An Effective Field Theory for Spinning Gravitating Objects in the Post-Newtonian Scheme

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Setup of EFT is Universal/What can EFT do for me?

Hierarchy of scales

1. r_s scale of internal structure, $r_s \sim m$
2. r orbital separation scale, $r \sim r_s/v^2$
3. λ radiation wavelength scale, $\lambda \sim r/v$

   $v \ll 1$, $nPN \equiv v^{2n}$ correction in GR to Newtonian Gravity

For an EFT of GWs proceed in 3 stages

Stage 1 Remove the scale of the isolated compact object

$$S\left[g_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{g}R$$

Integrate out the strong field modes $g_{\mu\nu} \equiv g^s_{\mu\nu} + \bar{g}_{\mu\nu}$

$$\Rightarrow S_{\text{eff}}\left[y^\mu, e^\mu_A, \bar{g}_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}\right] + \underbrace{\sum_i C_i \int d\sigma O_i(\sigma)}_{S_{pp}\equiv\text{point particle action}}$$

Setup of EFT is Universal/What can EFT do for me?

For an EFT of GWs proceed in 3 stages

Stage 2 Remove the orbital scale of the binary

$$\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \underbrace{H_{\mu\nu}}_{\text{orbital}} + \underbrace{\widetilde{h}_{\mu\nu}}_{\text{radiation}}$$

$$\partial_t H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}, \quad \partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu}, \quad \partial_\rho \widetilde{h}_{\mu\nu} \sim \frac{v}{r} \widetilde{h}_{\mu\nu}$$

$$S_{\text{eff}}\left[y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu, \bar{g}_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}\,R\left[\bar{g}_{\mu\nu}\right] + S_{(1)\text{pp}} + S_{(2)\text{pp}}$$

Integrate out the orbital field modes

$$\Rightarrow e^{iS_{\text{eff(composite)}}\left[y^\mu, e_A^\mu, \widetilde{h}_{\mu\nu}\right]} \equiv \int \mathcal{D}H_{\mu\nu}\; e^{iS_{\text{eff}}\left[y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu, \bar{g}_{\mu\nu}\right]}$$

Stop here for an *effective* action in the conservative sector, i.e. **WITHOUT** any remaining orbital scale field DOFs

Symmetries (for Bottom-Up EFT)

Couple DOFs according to symmetries to construct effective action

1. *General coordinate invariance*, and *parity invariance*
2. *Worldline reparametrization invariance*
3. *Internal Lorentz invariance of the local frame field*
4. *SO(3) invariance of the body-fixed spatial triad*
5. *Spin gauge invariance*, i.e. invariance under the choice of a completion of the body-fixed spatial triad through a timelike vector
   This is a gauge of the rotational variables, i.e. of the worldline tetrad + worldline spin
6. Assume the isolated object has no intrinsic permanent multipole moments beyond the mass monopole and the spin dipole

Degrees of Freedom

Specify DOFs at each stage

1. *The gravitational field*
   - The metric $g_{\mu\nu}(x)$
   - The tetrad field $\eta^{ab}\tilde{e}_a{}^{\mu}(x)\tilde{e}_b{}^{\nu}(x) = g^{\mu\nu}(x)$
2. *The particle worldline coordinate*

   $y^{\mu}(\sigma)$ a function of an arbitrary affine parameter σ
   The particle worldline position does not in general coincide with the 'center' of the object, i.e. the reference point within the actual extended object
3. *The particle worldline rotating DOFs*

   The worldline tetrad, $\eta^{AB}e_A{}^{\mu}(\sigma)e_B{}^{\nu}(\sigma) = g^{\mu\nu}$
   $\Rightarrow$ The worldline angular velocity $\Omega^{\mu\nu}(\sigma)$ + worldline spin $S_{\mu\nu}(\sigma)$
   Local tetrad, $\tilde{e}_a{}^{\mu}\left(y(\sigma)\right) = \Lambda_a{}^{A}(\sigma)e_A{}^{\mu}(\sigma)$, disentangled from worldline tetrad
   $\Rightarrow$ The worldline Lorentz matrices, $\eta^{AB}\Lambda_A{}^{a}(\sigma)\Lambda_B{}^{b}(\sigma) = \eta^{ab}$
   + conjugate worldline spin, $S_{ab}(\sigma)$, projected to the local frame

For an EFT of GWs: Fix gauge of rotational variables!

Effective action of a spinning particle

- $u^\mu \equiv dy^\mu/d\sigma$, $\Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma} \Rightarrow L_{\rm pp}\left[u_\mu, \Omega^{\mu\nu}, g_{\mu\nu}\right]$
- $S_{\mu\nu} \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$ spin as further worldline DOF – classical source

$$\Rightarrow S_{\rm pp} = \int d\sigma \left[-m\sqrt{u^2} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + L_{\rm SI}\left[u^\mu, S_{\mu\nu}, g_{\mu\nu}\left(y^\mu\right)\right]\right]$$

For an EFT the gauge of the rotational variables
should be fixed at the level of the action

[ML, 2×PRD 2010; ML & Steinhoff, JCAP 2014]

Start in the covariant gauge: $e_{[0]\mu} = \frac{p^\mu}{\sqrt{p^2}}$, $\qquad S_{\mu\nu}p^\nu = 0$

- Linear momentum $p_\mu \equiv -\frac{\partial L}{\partial u^\mu} = m\frac{u^\mu}{\sqrt{u^2}} + \mathcal{O}(S^2)$

Unfixing the gauge of the rotational variables

Introduce the gauge invariance in the rotational variables

Transform $e^{A\mu}$ from a gauge condition $e_{[0]\mu} = q_\mu$ to a condition $\hat{e}_{[0]\mu} = w_\mu$ with a boost-like transformation in the 4d covariant form:

$\hat{e}^{A\mu} = L^\mu{}_\nu(w,q) e^{A\nu}$, $\quad q_a$, w_a are timelike unit 4-vectors

$\Rightarrow$ Generic gauge: $\quad \hat{e}_{[0]\mu} = w_\mu, \quad \hat{S}^{\mu\nu}\left(p_\nu + \sqrt{p^2}\hat{e}_{[0]\nu}\right) = 0$

Extra term in action from minimal coupling

- For minimal coupling: $\frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} = \frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} + \frac{\hat{S}^{\mu\rho}p_\rho}{p^2}\frac{Dp_\mu}{D\sigma}$
- Extra term contributes to finite size effects, yet carries no Wilson coefficient
- Beyond minimal coupling: $S_{\mu\nu} = \hat{S}_{\mu\nu} - \frac{\hat{S}_{\mu\rho}p^\rho p_\nu}{p^2} + \frac{\hat{S}_{\nu\rho}p^\rho p_\mu}{p^2}$

Integrating out the orbital scale

Worldline tetrad $\eta_{AB}\hat{e}^{A\mu}\hat{e}^{B\nu} = g^{\mu\nu}$
contains both rotational and field DOFs

- $\hat{e}_A{}^\mu = \hat{\Lambda}_A{}^b \tilde{e}_b{}^\mu$: Tetrad field $\eta_{ab}\tilde{e}^a{}_\mu \tilde{e}^b{}_\nu = g_{\mu\nu}$, $\eta^{AB}\hat{\Lambda}_A{}^a\hat{\Lambda}_B{}^b = \eta^{ab}$
- For the minimal coupling: $\frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} = \frac{1}{2}\hat{S}_{ab}\hat{\Omega}^{ab}_{\text{flat}} + \frac{1}{2}\hat{S}_{ab}\omega_\mu{}^{ab}u^\mu$
  Ricci rotation coefficients $\omega_\mu{}^{ab} \equiv e^b{}_\nu D_\mu e^{a\nu}$
  $\Rightarrow$ New rotational variables: $\hat{\Omega}^{ab}_{\text{flat}} = \hat{\Lambda}^{Aa}\frac{d\hat{\Lambda}_A{}^b}{d\sigma}$, $\hat{S}_{ab} = \tilde{e}^\mu_a \tilde{e}^\nu_b \hat{S}_{\mu\nu}$

Separation of field from particle worldline DOFs is not complete!

- The gauge of the worldline temporal Lorentz matrix $\hat{\Lambda}_{[0]}{}^a = w^a = \tilde{e}^a{}_\mu w^\mu$ may contain further field dependence
- The temporal components of the local spin contain further field dependence
  $\Rightarrow$ The field is completely disentangled from the worldline DOFs
  only once a gauge for the rotational variables is fixed

Nonminimal couplings with spin: Construction

Spin-induced multipoles

- Recall we start in the covariant gauge ${e_{[0]}}^{\mu} = u^{\mu}/\sqrt{u^2}$, ${e_{[i]}}^{\mu} u_{\mu} = 0$
- Considering the body-fixed frame the spin multipoles are SO(3) irreps tensors
- Parity invariance, $S^{\mu} \equiv *S^{\mu\nu} \frac{p_{\nu}}{\sqrt{p^2}} \simeq *S^{\mu\nu} \frac{u_{\nu}}{\sqrt{u^2}}$, $*S_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\alpha\beta\mu\nu} S^{\mu\nu}$

  $\Rightarrow$ Independent combinations of the spin vector S^{μ}

  $\Rightarrow$ Spin-induced higher multipoles are symmetric, traceless, and spatial, constant tensors in the body-fixed frame

Curvature tensors

Electric and magnetic curvature components: $E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta} u^{\alpha} u^{\beta}$

$$B_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\alpha\beta\gamma\mu} {R^{\alpha\beta}}_{\delta\nu} u^{\gamma} u^{\delta}$$

- In vacuum they are symmetric, traceless, and orthogonal to u^{μ}, also when projected to the body-fixed frame, where they are spatial
- Their covariant derivatives also projected to the body-fixed frame

$$D_{[i]} = e^{\mu}_{[i]} D_{\mu}$$

Nonminimal couplings with spin

Curvature tensors

- Time derivative $D_{[0]} = u^\mu D_\mu \equiv D/D\sigma$ can be ignored
- Analogy to Maxwell's equations: $\epsilon_{[ikl]} D_{[k]} E_{[lj]} = \dot{B}_{[ij]}, \quad \epsilon_{[ikl]} D_{[k]} B_{[lj]} = -\dot{E}_{[ij]}$
  $\rightarrow D_{[i]} E_{[ij]} = D_{[i]} B_{[ij]} = 0, \quad \Box E_{[ij]} = \Box B_{[ij]} = 0$
  $\Rightarrow$ The indices of the covariant derivatives would be symmetrized with respect to the indices of the electric and magnetic tensors
  $\Rightarrow$ The covariant derivatives of these tensors are also traceless

LO nonminimal couplings to all orders in spin

New spin-induced Wilson coefficients:

$$L_{\text{SI}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

Conclusions

EFT for spin: Summary of Results

- An EFT formulation for spinning objects
- Spin induced non-minimal coupling to all orders in spin
- EOMs and Hamiltonians straightforward to derive

[ML & Steinhoff, JHEP 2015]

For the 1st time spinning sector is synced with non-spinning one!

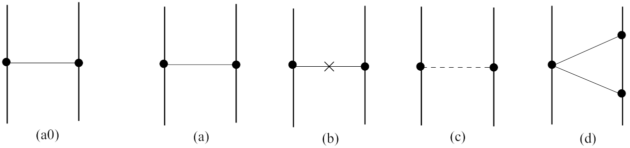
- NLO spin1-spin2, spin-orbit [ML, 2×PRD 2010], spin^2 [ML & Steinhoff, JHEP 2015]
- NNLO s1-s2 [ML, PRD 2011], spin-orbit, spin^2 [ML & Steinhoff, 2×JCAP 2016]
- LO cubic and quartic in spin [ML & Steinhoff, JHEP 2014]

Prospective work

- Radiative sector with spin
  - Formulation of EFT of radiation for spin
  - Implementation up to 4PN order
- Tidal and dissipative effects with spin

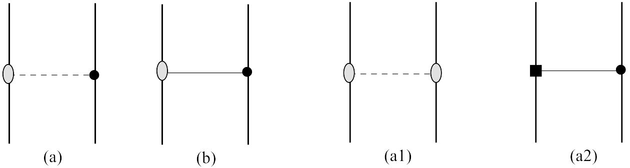
LO beyond Newtonian sectors

Feynman diagrams of non-spinning sector to 1PN order



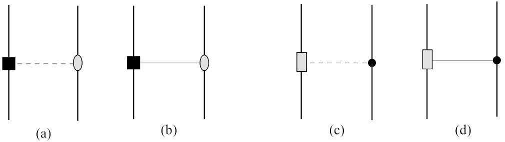
One-loop diagram – absent from 1PN order with NRG fields

Feynman diagrams of LO to quadratic in spin



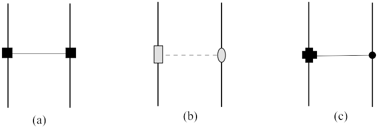
New results: LO cubic and quartic in spin sectors

Feynman diagrams of LO cubic in spin sector



- On the left pair – quadrupole-dipole, on the right – octupole-monopole
- Note the analogy of each pair with the LO spin-orbit sector

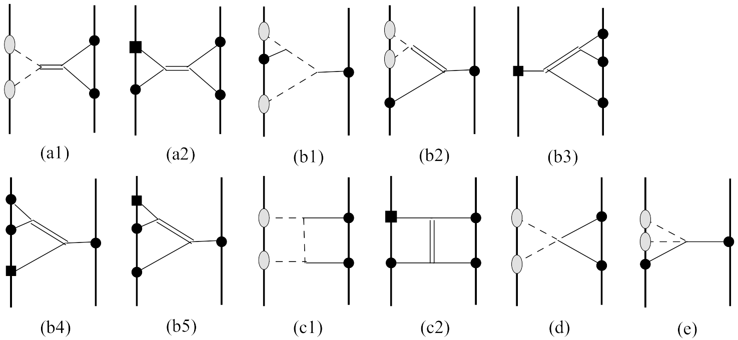
Feynman diagrams of LO quartic in spin sector



- On the left and right – quadrupole-quadrupole and hexadecapole-monopole
- Each of which is analogous to the LO spin-squared sector
- On the middle – octupole-dipole analogous to the LO spin1-spin2 sector

More new results: NNLO spin-squared sector

Feynman diagrams of order G^3 with two loops



- At NNLO 2-loop diagrams are the most complex
- The five 2-loop topologies divide into 3 kinds
- The irreducible kind – the H topology (c1,c2) – is the nasty one!