A New Area Law in General Relativity

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Based on:
arXiv: 1504.07627, 1504.07660
with R. Bousso

Surfaces with $\theta = 0$ as Local Black Hole Boundaries

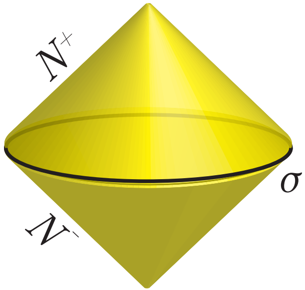
- Event horizon: teleological, nonlocal (whether past or future)
- Hypersurfaces foliated by surfaces with $\theta = 0$ (e.g. dynamical horizons) capture a "local" notion of an event horizon in two ways:
  1. Geometric: can be thought of as a way of characterizing the boundary of a trapped region
  2. Thermodynamic: the area of $\theta = 0$ surfaces gives a bound on the entropy of a null slice of the spacetime (Bousso '99)

The Bousso Bound

Let σ be a codim 2 surface with one vanishing expansion:

$$\theta_k(\sigma) = 0$$
$$\theta_l(\sigma) < 0 \quad \text{or} \quad \theta_l(\sigma) > 0$$

where k^a and l^a are the null normals to σ.



Let $N^\pm$ be the null congruences generated by $\pm k^a$.

Covariant Entropy Bound (Bousso '99):

$$S(N^\pm) \leq \frac{A(\sigma)}{4G\hbar}$$

In this sense, the surface σ is *holographic*.

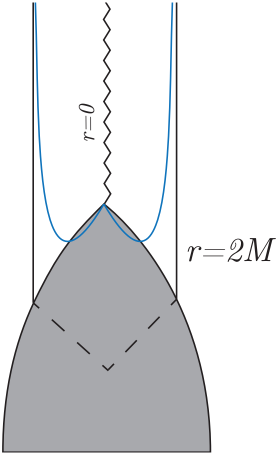
Holographic Screens

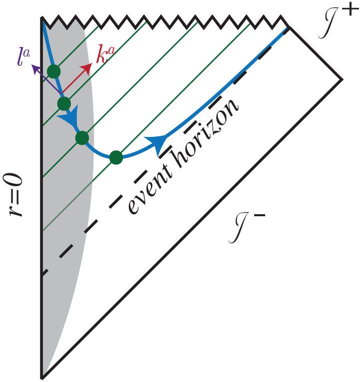
A *holographic screen* is a hypersurface foliated by leaves σ with $\theta(\sigma) = 0$. (Bousso '99)

A *future* holographic screen is foliated by marginally trapped surfaces (also known as MTT): $\theta_k(\sigma) = 0$, $\theta_l(\sigma) < 0$.

A *past* holographic screen is foliated by marginally anti-trapped surfaces: $\theta_k(\sigma) = 0$, $\theta_l(\sigma) > 0$.

Future Holographic Screen in Oppenheimer-Snyder





One Area Law to Rule Them All? ... and in the black hole, bind them

- Area increases "outwards" on a spacelike screens (Ashtekar, Krishnan; Hayward)
- Similar reasoning showed area increases towards the past on a timelike screens (Ashtekar, Krishnan; etc.)
- Is it possible that typical screens with signature changes could always have increasing area?

Do the spacelike and timelike components always conspire to stitch themselves together so that all holographic screens obey One Area Law?

Area Theorem for Holographic Screens

Area Theorem

Let H be a future holographic screen. Assuming the spacetime obeys

1. The Null Curvature Condition:

$$R_{ab}n^a n^b \geq 0$$

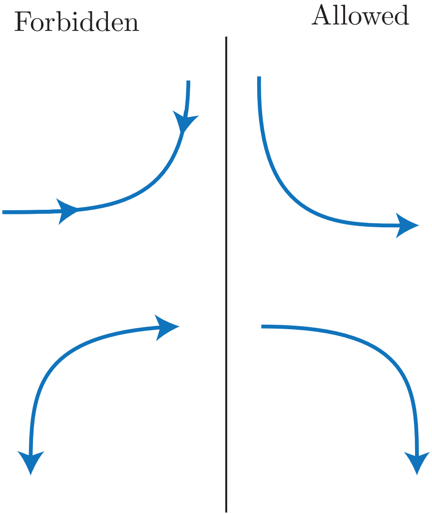
2. A generic condition:

$$\left.\frac{d\theta_k}{d\lambda}\right|_H < 0$$

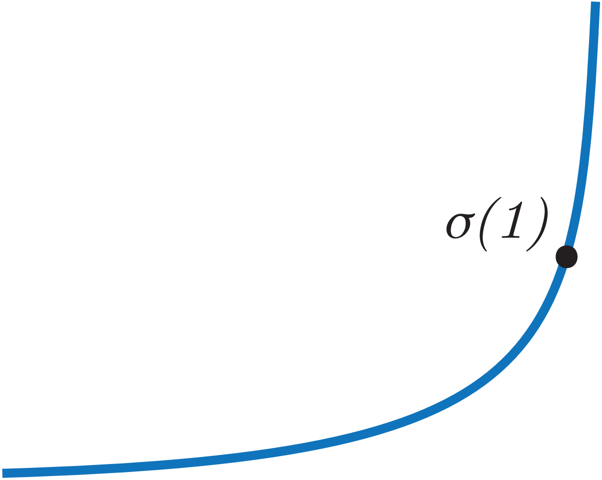
where $\theta_k(\sigma) = 0$ for any leaf σ of H.

and a set of minor technical assumptions, then the area of leaves of H increases monotonically with flow along H.

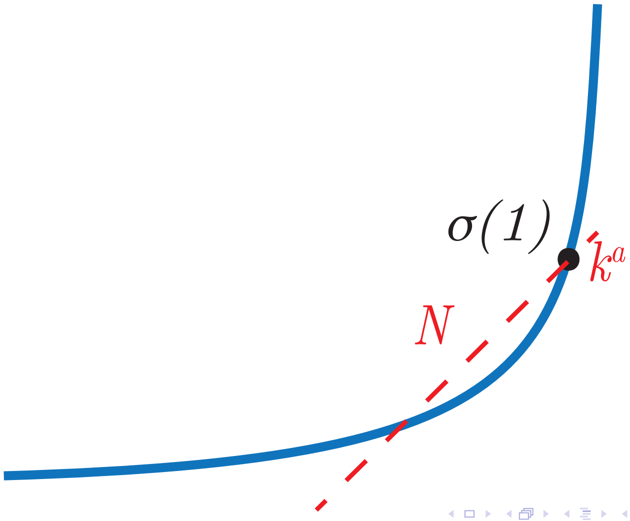
Towards a Proof for Indefinite Signature Screens



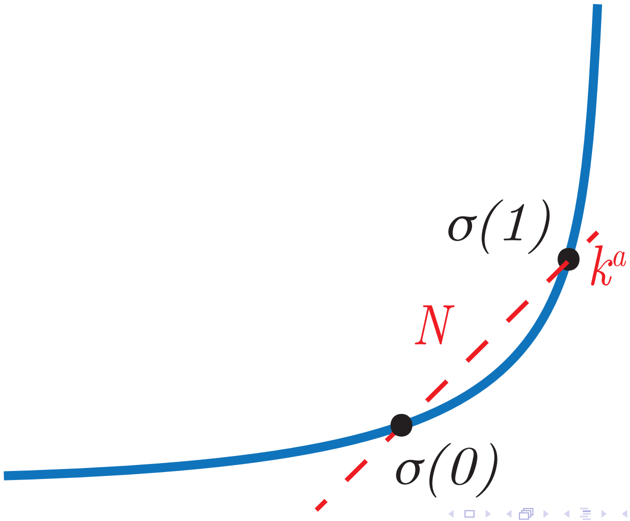
Proof (spherical symmetry) Proof w/o spherical symmetry in 1504.07627



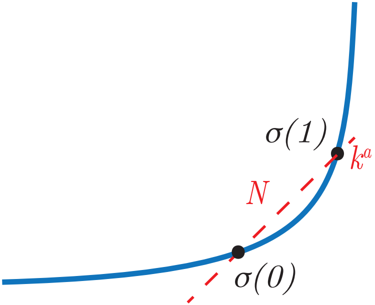
Proof (with spherical symmetry)



Proof (with spherical symmetry)



Proof (with spherical symmetry)



$$\theta(N)|_{\sigma(1)} = 0 \quad \& \quad \theta(N)|_{\sigma(0)} = 0$$

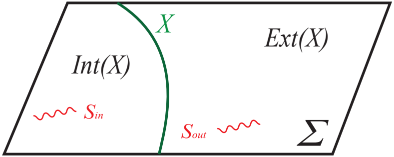
$$\text{NCC} \;\Rightarrow\; \theta(N) = 0$$

$$\text{Generic Condition} \;\Rightarrow \theta(N) < 0$$

$$\rightarrow\leftarrow$$

Quantum Generalization: Generalized Entropy

- Perturbative quantum corrections can violate NCC: Hawking area theorem violated, as is the area theorem for holographic screens
- Bekenstein: $A \to 4G\hbar S_{\mathrm{gen}} = A + 4G\hbar S_{\mathrm{out}}$



- GSL: S_{gen} increases
- Can formulate GSL analogue for holographic screens in the same way (requires changing definition of a marginally trapped surface to quantum marginally trapped surface); see 1510.02099.

Future work

- Better understanding of the thermodynamic connection, both in classical an quantum contexts
- Non-uniqueness of holographic screens
- Holographic screens in holography (AdS/CFT)?