{2}{B^2 \tilde{\varphi}^2} - \dfrac{3}{2}$	$\tilde{\varphi}_0,\ B$	$\tilde{\varphi}_0 \to 0$

Spontaneous scalarization

$$S = \int d^4x \frac{c^3\sqrt{-\tilde{g}}}{16\pi G}\left[\tilde{R} - 2\tilde{g}^{\mu\nu}\nabla_\mu\tilde{\varphi}\nabla_\nu\tilde{\varphi}\right] + S_m\left[e^{-\int 2d\tilde{\varphi}/\sqrt{3+2\omega(\tilde{\varphi})}}\tilde{g}_{\mu\nu}, \Xi\right]$$

For $B > 0$, the $\mathbb{Z}_2$ symmetry associated with $\tilde{\varphi} \to -\tilde{\varphi}$ can be *spontaneously broken* in the relativistic regime.

The corresponding phase transition allows the theory to behave very similarly to GR in the weak-field limit (satisfying current experimental constraints), but still generate detectable non-GR phenomena in regions of strong gravity.
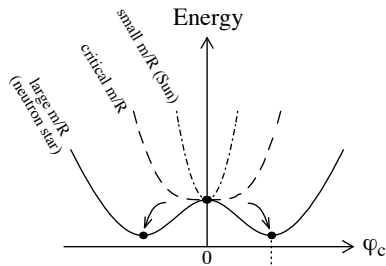
These phenomena include *spontaneous scalarization* and *dynamical scalarization*
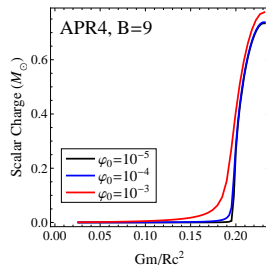
Spontaneous scalarization

The dominant contribution to the energy of an isolated body is given by

$$\text{Energy} \approx \int d^3x \left[\frac{1}{2}(\partial_i \varphi)^2 + \rho e^{-B\varphi^2/4} \right] \approx mc^2 \left(\frac{\varphi_c^2/2}{Gm/Rc^2} + e^{-B\varphi_c^2/4} \right)$$

Sufficiently compact neutron stars can "scalarize," [Damour & Esposito-Farèse 1993] developing a non-trivial *scalar charge* (akin to ferromagnetism below the Curie temperature).

[Esposito-Farèse 2004]

$$\varphi = \varphi_0 + \frac{GM_\varphi}{|\mathbf{x}|c^2} + \mathcal{O}\left(|\mathbf{x}|^{-2}\right)$$
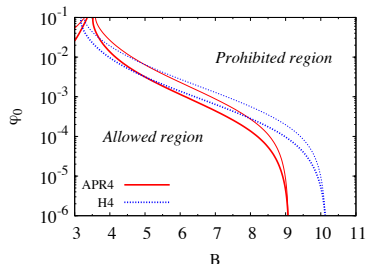
Binary pulsar constraints (on spontaneous scalarization)

Scalarization drastically affects the evolution of binary systems. In particular, scalarized systems emit *significant dipole radiation*, which shortens their inspiral.

To date, timing measurements of binary pulsars are consistent with GR (no dipole radiation). The absence of observed scalarized stars places a constraint on B. Currently, this constraint is

$$B \lesssim 9 - 10$$

depending on the NS equation of state.

Taken from [Shibata+ 2014]

No scalarized neutron stars have been observed, but the parameter space of this theory is not entirely ruled out.

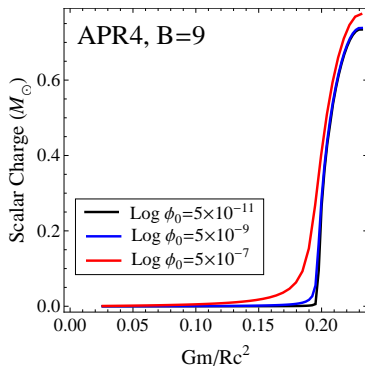# Spontaneous scalarization is a non-perturbative phenomena

We describe a phenomenom as *non-perturbative* if it cannot be found at any finite PN order. Spontaneous scalarization is an illustrative example.

The scalar charge of an isolated body placed in a background ϕ_0 can be expanded in powers $s = \frac{Gm}{Rc^2}$

$$M_\varphi = \sqrt{B \log \phi_0} \left(a_0 + a_1 s + a_2 s^2 + \cdots \right)$$

Truncated at any order in s, $M_\varphi \to 0$ as $\phi_0 \to 1$ (the GR limit).

However, beyond some critical compactness, there exists a solution with non-trivial M_φ not captured by the PN expansion.

APR4, B=9

Legend:
- Log $\phi_0 = 5 \times 10^{-11}$
- Log $\phi_0 = 5 \times 10^{-9}$
- Log $\phi_0 = 5 \times 10^{-7}$

Scalar Charge ($M_\odot$) vs Gm/Rc^2

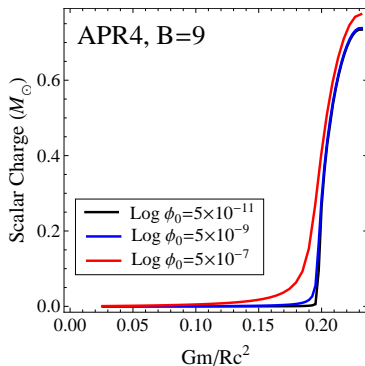# Spontaneous scalarization is a non-perturbative phenomena

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In order to accomodate spontaneous scalarization, *do not expand M_φ*. Instead, solve for it numerically.

APR4, B=9

Scalar Charge ($M_\odot$) vs Gm/Rc²

Log ϕ_0=5×10⁻¹¹
Log ϕ_0=5×10⁻⁹
Log ϕ_0=5×10⁻⁷

Resummation schemes: "what" to resum

As described above, a resummation of the PN expansion of $m_A(\phi)$ allows one to model dynamical scalarization. There's more than one way to do this—consider a trivial (but relevant) choice of quantity to resum:

1. Jordan frame mass: $\qquad\qquad m_A(\phi) \Rightarrow m_A(\phi, \xi) = m_A(\xi)$
2. Einstein frame mass: $m_A^{(E)} = m_A(\phi)/\sqrt{\phi} \Rightarrow m_A(\phi, \xi) = m_A(\xi)\sqrt{\phi/\xi}$

Similarly, our resummation was implemented by matching the field ξ defined in the body zone to ϕ defined in the PN region. More generically, we could have instead matched ξ to any function $F(\phi)$. We consider two natural choices:

1. $\xi = \phi \Rightarrow F(\phi) = \phi$
2. $\xi = \varphi_{\text{DEF}} \Rightarrow F(\phi) = \sqrt{2\log\phi/B}$

The choice of F determines how the scalar charge $\alpha(\phi, \xi)$ is resummed. Of the above options, the choice of F has a much greater impact on the model than the choice of m_A

Field equations

We define an action for the two particles

$$S_m = \sum_{A=1,2} \int d^4x \int d\tau_A \delta^{(4)}\Big(x - \gamma_A(\tau_A)\Big) \times \Big(m_A(\phi, \xi) + \lambda_A(\tau_A)\left(F(\phi) - \xi\right)\Big)$$

and derive the field equations from the full action

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{\omega(\phi)}{\phi^2}\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi\right)$$
$$+ \frac{1}{\phi}\left(\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\right) + \frac{8\pi G}{\phi c^4}T_{\mu\nu}$$

$$\Box\phi = \frac{1}{3 + 2\omega(\phi)}\left(\frac{8\pi G}{c^3}T - \frac{16\pi G}{c^3}\phi\frac{DT}{D\phi} - \frac{d\omega}{d\phi}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi\right)$$

$$u_A^\sigma\nabla_\sigma\left(m(\phi, \xi)u_A^\alpha\right) = -\frac{Dm}{D\phi}\partial^\alpha\phi$$

with $\frac{D}{D\phi} \equiv \frac{\partial}{\partial\phi} + \frac{dF}{d\phi}\frac{\partial}{\partial\xi}$