Study of Thin-Shell Wormholes Stability

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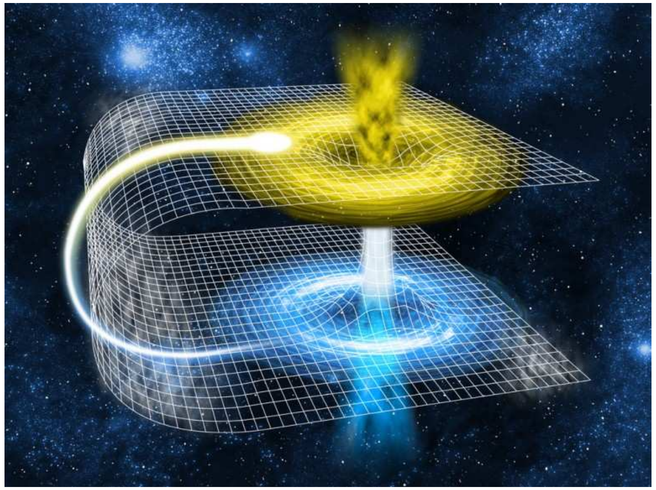
Format:

- **Thin-Shell Wormholes**
- **Stability Analysis**
- **Concluding Remarks**

Wormhole $\Rightarrow$ **One of the most fascinating features of general relativity which acts as a shortcut connecting different regions of the spacetime.**

[Misner, C.W. and Wheeler, J.A.: Ann. Phys. 2(1957)525]




- **To derive any physically reasonable source for regular BH, one needs to enlarge the class of electrodynamics to nonlinear ones.**

[Ayon-Beato, E. and Garcia, A.: Phys. Rev. Lett. 80(1998)5056.]

[Bronnikov, K.A.: Phys. Rev. D 63(2001)044005.]

- **These regular BHs behave as ordinary RN BH solutions and the existence of these solutions does not contradict with the singularity theorems.**

[Hawking, S.W. and Ellis, G.F.R.: *The Large Scale Structure of Spacetime*.]

- **This motivates us to discuss stability of viable thin-shell wormholes coupled with NLED.**

Action:

Gravity coupled with NLED and Λ is

$$S=\frac{1}{16\pi}\int\sqrt{-g}\,[(R-2\Lambda)-\mathcal{L}(F)]\,d^4x,$$

R **- scalar curvature**

$\mathcal{L}(F)$ **- Lagrangian for NLED**

$$\mathcal{L}(F)=F\left[1-\tanh^2\left(\frac{Q}{2M}\left(\frac{FQ^2}{2}\right)^{\frac{1}{4}}\right)\right],$$

which depends on the invariants of the

electromagnetic field $F=F^{\mu\nu}F_{\mu\nu}$ in nonlinear way,

where $F_{\mu\nu}=\Phi_{\nu;\nu}-\Phi_{\mu;\nu}$.

ABGB Metric:

$$ds^2 = -N(r)dt^2 + N^{-1}(r)dr^2 + r^2 d\Omega^2,$$

$$N(r) = 1 - \frac{2M}{r}\left[1 - \tanh\left(\frac{Q^2}{2Mr}\right)\right] - \frac{\Lambda r^2}{3}.$$

[Matyjasek, J., Tryniecki, D. and Klimek, M: Mod. Phys. Lett. A 23(2008)3377.]

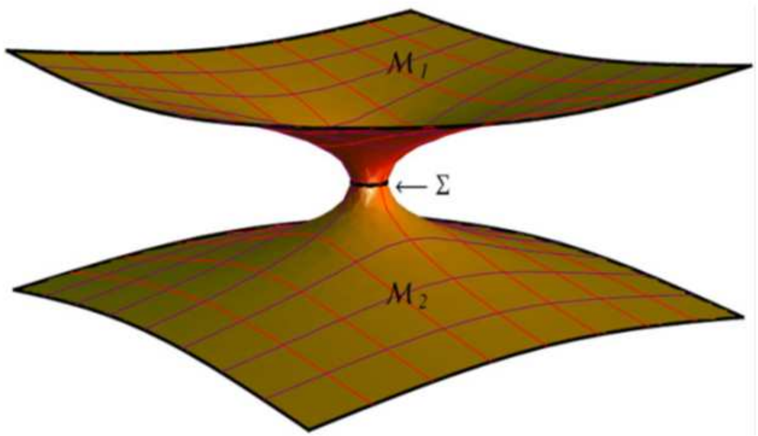
- **Event and cosmological horizons can be found by the real roots of $N(r) = 0$.**
- **Cosmological constant does not destroy regularity of the solution.**
- **For $\Lambda > 0$, it describes a regular ABGB-de Sitter BH which reduces to regular ABGB BH for $\Lambda = 0$.**

Wormhole Construction:

- **Standard cut and paste procedure to construct a regular ABGB thin-shell wormhole.**
- **Interior region is cut with $r < a$ which yields two identical copies $\mathcal{M}^{+}$ and $\mathcal{M}^{-}$.**
- **These 4D copies are glued at the hypersurface $\Sigma^{\pm} = \Sigma = \{r = a\}$.**
- **A 3D induced spacetime is considered at the shell**

$$ds^2 = -d\tau^2 + a^2(\tau)(d\theta^2 + \sin^2\theta d\phi^2).$$

Applying Israel Formalism:



Thin layer of matter on Σ causes the extrinsic curvature discontinuity.

Israel formalism is applied for the dynamical evolution of thin-shell which enables the joining of two regions of spacetime partitioned by Σ.
Unit normals to Σ in $\mathcal{M}$ are given by

$$n_{\beta}^{\pm}=\pm\left|g^{\mu\nu}\frac{\partial\eta}{\partial x^{\mu}}\frac{\partial\eta}{\partial x^{\nu}}\right|^{-\frac{1}{2}}\frac{\partial\eta}{\partial x^{\beta}}=\left(-\dot{a},\frac{\sqrt{N(r)+\dot{a}^{2}}}{N(r)},0,0\right).$$

Non-trivial components of the extrinsic curvature

$$K_{\tau\tau}^{\pm}=\mp\frac{N'(a)+2\ddot{a}}{2\sqrt{N(a)+\dot{a}^{2}}},\quad K_{\theta\theta}^{\pm}=\pm a\sqrt{N(a)+\dot{a}^{2}},\quad K_{\phi\phi}^{\pm}=\alpha^{2}K_{\theta\theta}^{\pm}.$$

Lanczos Equations

$$\Downarrow$$

$$S_{ij}=\frac{1}{8\pi}\{g_{ij}K-[K_{ij}]\},$$

leading to surface energy density $S_{\tau\tau}=\sigma$ **and surface pressures** $S_{\theta\theta}=p=S_{\phi\phi}$ **as**

$$\sigma = -\frac{1}{2\pi a}\sqrt{N(a)+\dot{a}^2}, \quad (1)$$

$$p = p_\theta = p_\phi = \frac{1}{8\pi}\frac{2\dot{a}^2+2a\ddot{a}+2N(a)+aN'(a)}{a\sqrt{N(a)+\dot{a}^2}}. \quad (2)$$

Violation of Energy Conditions

(Null and Weak)

$$\Downarrow$$

Presence of Exotic Matter

$\sigma < 0$ **and** $\sigma + p < 0 \Rightarrow$ **Exotic Matter**

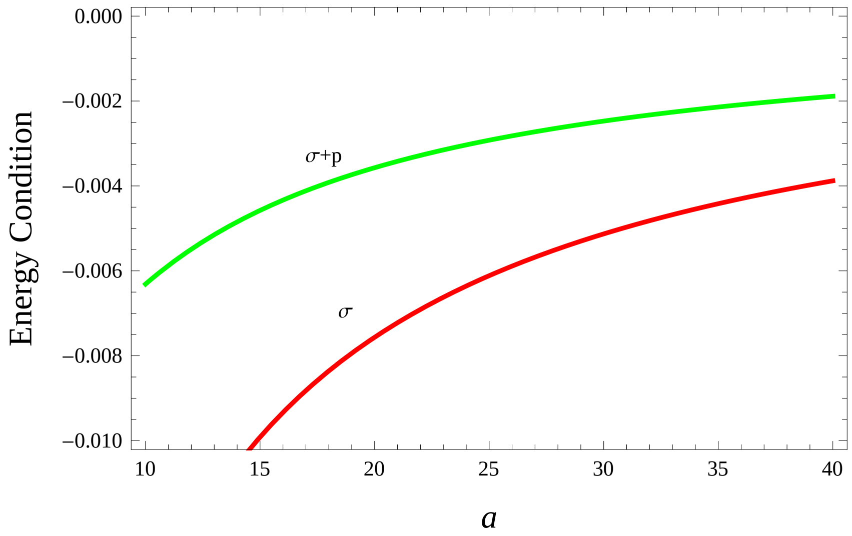


Figure 1: Plots for the violation of NEC and WEC when $\frac{Q}{M} = 0.99$**.**

Amount of Exotic Matter:

Total amount of exotic matter is quantified by the integral theorem

$$\Omega = \int (\rho + p_r)\sqrt{|g|}d^3x,$$
$$\Omega = \int_0^{2\pi}\int_0^{\pi}\int_{-\infty}^{+\infty}(\rho + p_r)\sqrt{|g|}dR\sin\theta d\theta d\phi.$$

Wormhole shell, being thin, does not apply any radial pressure ($p_r = 0$). Considering $\rho = \delta(R)\sigma(a)$

$$\Omega_a = -\frac{1}{\sqrt{3r}}\sqrt{3r - 6M\left(1 - \tanh\left(\frac{Q^2}{2Mr}\right)\right) - \Lambda r^3}.$$

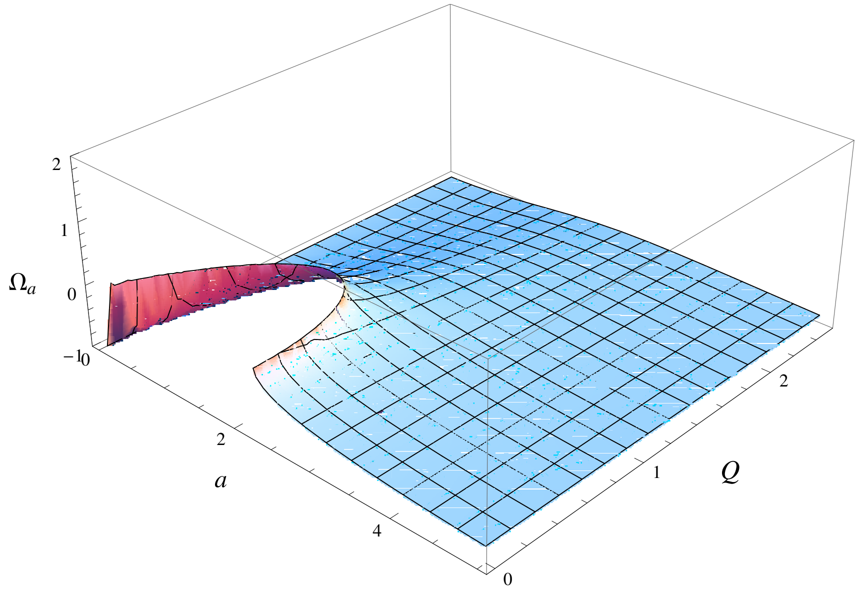


Figure 2: Plot for the total amount of exotic matter with $\Lambda = 0.1$ and different values of charge.

Stability Analysis:

Stability of ABGB thin-shell wormholes is discussed through linear perturbations. Surface energy density and pressure corresponding to static wormhole configuration ($a = a_0$) yield

$$\sigma_0 = -\frac{\sqrt{N(a_0)}}{2\pi a_0}, \quad p_0 = \frac{1}{4\pi}\left[\frac{\sqrt{N(a_0)}}{a_0} + \frac{N'(a_0)}{2\sqrt{N(a_0)}}\right].$$

Surface stresses satisfy the conservation identity $S^{ij}_{\ ;j} = 0$, which becomes

$$\frac{d}{d\tau}(\sigma\Delta) + p\frac{d\Delta}{d\tau} = 0,$$

$\Delta = 4\pi a^2$ **corresponds to wormhole throat area.**
First term $\Rightarrow$ **change in internal energy of the shell**
Second term $\Rightarrow$ **work done by internal forces.**

Thin-shell equation of motion can be obtained by rearranging Eq.(1) as $\dot{a}^2 + \Phi(a) = 0$**, which provides wormhole dynamics while the potential function is**

$$\Phi(a) = F(a) - [2\pi a\sigma(a)]^2, \quad (3)$$

σ **represents the perturbed energy density.**

To study wormhole stability, we assume a linear perturbation in the form of barotropic EoS

$$p = \Psi(\sigma). \tag{4}$$

Basic condition for stability of wormhole static solution yields $\Phi'(a_0) = 0 = \Phi(a_0)$ and $\Phi''(a_0) > 0$.

Using Eq.(4) and $\sigma' = \frac{\dot{\sigma}}{\dot{a}}$ in the conservation equation

$$\sigma'' = \frac{2}{a^2}(\sigma + \Psi)(3 - a\Psi').$$

The first derivative of Eq.(3) turns out to be

$$\Phi'(a_0) = N'(a_0) + 8\pi^2 a_0\sigma_0[\sigma_0 + 2p(\sigma_0)],$$

leading to

$$\Phi''(a_0) = N''(a_0) - 8\pi^2 \left\{ [\sigma_0 + 2p_0]^2 + 2\sigma_0[\sigma_0 + p_0][1 + 2\Psi'(\sigma_0)] \right\},$$

where $\Psi_0 = p_0$**.**

Conditions for Stability:

- **Stable static solution exists if $a_0 > r_h$ and $\Phi'' > 0$.**
- **Unstable static solution for $a_0 > r_h$ and $\Phi'' < 0$.**
- **For $a_0 \leq r_h$, no static solution exists which corresponds to the non-physical region.**

Models for Exotic Matter:

Here we take the following candidates for exotic matter at the shell to study the stability formalism of regular ABGB thin-shell wormholes.

- **Chaplygin Gas**
- **Generalized Chaplygin Gas**
- **Modified Generalized Chaplygin Gas**
- **Linear Gas**
- **Logarithmic Gas**

1. Chaplygin Gas:

$$\Psi(\sigma) = p_0 + \mu\left(\frac{1}{\sigma} - \frac{1}{\sigma_0}\right), \qquad (5)$$

[Kamenshchik, A.Y., et al.: Phys. Lett. B 487(2000)7.]

μ is EoS parameter. Differentiating w.r.t σ gives

$$\Psi'(\sigma_0) = -\frac{\mu}{\sigma_0^2}.$$

We are interested to explore the possibility of stable wormhole solutions and check the role of increasing charge in stability regions.

- $\Phi(a)$ and $\Phi'(a)$ disappear by inserting the values of $\sigma(a_0)$ and $p(a_0)$.
- $N(r)$ is also plotted to estimate the location of event horizon and wormhole throat. Stable regions and the metric function are represented by red and blue curves, respectively.
- We assume $a_0 > r_h$ for the viability of thin-shell wormholes without event horizons.
- Stability regions corresponding to positive and negative values of parameter μ and $\frac{Q}{M} = 0$, 0.707, $0.999, \ 1.1$. Here $\frac{Q}{M} = 0$ corresponds to S BH.

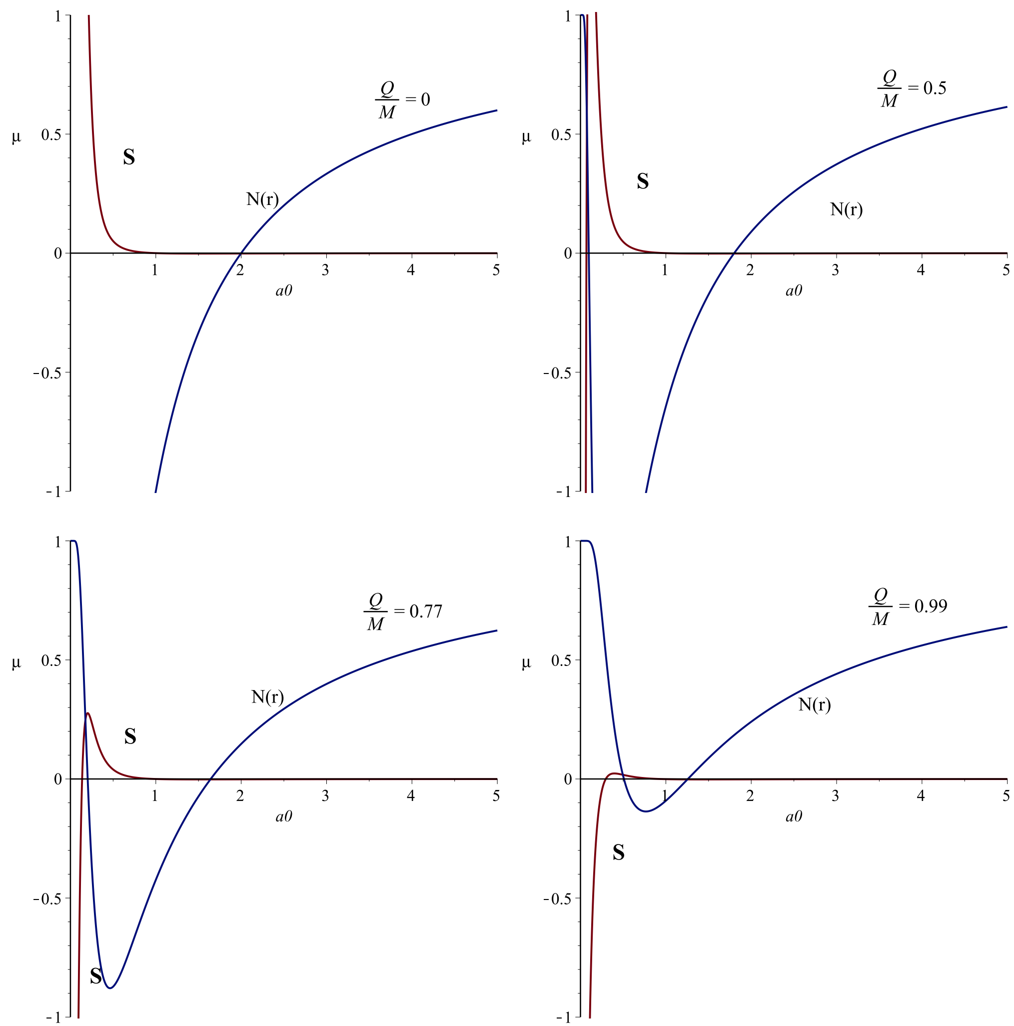


Figure 3: Plots for regular ABGB thin-shell wormholes by taking CG EoS and $\frac{Q}{M} = 0,\ 0.5,\ 0.77,\ 0.99.$

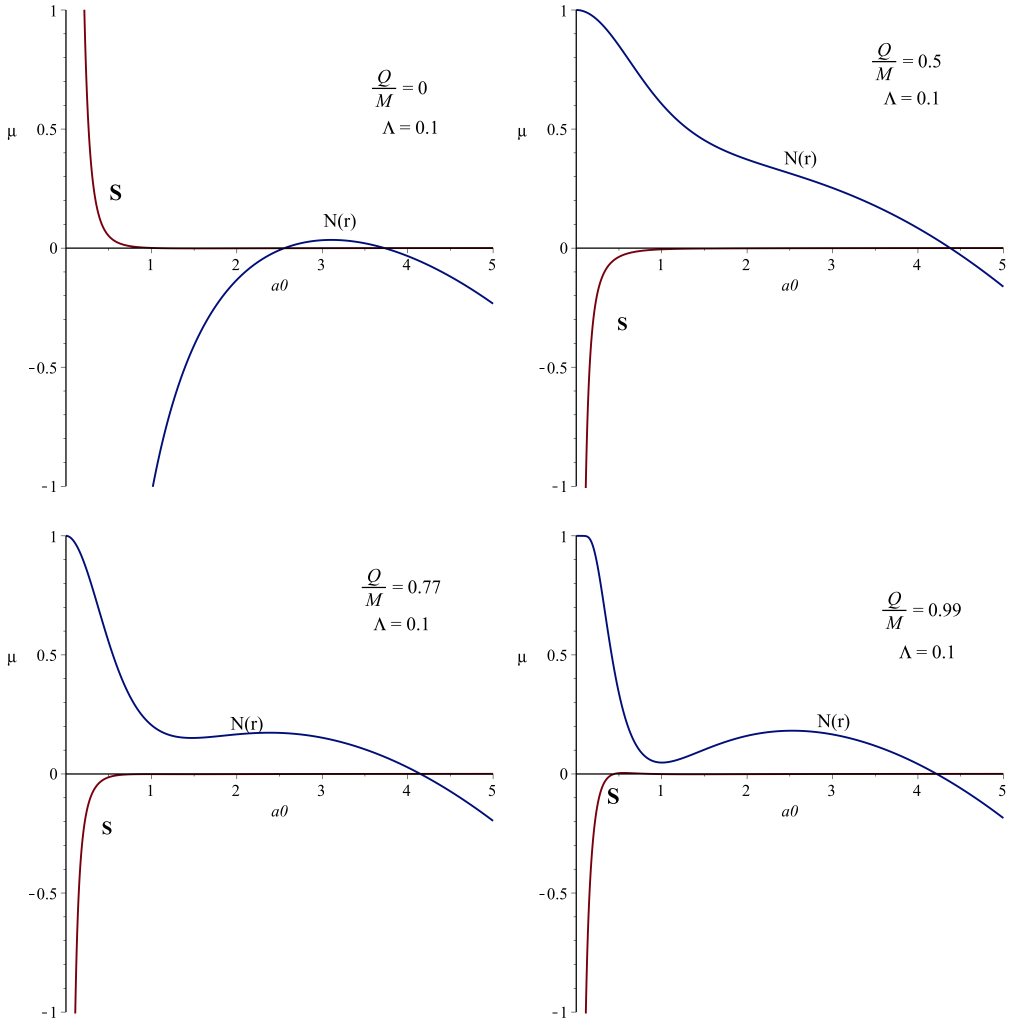


Figure 4: Plots for regular ABGB thin-shell wormholes for CG EoS in de Sitter background.

2. Generalized Chaplygin Gas:

EoS

$$\Psi(\sigma) = p_0 + \mu\left(\frac{1}{\sigma^\gamma} - \frac{1}{\sigma_0^\gamma}\right), \qquad (6)$$

[Barreiro, T. and Sen, A.A.: Phys. Rev. D 70(2004)124013.]

γ denotes EoS parameter. We set $\mu = p_0\sigma^\gamma$ such that the above EoS becomes

$$\Psi(\sigma) = p_0\left(\frac{\sigma_0}{\sigma}\right)^\gamma, \qquad (7)$$

which yields $\Psi'(\sigma_0) = -\frac{p_0}{\sigma_0}\gamma$.

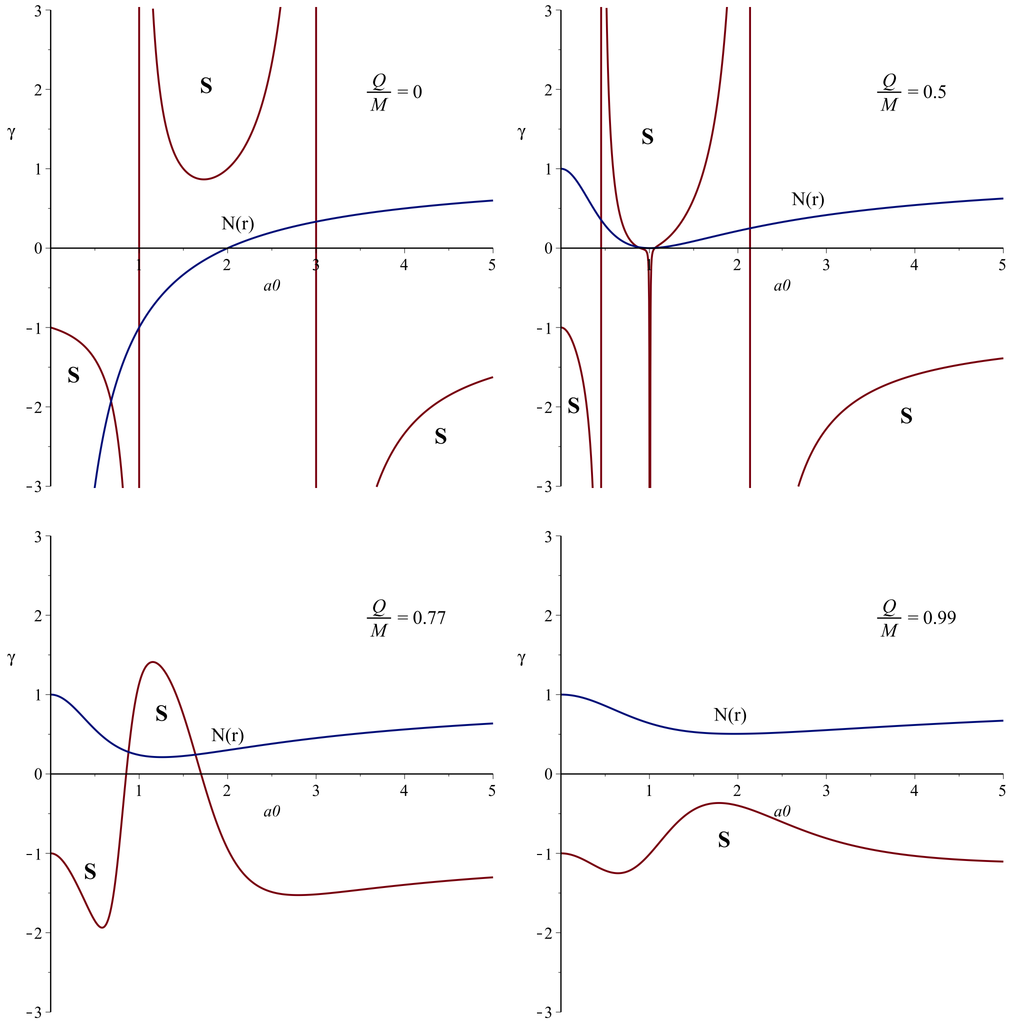


Figure 5: Plots for stability of regular ABGB thin-shell wormholes in terms of γ by taking GCG.

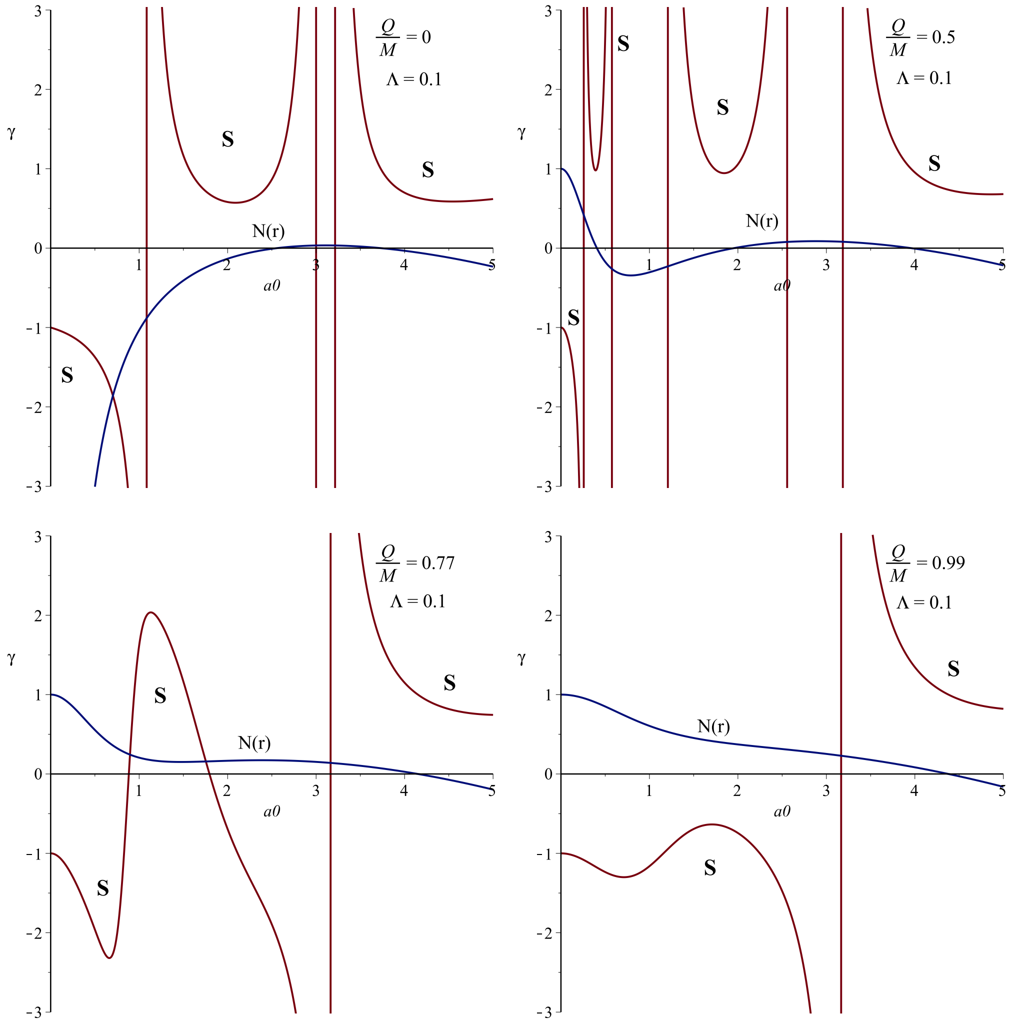


Figure 6: Plots for the stability of regular ABGB thin-shell wormholes in terms of γ by taking GCG gas and $\Lambda = 0.1$.

3. Modified Generalized Chaplygin Gas:

Commonly known extension of GCG is called MGCG defined as

$$\Psi(\sigma) = p_0 + \xi_0(\sigma - \sigma_0) - w\left(\frac{1}{\sigma^\gamma} - \frac{1}{\sigma_0^\gamma}\right), \tag{8}$$

[Xu, L., Wang, Y. and Noh, H.: Eur. Phys. J. C 72(2012)1931.]

ξ_0 and w are free parameters. Its differentiation yields

$$\Psi'(\sigma_0) = \xi_0 + \frac{w\gamma}{\sigma_0^{\gamma+1}}.$$

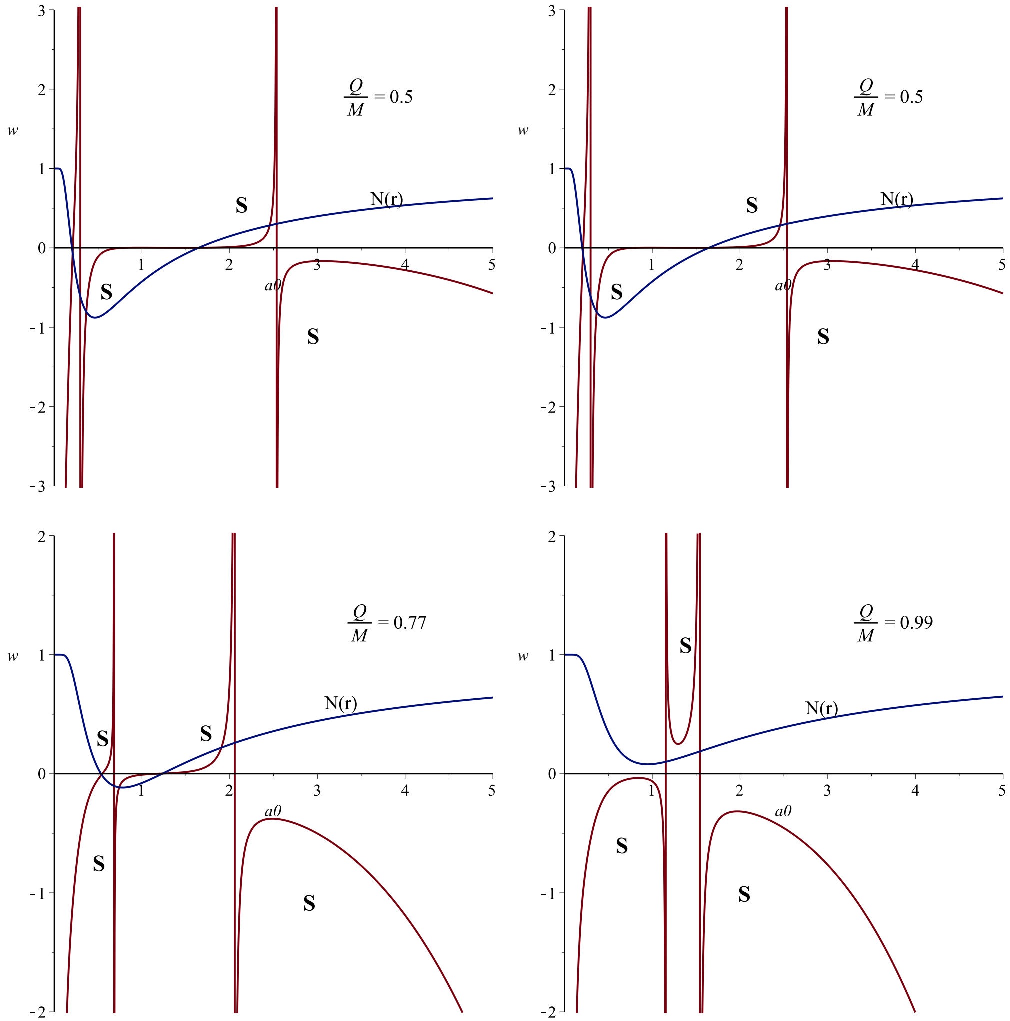


Figure 7: Plots for stable regular ABGB thin-shell wormholes by taking MGCG gas with $\xi_0 = \gamma = 1$ and different values of charge.

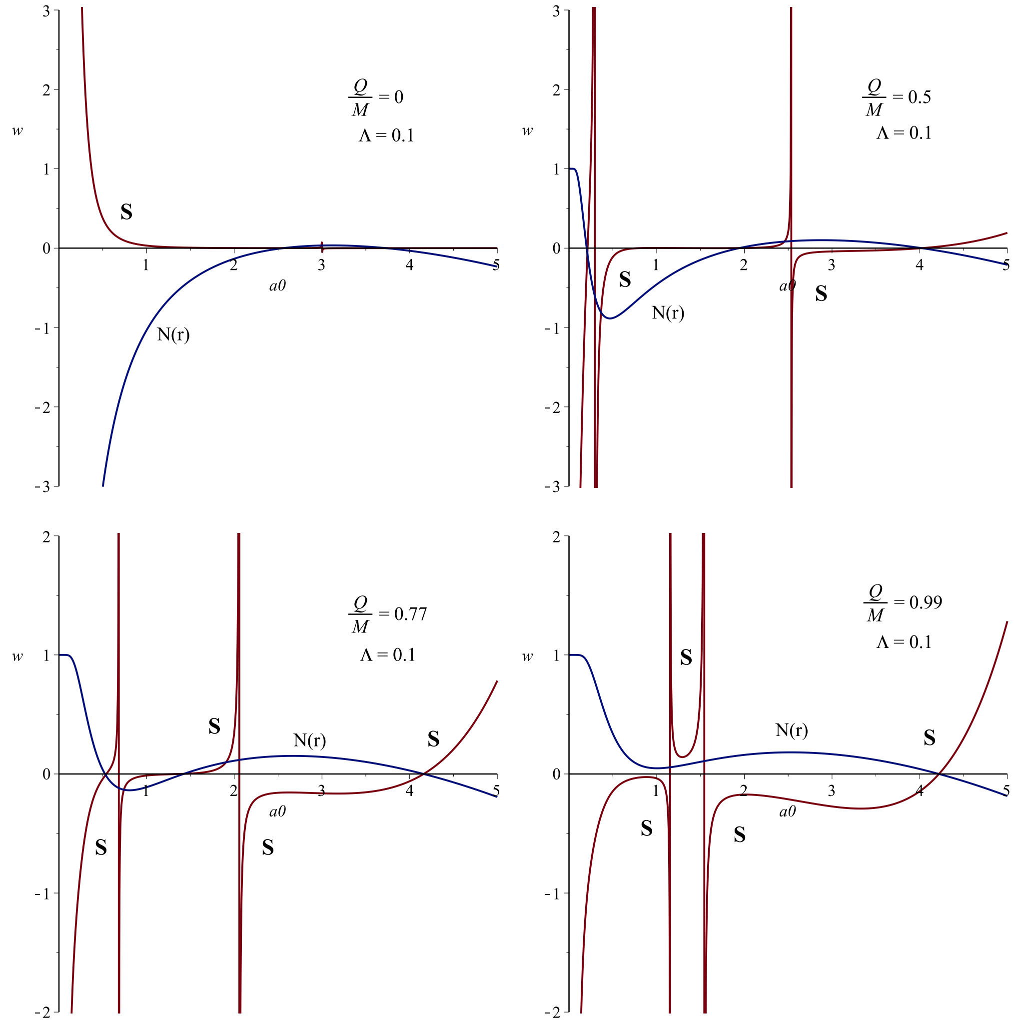


Figure 8: Plots for stability regions of regular ABGB thin-shell wormholes by taking MGCG gas with $\xi_0 = \gamma = 1$ in de Sitter background.

4. Linear Gas:

$$\Psi = p_0 + \mu(\sigma - \sigma_0), \qquad (9)$$

[Halilsoy, M., Ovgun, A. and Mazharimousavi, S.H.: Eur. Phys. J. C 74(2014)2796.]

Differentiating this equation with respect to σ, we obtain $\Psi'(\sigma_0) = \mu$.

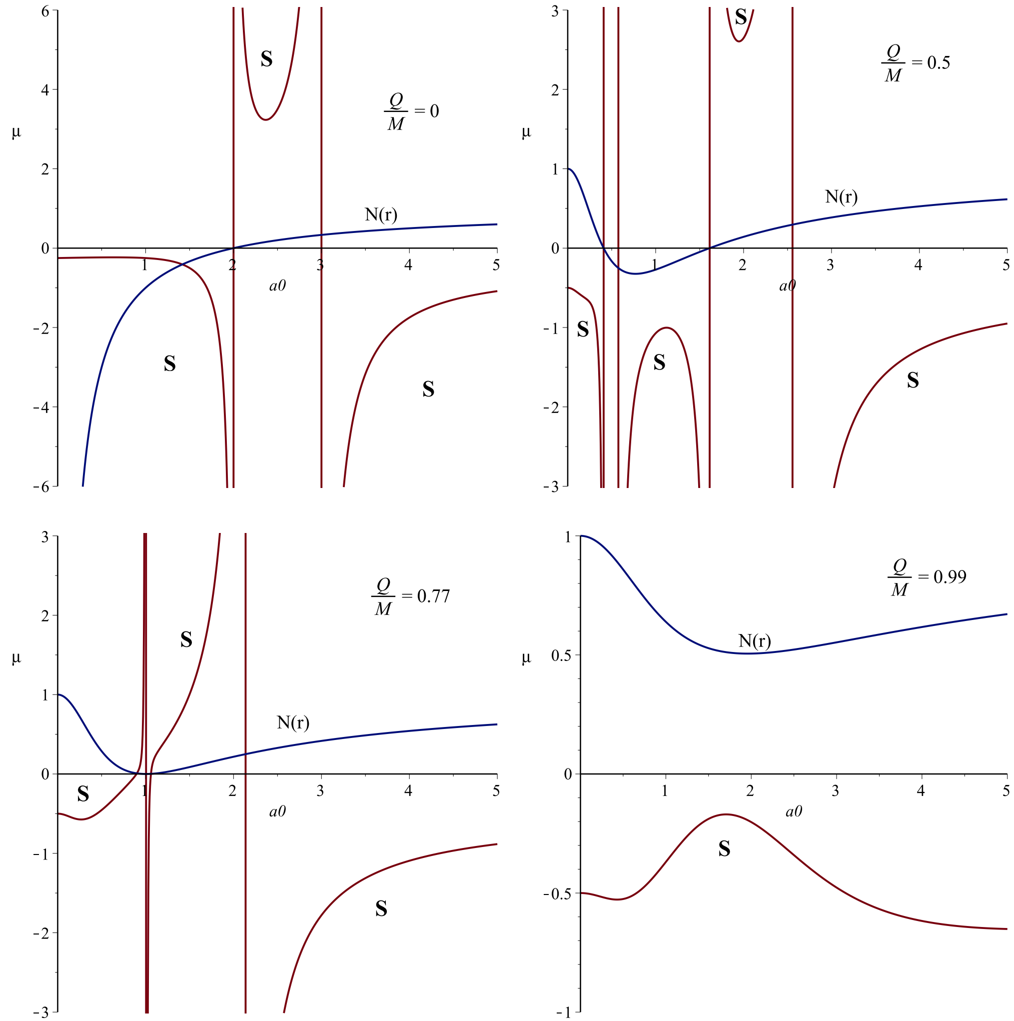


Figure 9: Plots for regular ABGB thin-shell wormholes corresponding to linear gas EoS with $\frac{Q}{M} = 0,\ 0.5,\ 0.77,\ 0.99$.

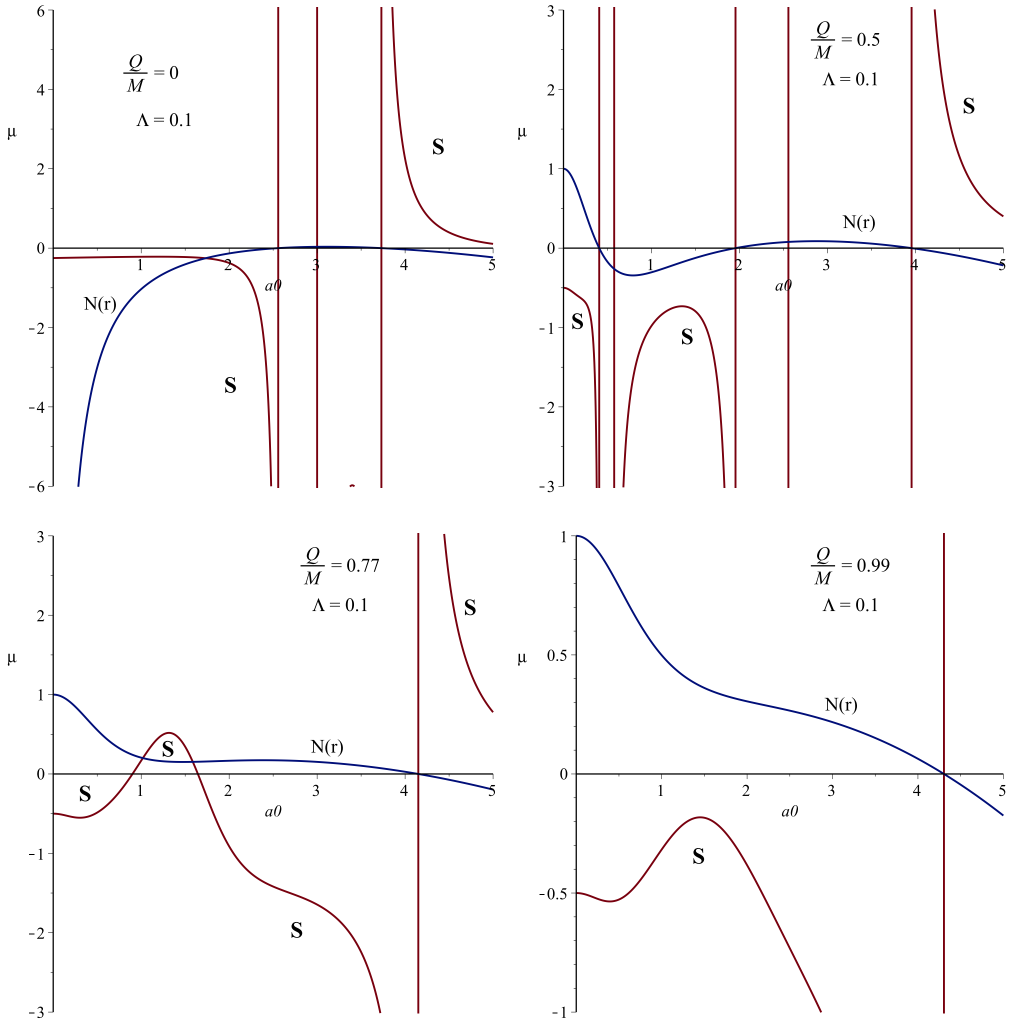


Figure 10: Plots for regular ABGB thin-shell wormholes corresponding to linear gas EoS in de Sitter background.

5. Logarithmic Gas:

$$\Psi(\sigma) = p_0 + w \ln \left| \frac{\sigma}{\sigma_0} \right|, \qquad (10)$$

[Halilsoy, M., Ovgun, A. and Mazharimousavi, S.H.: Eur. Phys. J. C 74(2014)2796.]

where $\Psi'(\sigma_0) = \frac{w}{\sigma_0}$.

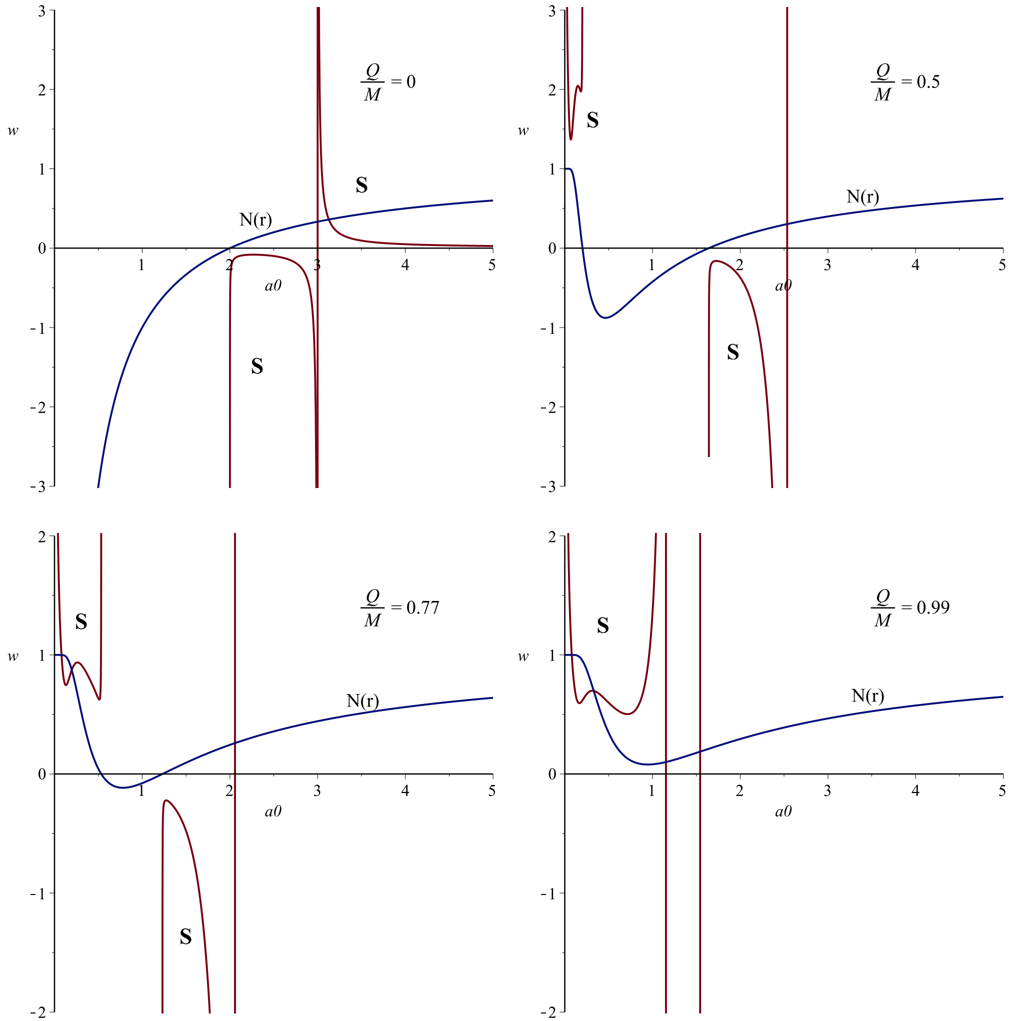


Figure 11: Plots for regular ABGB thin-shell wormholes by taking logarithmic gas EoS and $\frac{Q}{M} = 0,\ 0.5,\ 0.77,\ 0.99$.

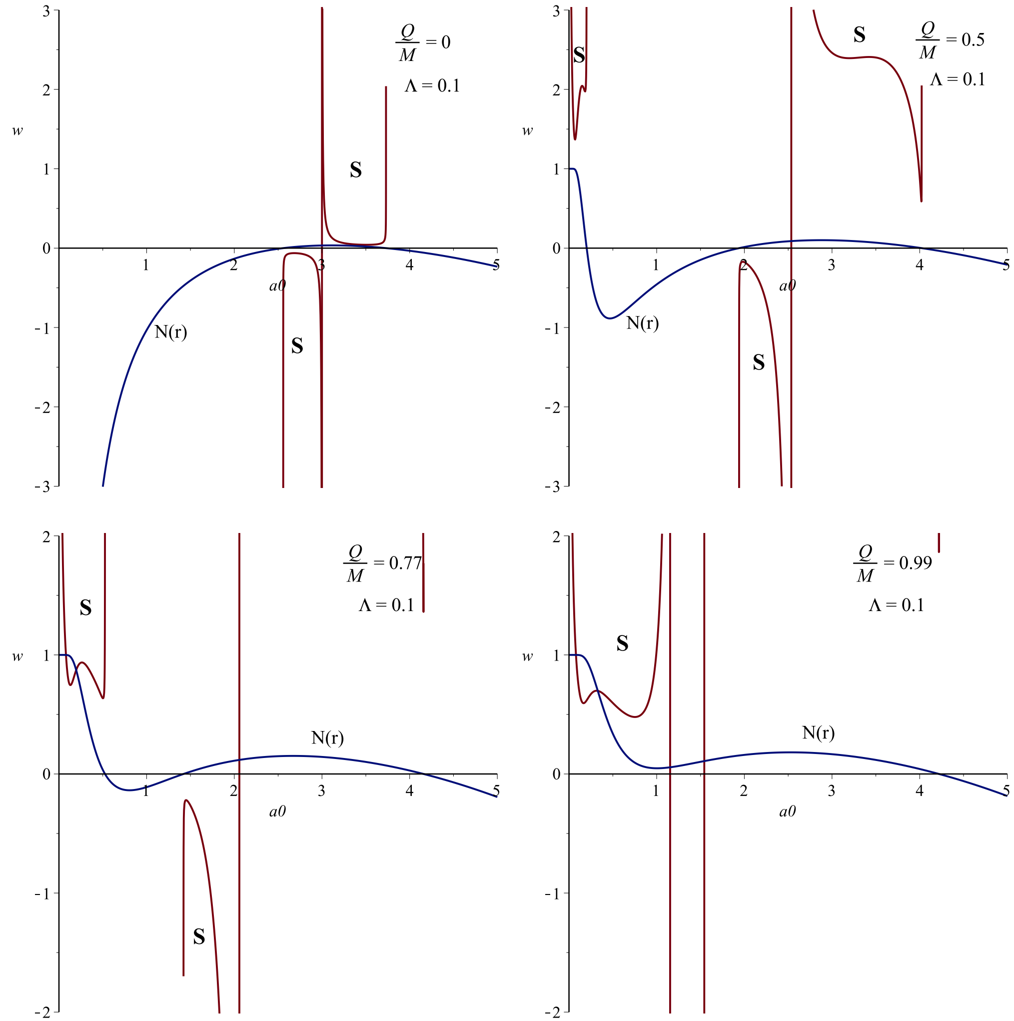


Figure 12: Plots for regular ABGB thin-shell wormholes by taking logarithmic gas EoS with $\frac{Q}{M} = 0,\ 0.5,\ 0.77,\ 0.99$ and $\Lambda = 0.1$.

- **ABGB thin-shell wormhole remains attractive for $\Lambda = 0$ while it shows both attractive and repulsive characteristics for different throat radii in de Sitter background.**

- **Stable regular ABGB wormhole solutions are possible against radial perturbations with arbitrarily small amount of the fluid describing cosmic expansion and acceptable range of charge and other parameters.**

- **Chaplygin gas does not appear significant as it provides least stable solutions than other models.**

- **MGCG and linear gas have remarkable significance as they provide maximum stable regions for wormhole configurations.**
- **Stable regions may expand or shrink accordingly depending on the tuning of charge and other parameters. More stability regions for de Sitter case as compared to the general case without Λ.**

Thank you