Bootstrapping a Lorentz-violating gravity theory

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Gravity & the SME

- Gravity sector of Standard Model Extension **(SME)**:

$$S = \int d^4x \sqrt{-g}\Big(\underbrace{R + \Lambda}_{\text{E-H}} + \underbrace{\nabla\psi^{\cdots}\nabla\psi^{\cdots} - V(\psi^{\cdots})}_{\text{Lorentz-violating field}} + \underbrace{uR + s^{ab}R_{ab} + t^{abcd}C_{abcd}}_{\text{Lorentz-violating couplings}}\Big)$$

  - Potential $V(\psi)$ minimized when $\psi \neq 0$

    ➡ **Spontaneous breaking** of Lorentz symmetry

  - Couplings u, s^{ab}, t^{abcd} depend on "vacuum values" of dynamical tensor fields ψ

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- Most work so far: linearized perturbations
  - Background solution: Minkowski space, constant LV tensor ψ
    - Static-field analysis: Bailey & Kostelecký (2006)
    - Wave analysis: Tasson & Kostelecký (2015); Mewes & Kostelecký (2016)
  - Exception: recent work by Y. Bonder

Bailey & Kostelecký: PRD **74**, 045001 (2006)
Tasson & Kostelecký: PLB **749**, 551 (2015)
Mewes & Kostelecký: PLB **757**, 510 (2016)

"Bootstrapping" LI gravity

- 1950s: "bootstrapping" GR from linear field theory
  - Central idea: **gravity gravitates**

Kraichnan, Gupta, Feynman, Thirring, Papapetrou, ...

- Start w/ linear **Lorentz-invariant (LI)** massless rank-2 tensor field

$$S = \int d^4x\, \mathcal{P}^{abcdef}\, \partial_a h_{bc} \partial_d h_{ef}$$

$$\mathcal{P}^{abcdef} = \eta^{a(b}\eta^{c)d}\eta^{ef} + \eta^{a(e}\eta^{f)d}\eta^{bc} - \eta^{a(b}\eta^{c)(e}\eta^{f)d} - \eta^{a(e}\eta^{f)(b}\eta^{c)d} - \eta^{ad}\eta^{bc}\eta^{ef} + \eta^{ad}\eta^{b(e}\eta^{f)c}$$

- Add coupling term between h_{ab} & its own stress-energy
  - Resulting ∞ series sums up to Einstein-Hilbert action

- Standard "bootstrap" construction assumes **Lorentz invariance (LI)**
    - **What happens when we relax this?**
- Sub-questions:

1) What kinds of propagators $\mathcal{P}^{abcdef}$ can we construct?
2) Can "bootstrap" procedure be extended to **Lorentz-violating (LV)** models?
3) Does the procedure constrain the LV field dynamics?

Constructing the LV propagator

Symmetries of the propagator

- Linear model for free "massless" tensor field:

$$S = \int d^4x \, \mathcal{P}^{abcdef} \partial_a h_{bc} \partial_d h_{ef}$$

$$\Rightarrow \quad \mathcal{E}^{bc} \equiv \mathcal{P}^{abcdef} \partial_a \partial_d h_{ef} = 0$$

- Symmetric under $b \leftrightarrow c, e \leftrightarrow f, a \leftrightarrow d, \{abc\} \leftrightarrow \{def\}$
- Eventually want to couple to conserved stress-energy T^{bc}

$$\partial_b \mathcal{E}^{bc} = 0 \quad \Rightarrow \quad \mathcal{P}^{abcdef} k_a k_b k_d = 0 \, \forall \, k_a$$

Constructing the LV propagator

- **Plan of attack:** build $\mathcal{P}^{abcdef}$ from simpler tensors
- Lorentz-invariant **(LI)**: build from η^{ab} only
    - Unique propagator (conventional $(\mathcal{P}_{LI})^{abcdef}$)
- Lorentz-violating **(LV)**: build from η^{ab} + tensor field
    - Vector field A^a or AS rank-2 tensor field B^{ab}
    - Unique propagator in both cases: $(\mathcal{P}_{LV})^{abcdef}$
    - In both cases, equiv. to LI propagator w/ "effective metric": e.g. $\eta^{ab} \to \tilde{\eta}^{ab} \equiv \eta^{ab} + \xi A^a A^b$

➡ **No gravitational birefringence**

Non-linear LV gravity

LI "bootstrap" procedure

- Write down **first-order** linear theory w/ correct EOMs
  - Use perturbed **metric density** $\mathfrak{h}^{ab}$ instead of h^{ab}

$$S = \int d^4x \left[2\mathfrak{h}^{ab} \partial_{[c} \Gamma^c{}_{b]a} + 2\eta^{ab} \left(\Gamma^c{}_{d[c} \Gamma^d{}_{a]b} \right) + \mathcal{L}_{\text{mat}} \left(\eta, \psi, \partial\psi \right) \right]$$

- Couple $\mathfrak{h}^{ab}$ to (trace-reversed) stress-energy
  - Matter terms may require ∞ series

Deser: Gen. Rel. Grav. **1**, 9 (1970)
Kostelecký & Potting: PRD **79**, 065018 (2009)

$$S \to S + \int d^4x \mathfrak{h}^{ab} \left[2\Gamma^c{}_{d[c} \Gamma^d{}_{a]b} + (\tau_{\text{mat}})_{ab} \right]$$

- Recombine to get **Palatini action** for GR:

$$S = \int d^4x \left[\mathfrak{g}^{ab} R_{ab} \left[\Gamma \right] + \mathcal{L}_{\text{mat}} \left(\mathfrak{g}, \psi, \partial\psi \right) \right] \qquad \mathfrak{g}^{ab} = \eta^{ab} + \mathfrak{h}^{ab}$$

LV ~~LI~~ "bootstrap" procedure

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- Recombine to get **Palatini action** for GR:

$$S = \int d^4x \left[\tilde{\mathfrak{g}}^{ab} R_{ab} [\Gamma] + \mathcal{L}_{\text{mat}} \left(\mathfrak{g}, \psi, \partial\psi \right) \right]$$

$$\mathfrak{g}^{ab} = \eta^{ab} + \mathfrak{h}^{ab}$$
$$\tilde{\mathfrak{g}}^{ab} = \tilde{\eta}^{ab} + \mathfrak{h}^{ab}$$

metric in grav. action ≠ metric for matter

Conclusions

- Answers to earlier questions:

1) Using simple LV tensors, can change metric for linear wave propagation (but no polarization effects)

2) Models w/ differing “matter metric” & “gravity metric” can be bootstrapped, so long as…

3) … the flat-space Lagrangian for the LV field can be successfully bootstrapped

- Work is ongoing!

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