PRIMORDIAL FLUCTUATIONS IN QUANTUM COSMOLOGY

[In coll. with L. Castelló Gomar and G.A. Mena Marugán]

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INTRODUCTION

- Planck data indicates **possible anomalies** at large angular scales

- They could be the result of **genuine QG effects** that would have affected the power spectrum of primordial perturbations

$\longrightarrow$ **Quantum Cosmology**

- Realistic hopes of confronting QC with future observations

- Tool: **Effective Mukhanov-Sasaki equations** for cosmological perturbations over FRW, that incorporate **QG corrections**

- Framework: **hybrid LQC**

CLASSICAL SYSTEM

[Halliwell, Hawking]

- **Flat** homogeneous and isotropic **FRW** cosmology, with compact spatial sections

- We include a **scalar field** subject to a potential (e.g. mass term)

- For simplicity, we analyze only **scalar perturbations**

- We expand the fields in (real) **Fourier modes**

- We truncate the action at **quadratic perturbative order**

- Symplectic structure for entire system: zero-modes + perturbations

- Zero-mode of scalar constraint + linear perturbative constraints

  contains corrections quadratic in perturbations

GAUGE-INVARIANT FORMULATION

[L. Castelló Gomar, M. M-B, G.A. Mena Marugán, JCAP 06, 045, 2015]

- We find a canonical transformation to express the system in terms of **gauge-invariants** (at our truncation order), both for the **perturbations** and for the **zero-modes**

- **Zero-modes**: that of the scalar fiel (ϕ) , and of the geometry (v)

$$\{\phi, \pi_\phi\} = 1 \qquad\qquad \{b, v\} = 2$$

- **Perturbations**: Modes of the Mukhanov-Sasaki field and **abelianized** linear perturbative constraints.

$$\{v_{\vec{n}}, \pi_{v_{\vec{n}'}}\} = \delta_{\vec{n},\vec{n}'} \quad , \quad \cdots$$

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- **Zero-modes**: that of the scalar fiel (ϕ) , and of the geometry (v)

$\longrightarrow$ LQC representation

- **Perturbations**: Modes of the Mukhanov-Sasaki field and **abelianized** linear perturbative constraints.

$\longrightarrow$ Fock quantization

$\boxed{\textbf{Hybrid LQC}}$

QUANTUM CONSTRAINTS

- We represent the **linear perturbative constraints** (or an integrated version of them) as derivatives (or as translations)

- We pass to a space of states $\mathcal{H}_{\text{hom}} \otimes \mathcal{F}$, that depend on the **zero-modes** and the **Mukhanov-Sasaki modes**, with **no gauge fixing**

- Physical states still must satisfy the scalar constraint

$$\boxed{\hat{\mathcal{C}} = \hat{\mathcal{C}}^{(0)} - \hat{\Theta}_e - \frac{1}{2}\left(\hat{\Theta}_o \hat{\pi}_\phi + \hat{\pi}_\phi \hat{\Theta}_o\right)}$$

$$\hat{\mathcal{C}}^{(0)} = \hat{\pi}_\phi^2 - \frac{3\pi G}{4}\hat{\Omega}^2 - 2W(\hat{\phi})\hat{V}^2$$

represents $(vb)^2$ potential homogeneous volume

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$$\hat{\Theta}_e = \sum_{\vec{n}} \left[(\hat{\vartheta}_e\, \omega_n^2 + \hat{\vartheta}_e^q)\hat{v}_{\vec{n}}^2 + \hat{\vartheta}_e \hat{\pi}_{v_{\vec{n}}}^2 \right] \qquad \hat{\Theta}_o = \sum_{\vec{n}} \hat{\vartheta}_o \hat{v}_{\vec{n}}^2$$

Eigenvalue of the Laplacian M-S modes operators of the homogeneous FRW geometry and of $W(\hat{\phi})$

MODIFIED M-S EQUATIONS

- **Born-Oppenheimer ansatz**: Consider states whose dependence on the FRW geometry and the inhomogeneities ($\mathcal{N}$) **split**

$$\Psi = \chi(v,\phi)\psi(\mathcal{N},\phi)$$

- Considering that we can disregard transitions from χ to other FRW states, and treating the perturbations as classical:

$$\boxed{d^2_{\eta_\chi} v_{\vec{n}} = -\left[\omega_n^2 + \frac{\langle \hat{\vartheta}_e^q + (\hat{\vartheta}_o \hat{h}_0)_{sym} + \frac{1}{2}[\hat{\pi}_\phi - \hat{h}_0, \hat{\vartheta}_o]\rangle_\chi}{\langle \hat{\vartheta}_e \rangle_\chi}\right] v_{\vec{n}}}$$

State-dependent conformal time

Master equation to predict QG modifications to observables

FRW PHYSICAL STATES

- Evolution picture: Scalar plays the role of time , Hamiltonian

$$\chi(v,\phi) = \hat{U}(v,\phi)\chi_0(v) \quad , \quad \hat{U}(v,\phi) = \mathcal{P}\left[\exp\left(i\int_{\phi_0}^{\phi} d\tilde{\phi}\,\hat{h}_0(v,\tilde{\phi})\right)\right]$$

$$\left(\hat{\mathcal{C}}^{(0)}\chi \simeq 0 \quad \rightarrow \quad -\partial_\phi^2\chi = \hat{h}_0^{(2)}\chi \quad , \quad \hat{h}_0^{(2)} = \frac{3\pi G}{4}\hat{\Omega}^2 + 2W(\phi)\hat{V}^2\right)$$

- $\hat{h}_0$ is (close to) the square root of $\hat{h}_0^{(2)}$

- **Main obstacle**: dealing with the **quantum evolution** of the **FRW states,** since the **Hamiltonian** is **time-dependent**
(due to non-constant field potential)

QUANTUM EVOLUTION OF FRW STATES

[L. Castelló Gomar, M. M-B, G.A. Mena Marugán, Phys.Rev. D93, 104025, 2016]

- To retain **quantum corrections** up to the **maximum practical extent** beyond usual effective or semiclassical approximations:

1. From the generator of the FRW dynamics, extract its **free geometric part** (vanishing potential) $\longrightarrow$ **Interaction image**

$$\hat{h}_1 = \hat{h}_0 - \hat{h}_0^F \quad , \quad \hat{h}_0^F = \sqrt{\frac{3\pi G}{4}\hat{\Omega}^2}$$

$$\chi_I(v,\phi) = e^{-i\hat{h}_0^F(\phi-\phi_0)}\chi(v,\phi) \quad , \quad \hat{A}_I = e^{-i\hat{h}_0^F(\phi-\phi_0)}\hat{A}e^{i\hat{h}_0^F(\phi-\phi_0)}$$

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2. Integrate explicitly this free evolution (sLQC)

$$\hat{A} \to \hat{A}_I(\phi)$$

$$\langle \hat{A}(\phi) \rangle_\chi = \langle \hat{A}_I(\phi) \rangle_{\chi_I} = \langle \hat{U}_I^\dagger(\phi) \hat{A}_I(\phi) \hat{U}_I(\phi) \rangle_{\chi_0}$$

generated by $\hat{h}_{1I}$

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3. Extract the **dominant contribution** of the scalar field **potential**

$\longrightarrow$ **New interaction image** $\quad \hat{h}_{1I} - \hat{h}_{2I} = \hat{h}_{3I} \longleftarrow$ remaining part

dominant contribution in the field potential (regarded as a perturbation)

$$\hat{A}_J = \hat{U}_{2I}^\dagger \hat{A}_I \hat{U}_{2I} \quad , \quad \langle \hat{A}_I \rangle_{\chi_I} = \langle \hat{U}_J^\dagger \hat{A}_J \hat{U}_J \rangle_{\chi_0}$$

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5. Treat **semiclassically** the **remaining interaction** terms $\left(\hat{h}_{3J}\right)$

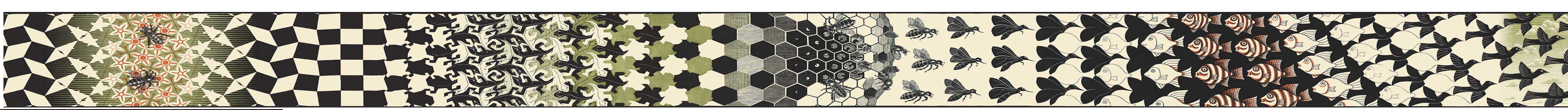
CONCLUSIONS

- We have considered a flat FRW universe with a scalar field perturbed at **quadratic** order in the action

- Formulation for the **full system** that respects **covariance** at the perturbative level of truncation, and hybrid quantization of it.

- Effective **Mukhanov-Sasaki equations** include quantum corrections encoded in expectation values of quantum operators on the state that describe the FRW geometry

- We have proposed a number of steps to **compute** those **expectations values**, by permitting some approximations while still **retaining** a significant part of their **quantum features**



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OUTLOOK

- Study the **consequences of quantum corrections** to the MS equations in the power spectrum of the **CMB**

- Validity of the semiclassical and/or effective approximation adopted in previous studies, and **reveal quantum phenomena** that might have been ignored

- The final hope is that **confrontation of predictions** with cosmological observations may eventually falsify the theoretical models

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THANK YOU FOR YOUR ATTENTION

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