Long range correlations in loop quantum gravity

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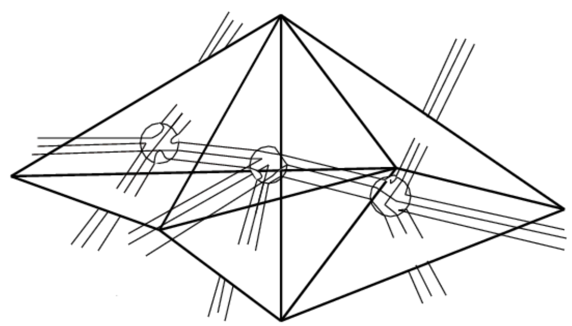
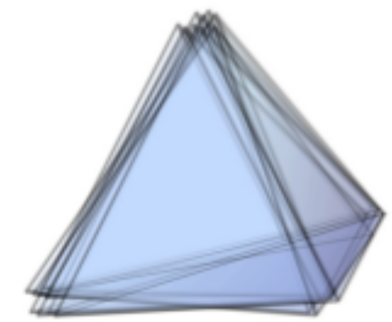
Introduction

In the spatial geometry of LQG as described by spin network states:





Geometrical observables $\hat{\mathcal{O}}$ are discrete

Geometry has quantum fluctuations

Semiclassical states should satisfy:

Averages form classical geometry	$\langle\alpha\|\hat{\mathcal{O}}_i\|\alpha\rangle = \mathcal{O}_i$	$\xrightarrow{\textit{large scales}}$	g_{ab}
Fluctuations are minimized	$\langle\alpha\|\hat{\mathcal{O}}_i\hat{\mathcal{O}}_i\|\alpha\rangle \sim \hbar$		
Correlations	$\langle\alpha\|\hat{\mathcal{O}}_i\hat{\mathcal{O}}_j\|\alpha\rangle = ?$		

Introduction

How to recognize semiclassical states in quantum gravity?

- Fluctuations of geometry as vacuum of QFT in classical background
- Long range correlations: $\langle \hat{h}(x)\hat{h}(y)\rangle \sim 1/d(x,y)^2$

How to construct semiclassical states?

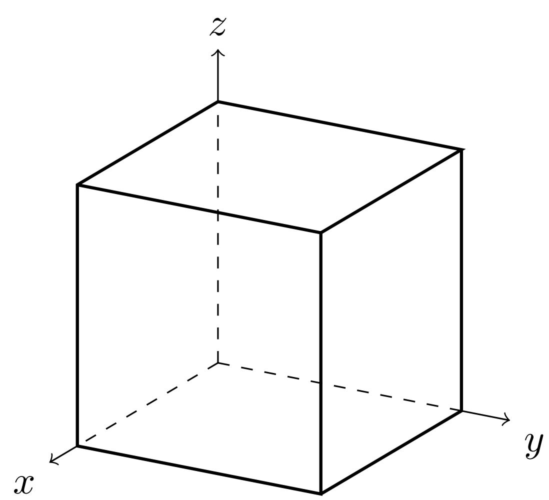
- Coherent states and squeezed vacua in loop quantum gravity **[1]**

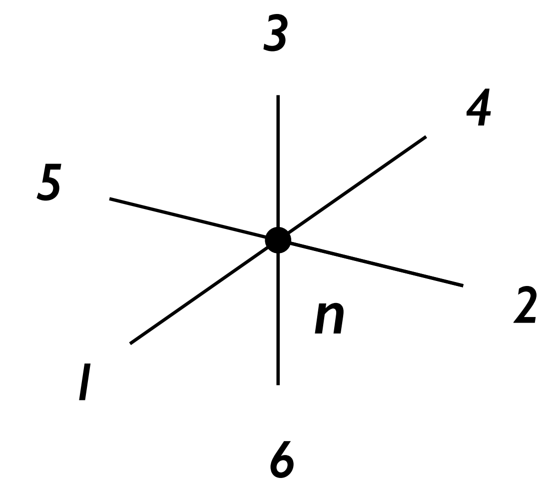
In this talk, focus on area-area correlations for homogeneous states in graphs of cubic structure.

[1] E. Bianchi, J. Guglielmon, L. Hackl, NY, arXiv:1605.05356 and PRD92 (2015) 085045

Cubic lattice and spinors

Geometry of a cubic lattice described with spinor variables



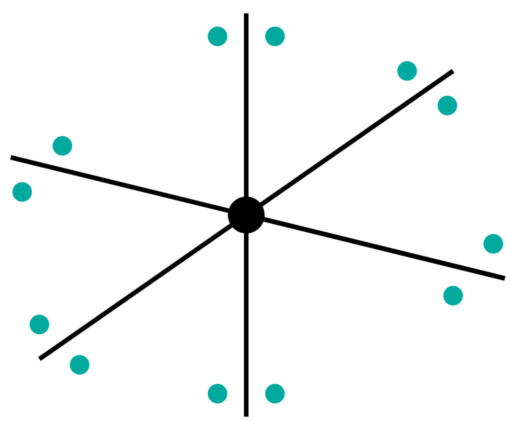


3d vectors from spinors: $\vec{v}(z) = \frac{1}{2}\vec{\sigma}_{AB}\,\bar{z}^A z^B\,, \quad z \in \mathbb{C}^2$

For cubic lattice: $z_{n\mu} = z_\mu\,, \quad \vec{v}(z_\mu)$ are the $\pm\,\hat{x}^i$ directions

- *Spinors have extra information:* $\vec{v}(z_\mu)$ *determines z up to phase* $e^{i\xi_\mu}$ *(framed cube). The phases will affect holonomies, but not spin fluctuations in the quantum theory.*

Bosonic representation of LQG



Bosonic variables: $a^A_{n\mu}, a^{A\dagger}_{n\mu}$, $A = 0, 1$ [2]

Hilbert space: $\mathcal{H}_\Gamma = P_G\, P_A \bigotimes_{i=1}^{4L} \mathcal{H}^{(i)}_{osc}$

Gauss constraint *Area matching constraint*

$\boxed{\text{Spin-spin correlations:} \quad C_{\ell\ell'} = \langle I_\ell I_{\ell'}\rangle - \langle I_\ell\rangle\langle I_{\ell'}\rangle\,, \quad I_\ell = \frac{1}{2}\delta_{AB}\, a^{A\dagger}_{s(\ell)} a^B_{s(\ell)}}$

1. Coherent states $|z\rangle = P_G\, P_A \exp[\lambda\, z^{n\mu}_A a^{A\dagger}_{n\mu}]|0\rangle$

2. Squeezed vacua $|\gamma\rangle = P_G\, P_A \exp\left\{\lambda\,[\gamma(z)]^{(m\mu)(n\nu)}_{AB}\, a^{A\dagger}_{m\mu} a^{B\dagger}_{n\nu}\right\}|0\rangle$ [3]

[2] F. Girelli and E. Livine ’05; L. Freidel and E. Livine ’10 and ’11; Borja et al ’11
[3] E. Bianchi, J. Guglielmon, L. Hackl, NY, arXiv:1605.05356

Correlations for coherent states

Spin-spin correlations in the limit of large spins, $j_\ell \gg 1$

At each link, after area matching:

$$|z_\ell\rangle = \sum_j \frac{\lambda^{4j}}{(2j)!} |j, z_{m\mu}\rangle |j, z_{n\nu}\rangle$$

$$P(j) \simeq \frac{1}{\sqrt{\pi}\lambda} \exp\left[-\frac{4}{\lambda^2}\left(j - \frac{\lambda^2}{2}\right)^2\right]$$

At each node, after gauge-averaging:

$$|LS, j_\mu\rangle = P_G \bigotimes_{\mu=1}^{6} |j_\mu, z_\mu\rangle$$

$$P(j_\mu) \simeq \frac{1}{\sqrt{\pi}(2\lambda^2)^{3/2}} \prod_{a=1}^{3} e^{-(j_{a+3}-j_a)^2/2\lambda^2}$$ [4]

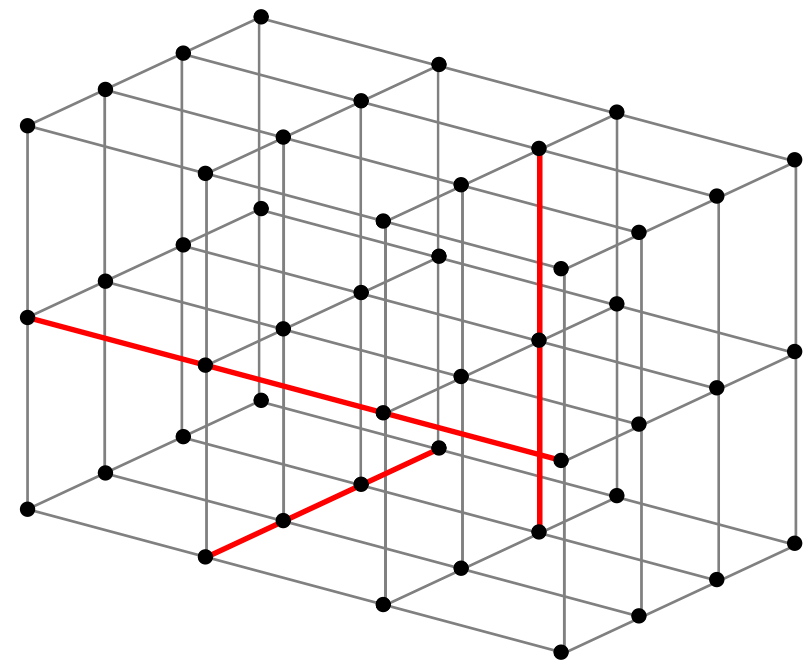
Spin fluctuations follow normal distribution peaked at $j_0 = \lambda^2/2$ with $\sigma_j = \sqrt{j_0}/2$.
The Gauss projection introduces correlations of fluctuations at distinct links.

$$P_\lambda(\{j_\ell\}) \propto \prod_{\text{links}} \exp\left[-\frac{4}{\lambda^2}\left(j_\ell - \frac{\lambda^2}{2}\right)^2\right] \prod_{\text{nodes}} \left(\prod_a e^{-(j_{a+3}-j_a)^2/2\lambda^2}\right)$$

[4] E. Livine and S. Speziale '07

Correlations for coherent states

1. Probability distribution factorizes over one-dimensional sublattices



2. Correlations within each sublattice:

$$C(R) \equiv C_{j_i, j_{i+R}} = \frac{j_0}{5} e^{-2.3R} \quad \Longrightarrow \quad \text{Correlation length: } \xi = 0.43$$

(in lattice units)

Similar result holds for heat kernel states, $|\{H_\ell\}, t\rangle$, $H_\ell \in SL(2, \mathbb{C})$ [5]

$$C(R) \simeq \frac{j_0}{(1+4tj_0)} [2(1+4tj_0)]^{-R} \quad \Longrightarrow \quad \xi < \frac{1}{\log 2} \simeq 1.44$$

[5] T. Thiemann '01; E. Bianchi, E. Magliaro and C. Perini '10

Correlations in the limit of small spins [6]

A state $|\psi\rangle \in \mathcal{H}_\Gamma$ is factorizable if in the loop expansion

$$|\psi\rangle = \sum_\Phi c_\Phi F_\Phi^\dagger |0\rangle$$

the expansion coefficients satisfy: (i) $c_{\Phi_1 \sqcup \Phi_2} = c_{\Phi_1} c_{\Phi_2}$. Assume that the amplitudes decay with the length of the loop: (ii) $c_\Phi \leq A\lambda^{\beta|\Phi|}$.

$\longrightarrow$ ***Loop expansion is dominated by contributions from small loops.***

$$C_{\ell_1 \ell_2} \simeq 0 \quad \text{at order } \lambda^{2\beta d_{12}}$$

Coherent states satisfy (i) and (ii), with $\beta = 2$. Exponential decay of correlations in the limit of small spins (= small λ).

[6] E. Bianchi, J. Guglielmon, L. Hackl, NY, arXiv:1607.XXXX

Correlations for squeezed states

Squeezed states: $|\gamma\rangle = P_G P_A \exp[\lambda (a^\dagger \gamma a^\dagger)]|0\rangle \,, \quad \lambda < 1$

Choose: $\gamma^{AB}_{(m\mu),(n\nu)} = \lambda \, \epsilon^{AB} \epsilon_{CD} z^C_\mu z^D_\nu \, (\delta_{mn} + \varepsilon \, f_{mn})$

fixes scale factor

areas peaked along Euclidean directions

$\varepsilon \ll 1$, *encodes correlations*

Average values of local observables fixed by λ and z's. For small spins:

$$\langle j \rangle = \frac{2}{3^8}\lambda^8 \,, \qquad \langle W \rangle = \frac{\lambda^4}{2 \cdot 3^4} \cos[2(\xi_i + \xi_j)]$$

For $\varepsilon = 0$, the state is factorizable. Correlations are short ranged.

Correlations for squeezed states

For nonzero ε, we find for the spin-spin correlations:

$$C_{\ell\ell'} = \frac{2^6}{3^{16}} \lambda^{16} \varepsilon^2 f^2_{s(\ell)s(\ell')}$$

The function f can be chosen to scale with the inverse of the distance, yielding an inverse square law for the correlations:

$$C_{\ell\ell'} \propto 1/(d_{s(\ell)s(\ell')})^2$$

✦ *The distance d does not refer to a background geometry, but is encoded in the state, being determined by the diagonal part of the squeezing matrix.*

✦ *In general, states with polynomially decaying correlations on a bosonic lattice satisfy an area law bound for the entanglement entropy.* [Cramer et al '05]

Conclusion

- For coherent and heat kernel states, correlations in the fluctuations of spins on a cubulation decay exponentially with the distance in the limits of both small and large spins.
- For squeezed states, squeezing matrix can be decomposed into a diagonal part, which fixes the local geometry (intrinsic and extrinsic), and an off-diagonal part, which encodes correlations.
- Off-diagonal part can be chosen so that spin-spin correlations decay as the squared distance, yielding the characteristic decay of equal time vacuum fluctuations of massless fields in Minkowski spacetime.
- Area law for the entanglement entropy.