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Renormalizing the Non-Renormalizable: Asymptotic Safety in Quantum Gravity

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Outline

- Introduction: Quantum Gravity and Asymptotic Safety
- Renormalization: Scale Evolution of Vertices
- The Functional Renormalization Group
- Fixed Points in Quantum Gravity
- Outlook

Perturbative Quantization

- relevant energy scale: Planck scale $M_{\text{Pl}} \approx 10^{19} \text{ GeV}$
- expansion parameter for n-point Greens functions:

dimensionless \rightarrow $g \equiv G_N E^2 = \frac{E^2}{M_{\text{Pl}}^2}$ \leftarrow energy scale

- \rightarrow higher loop orders require higher derivative counterterms !
- \rightarrow full theory is either divergent or includes infinitely many free parameters

(perturbatively) non-renormalizable

Asymptotic Safety in a Nutshell

- Non-perturbative renormalization in Quantum Gravity

- (i) d.o.f. carried by the metric field
- (ii) diffeomorphism invariance
- (iii) quantum field theory of point particles

- Quantum Fluctuations  scale dependent couplings

$$g_i \longrightarrow g_i(k)$$

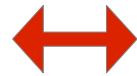
k = energy scale g dimensionless couplings

UV fixed point:

$$\lim_{k \rightarrow \infty} g_i(k) = g_{i,*} < \infty$$

+ finite number of free parameters (predictive)

Asymptotic Safety



UV completion

S. Weinberg (1979)

- example: Asymptotic Freedom : $g_* = 0$ (perturbative)

Functional Renormalization Group

- Functional Renormalization: non-perturbative!

→ scale evolution of the quantum effective action $\Gamma_k[\phi]$

→ generates the non-pert. vertex functions

$$\Gamma_k^{(n)} = \frac{\delta^n \Gamma_k[\phi]}{\delta \phi^n}$$

Flow equation/
Wetterich equation:

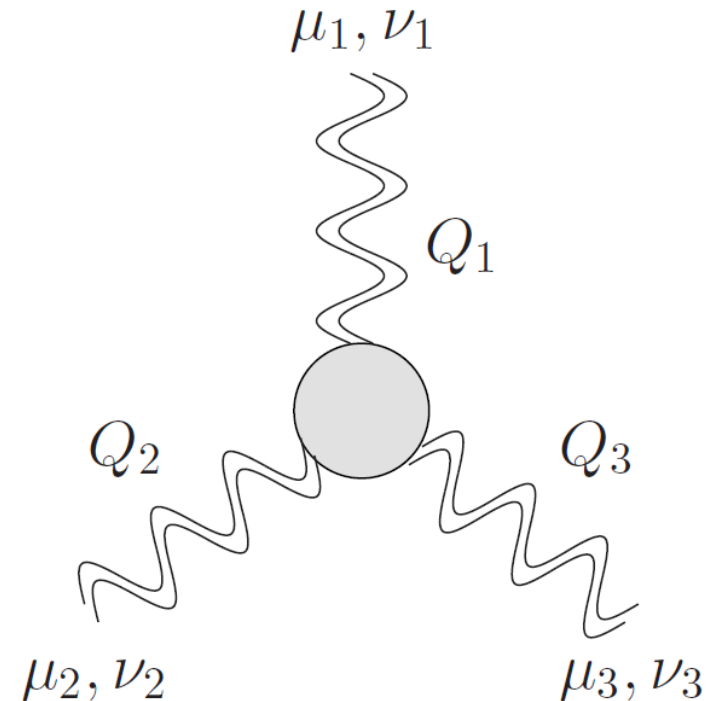
Wetterich (1993), Application in QG: Reuter (1996)

$$k \frac{d}{dk} \Gamma_k[\phi] = \text{STr} \left(\frac{\delta^2}{\delta \phi^2} \Gamma_k + R_k \right)^{-1} [\phi] k \frac{d}{dk} R_k$$

k : RG scale

R_k : regulator

→ describes physics at scale k



more on FRG and asymptotic safety: plenary talk by M. Reuter

The Running Couplings I

- Fixed point condition of *Asymptotic Safety*

→
$$\beta_{g_i} = k \frac{d}{dk} g_i(k) \Big|_{g=g_*} \stackrel{!}{=} 0$$

UV attractive directions



free parameter

UV repulsive directions



parameter fixed from theory

- Beta Functions from Flow Equations:

→ parameterization of vertices $\Gamma^{(n)}$ with running couplings

→ choose tensor structures of $\Gamma^{(n)}$ e.g. from

$$\Gamma^{(n)} \sim \frac{\delta^n}{\delta \phi^n} \int d^4x \sqrt{g} (-R + 2\Lambda) \quad , \quad R = \text{Ricci scalar}$$

→ coupled system of beta functions for the running couplings

The Running Couplings II

- Systematic expansion and running two- and three-point function

NC, Knorr, Meibohm, Pawłowski, Reichert 2015

two point function $\Gamma^{(2)}$
/inverse Propagator

$$\eta(k, p) , \mu(k) = -2\lambda_2(k)$$

graviton anomalous
dimension

„mass gap“

scale dependent, running Newton constant

three point function $\Gamma^{(3)}$

$$g(k) , \lambda_3(k)$$

- With „Einstein-Hilbert-like“ tensor structures (quadratic in derivatives)

Fixed point with two relevant, one irrelevant direction

$$(g^*, \mu^*, \lambda_3^*) = (0.66, -0.59, 0.11)$$

Asymptotic Safety:



Higher Derivative Gravity (preliminary)

- Full basis of four-derivative operators (NC 2016, in preparation)

$$\Gamma^n \sim \frac{\delta^n}{\delta \phi^n} \int d^4x \sqrt{g} \left(-\frac{R}{G_N} + B R^2 + F R_{\mu\nu} R^{\mu\nu} \right)$$

- From two-point function $\Gamma^{(2)}$

➔ Beta-functions of $B(k)$, $F(k)$, $\mu(k)$ and algebraic equation for $\eta(k)$

➔ Newtons constant: free parameter

➔ assume fixed point g_*

➔ Most viable fixed point in B and F:
 $F(k) \approx -2.9$ $B(k) \approx 0.6$

Asymptotic Safety:



➔ Irrelevant couplings! i.e. fixed from theory

Summary and Outlook

- Non-Pert. Renormalization of Quantum Gravity

→ Asymptotic Safety Scenario

- Vertex Expansion and Vertex Flows in Quantum Gravity
- Two- and Three-Point Function: UV-fixed point with one irrelevant direction
- Including all four-derivative operators:
UV fixed point, no further free parameters

Outlook

- Three-point function with higher derivative operators
- Systematics !?
- Unitarity

Thank You!!!

Vertices and Flows

- Usually no exact solution to flow equation

→ Approximation schemes

$$\Gamma[g] = \sum \frac{1}{n!} \frac{\delta^n \Gamma}{\delta g^n} [\bar{g}] (g - \bar{g})^n$$

full propagators!
full vertices!

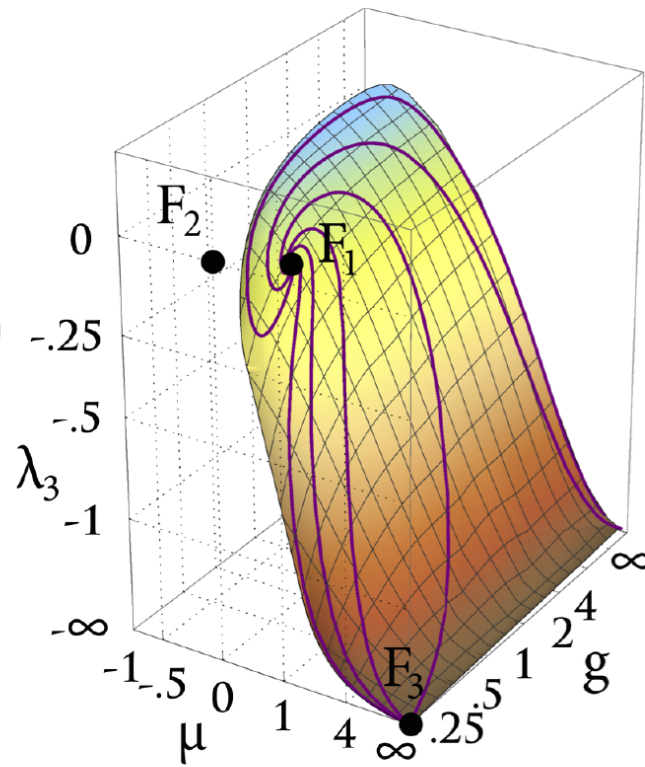
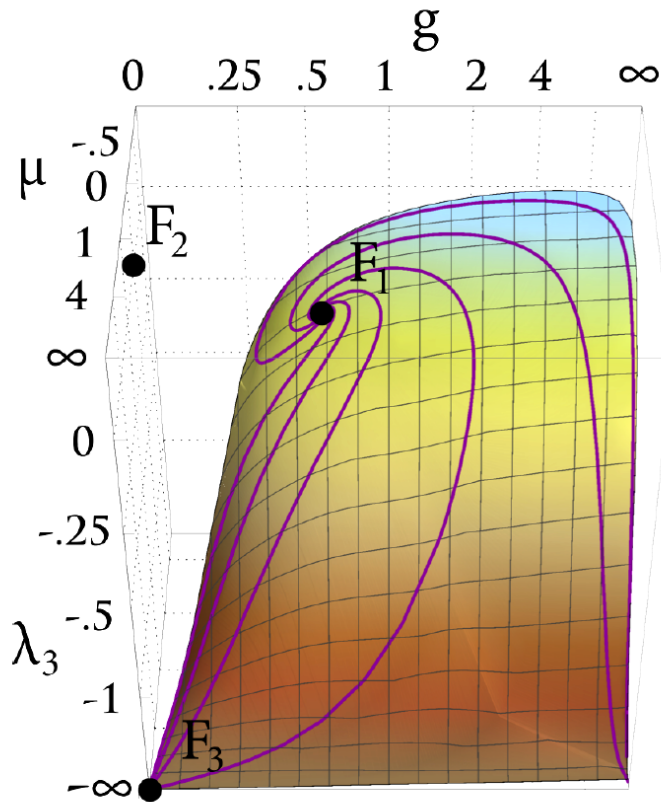
$$k \frac{d}{dk} \Gamma^{(2)} = -\frac{1}{2} \text{ (diagram 1)} + \text{ (diagram 2)} - 2 \text{ (diagram 3)}$$

$$k \frac{d}{dk} \Gamma^{(3)} = -\frac{1}{2} \text{ (diagram 1)} + 3 \text{ (diagram 2)} - 3 \text{ (diagram 3)} + 6 \text{ (diagram 4)}$$

$$k \frac{d}{dk} \Gamma^{(n)} = \text{Flow}[\Gamma^{(2)}, \dots, \Gamma^{(n+2)}] \rightarrow \text{infinite hierarchy of flow equations}$$

The Phase Diagram

- Phase Diagram: Solutions for different initial conditions



F_1 : Non-Gaussian UV-FP

F_2 : Gaussian FP

F_3 : Non-Gaussian IR-FP

→ At the UV-FP: two attractive, one repulsive direction

→ Further evidence for asymptotic safety in quantum gravity