Asymptotic Safety

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Nature's fundamental interactions

- electro-magnetic, weak, strong interactions

<u>class.:</u> Yang-Mills type gauge theories

<u>quantum:</u> perturbatively renormalizable
local quantum field theories

- gravity

<u>class.:</u> General Relativity

<u>quantum:</u> effective field theory,
not renormalizable within perturbation theory

$\rightsquigarrow$ Quantum corrections computable
(no <u>conceptual</u> clash !!),
but has little / no predictive power
at short distances:

increasing order of perturbation theory $\Rightarrow$
" number of counter terms $\Rightarrow$
" " " undetermined parameters

Beyond effective QFT : options

- Leave the framework of Quantum Field Theory:
  LQG, Spin Foams, String Theory, ...

- Stay within (non-perturbative!) QFT:

Asymptotic Safety

continuum approach:

Functional Renormalization Group Equation (FRGE),

Effective Average Action

statistical mech. appr.:

Dynamical Triangulations,
Regge calculus, ...

modern Wilsonian concept of
renormalization and
non-perturbative renormalizability

Asym. safe Quantum Einstein Gravity

- degrees of freedom carried by $g_{\mu\nu}(x)$
- quantization / renormalization is non-perturbative in an essential way
- bare ("classical") action is not an assumption but a prediction:

  $\int d^4x \sqrt{-g}\, R$ + more $\sim$ non-Gaussian fixed point of a functional (∞ dim.!) RG flow
- UV limit taken at NGFP $\Longrightarrow$

  non-perturbative renormalizability despite perturbative non-renormalizability
- predictive at shortest distances:

  NGFP "tames" plethora of parameters that would be undefined in effective theories

The fundamental problem:

Give a meaning to ("define", "renormalize", "take the continuum limit of", ...) a functional integral over all metrics on a space time $\mathcal{M}$:

$$\int \mathcal{D}\hat{g}_{\mu\nu}\, e^{-S[\hat{g}_{\mu\nu}]}$$

S: diff $(\mathcal{M})$ - invariant bare action,

e.g. S_{EH} + counter terms

$$\mathcal{D}\hat{g}_{\mu\nu} \equiv \prod_{x\in\mathcal{M}} \prod_{\mu,\nu} d\hat{g}_{\mu\nu}(x)$$

requires regularization (UV Cutoff)

1. *Effective Average Action and its Functional Renormalization Group Equation*

2. *The idea of Asymptotic Safety*

3. *"Quantum Einstein Gravity": results and general properties*

The Effective Action $\Gamma[\Phi]$

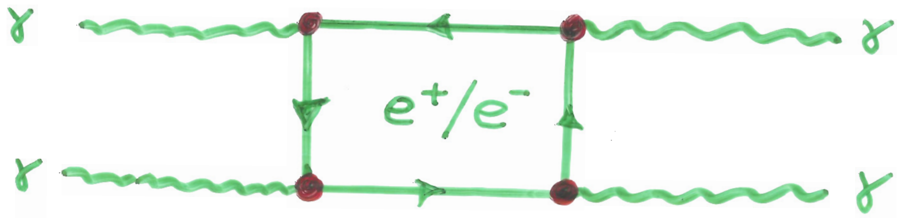
Q: How do the not directly observable vacuum fluctuations (virtual particle-/antiparticle creation, ...) manifest themselves in the dynamics of the observed particles?

A: Classical field equation $\quad \delta S[\Phi] = 0$

$\longrightarrow$ effective eq. $\quad \delta \Gamma[\Phi] = 0$

$\langle 0 | \hat{\Phi} | 0 \rangle$

E.g.:



$$\Gamma[\vec{E}, \vec{B}] = \int d^4x \left\{ \frac{1}{2} \left(\vec{E}^2 - \vec{B}^2 \right) \right.$$

Maxwell

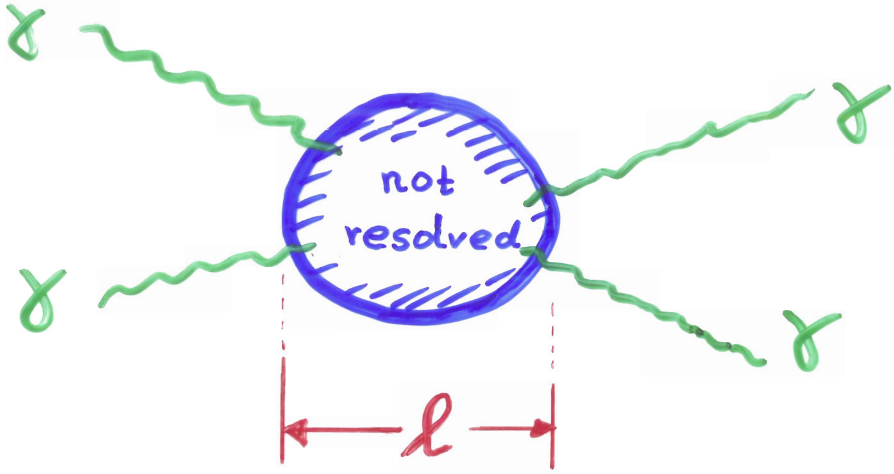
$$+ \frac{2\alpha^2}{45 m^4} \left[\left(\vec{E}^2 - \vec{B}^2 \right)^2 + 7 \left(\vec{E} \cdot \vec{B} \right)^2 \right]$$

$$\left. + \dots \right\}$$

Heisenberg-Euler

The Effective Average Action $\Gamma_k[\Phi]$

C. Wetterich, MR, ≥ 1990



Q: How does the effective dynamics of the observed particles depend on the resolving power $k \equiv 1/\ell$ of the "microscope" used in the experiment ?

A: Scale-dependent eff. field equation

$$\delta \Gamma_k [\Phi] = 0$$

$\Phi \equiv$ average* of fundamental field over Euclidean ball of radius $\ell = 1/k$

*: in the sense of Feynman's functional integral

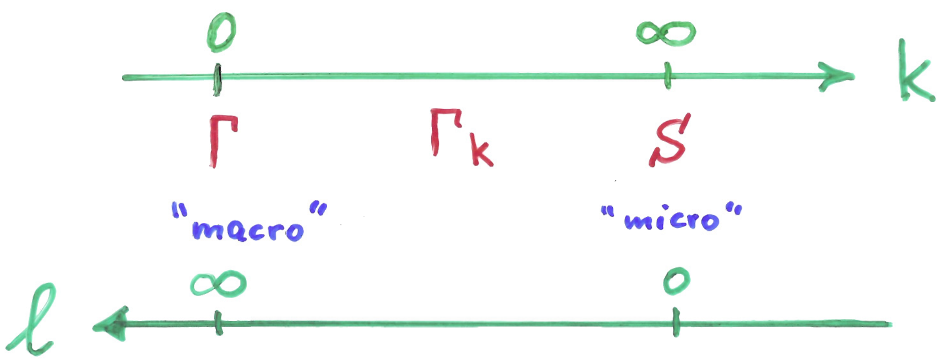
$$\Rightarrow \quad \lim_{k \to 0} \Gamma_k = \Gamma \qquad \text{standard eff. act.}$$

$\hat{=} \quad \ell \to \infty$

- In a theory with the classical action S, the Effective Average Action satisfies the Functional Renormalization Group Equation

$$\frac{d}{dk}\Gamma_k[\Phi] = \frac{1}{2}\,\mathrm{STr}\left[\left(\frac{\delta^2\Gamma_k[\Phi]}{\delta\Phi^2} + R_k\right)^{-1}\frac{dR_k}{dk}\right]$$

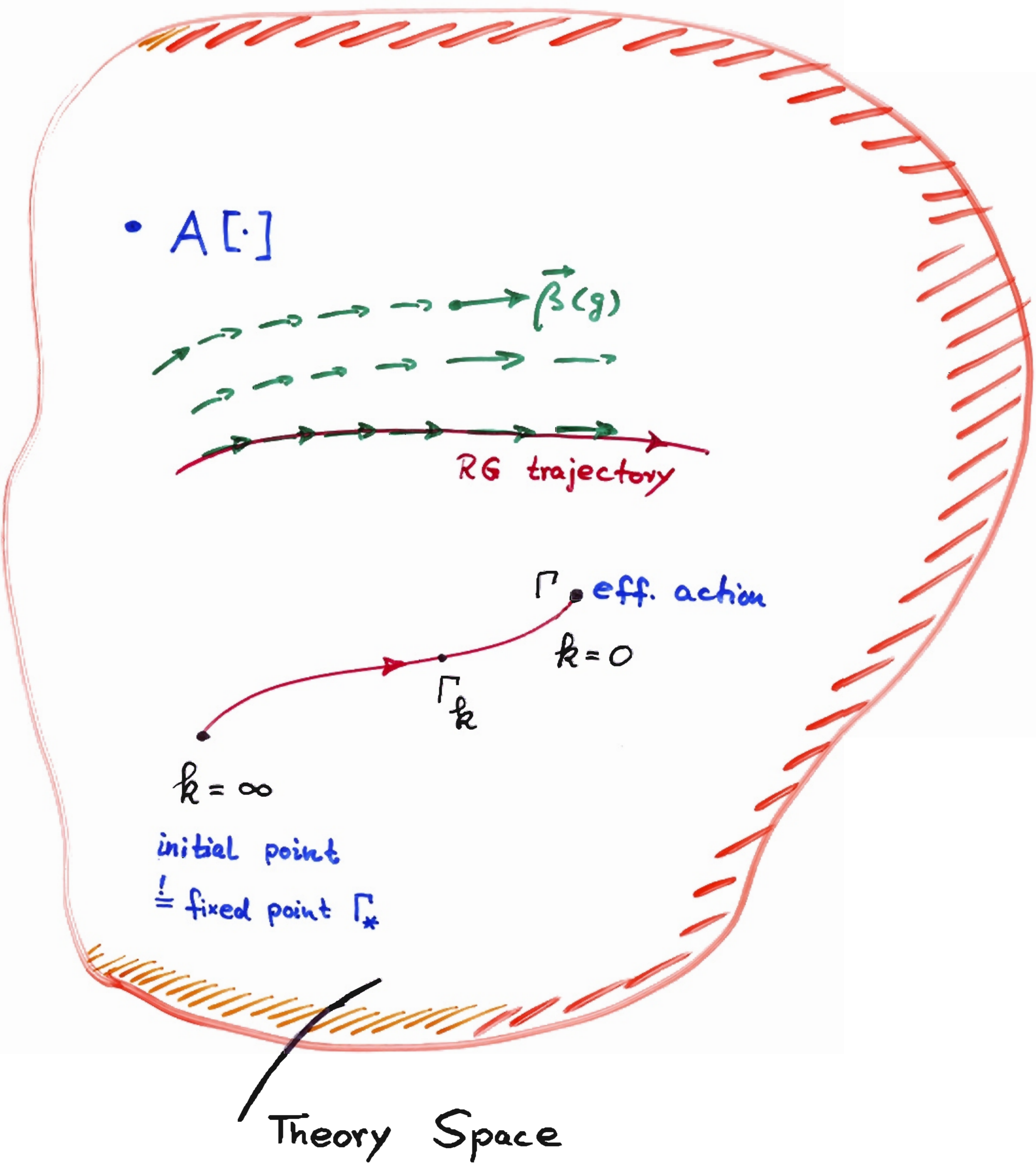
with initial condition $\Gamma_{k=\infty} = S$.

- 

- "Knowing Γ is knowing everything"

$\Longrightarrow$ "Quantizing" / "solving" a theory becomes an evolution problem on the infinite-dimensional Theory Space $\equiv$ $\{$ all possible action functionals $A[\Phi]$ $\}$

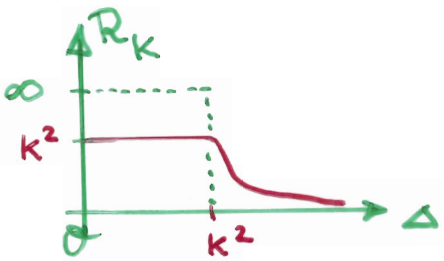
- Interpret solution $k \mapsto \Gamma_k$ as a curve on theory space: "RG trajectory".



• A[·]
$\vec{\beta}(g)$
RG trajectory
Γ eff. action
$k = 0$
Γ_k
$k = \infty$
initial point
$\overset{!}{=}$ fixed point Γ_*
Theory Space

The Effective Average Action

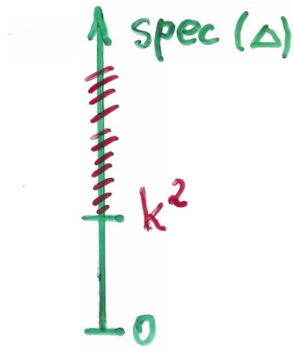
$$e^{W_k[J]} := \int \mathcal{D}\hat{\phi}\, e^{-S[\hat{\phi}]} \cdot e^{\int dx\, J\hat{\phi}} \cdot e^{-\frac{1}{2}\int \hat{\phi} R_k(\Delta)\hat{\phi}}$$



suppresses low eigenvalue ("low momentum", "large wave length") eigen-modes of Δ :

IR cutoff at (mass) scale $k \in [0, \infty)$

$$\Gamma_k[\phi] := \left(\text{Legendre transf. of } W_k[J]\right) - \frac{1}{2}\int \phi R_k(\Delta)\phi$$



Interpolating property:

$$\Gamma \xleftarrow{k\to 0} \Gamma_k \xrightarrow{k\to\infty} \sim S$$

ordinary eff. action — bare action

Functional Renormalization Group Equation (FRGE):

$$\partial_k \Gamma_k = \frac{1}{2}\operatorname{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k\right]$$

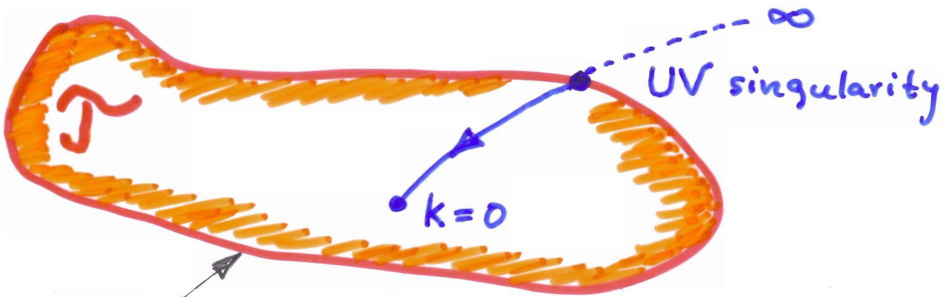
UV and IR finite !

$$\int \mathcal{D}\hat{\phi}(\cdot)\, e^{-S[\hat{\phi}(\cdot)]} = \text{" sum " over fields with arbitrarily high momenta :}$$

$$\hat{\phi}(x) = \int d^4p \; a(p)\, e^{ipx}$$

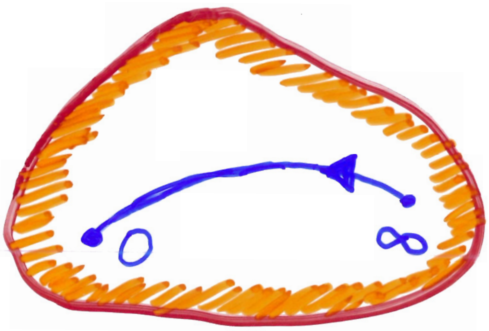
$$\int \mathcal{D}\hat{\phi}(\cdot) \;\hat{=}\; \prod_{p \in \mathbb{R}^4} \int_{-\infty}^{\infty} da(p)$$

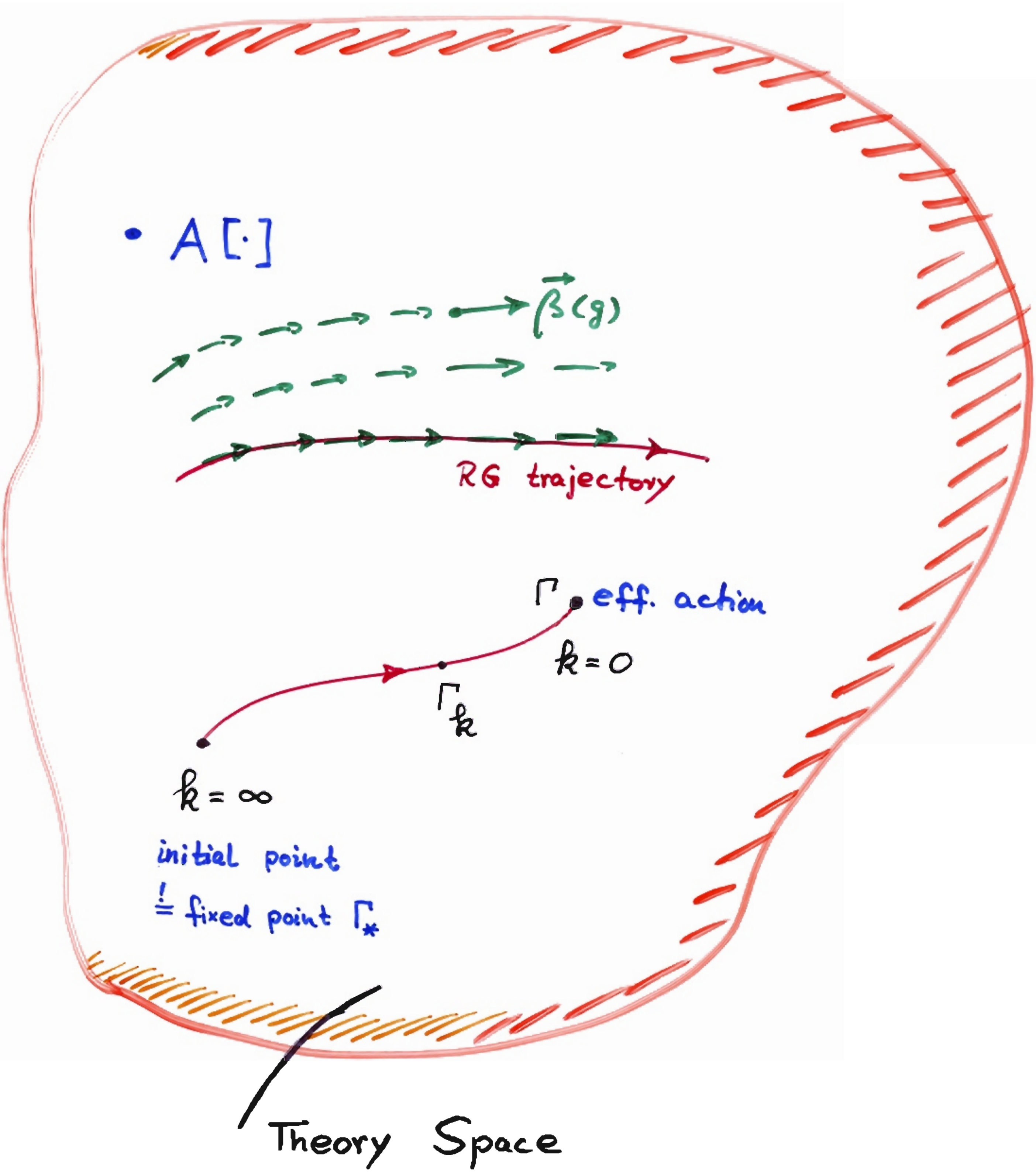
UV renormalization problem



boundary of space of "well behaved" action functionals

$$\boxed{\text{fundamental QFT}} \longleftrightarrow \boxed{\text{fully extended RG trajectory: } \Gamma_k\,,\; 0 \leq k < \infty\,.}$$





• A[·]
β(g)
RG trajectory
Γ eff. action
k=0
Γk
k = ∞
initial point
≟ fixed point Γ*
Theory Space

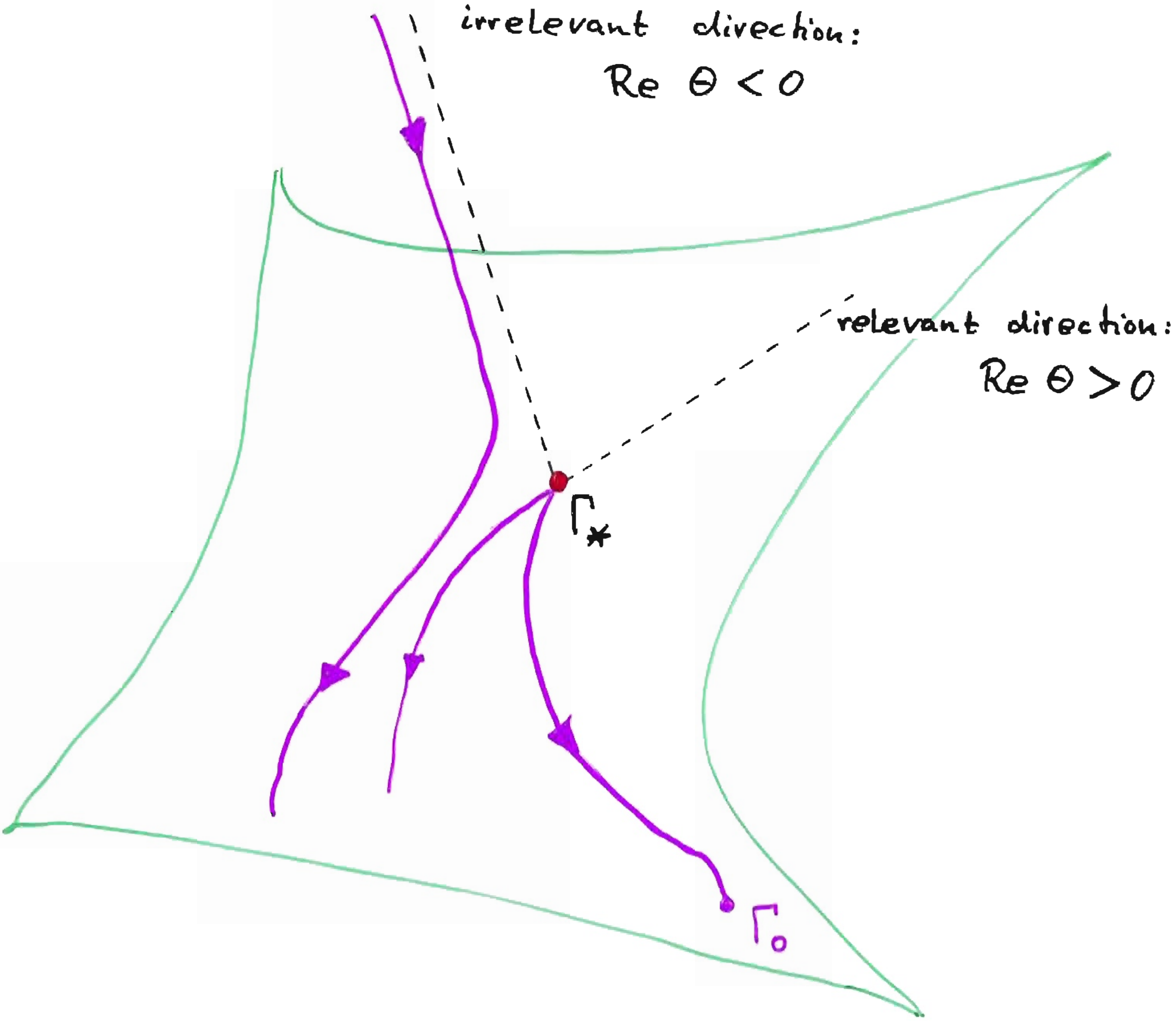
The Asymptotic Safety conjecture

S. Weinberg, 1979

- The RG flow of metric quantum gravity possesses a non-trivial fixed point $\Gamma_* \subset \mathcal{T}$, i.e. $\beta(\Gamma_*) = 0$, with a low-dimensional UV-critical hypersurface.

- All RG trajectories which do <u>not</u> hit Γ_* for $k \to \infty$ will leave $\mathcal{T}$ ultimately.

The UV-critical hypersurface $\mathcal{S}_{UV}$:



$\Delta_{UV} \equiv \dim \mathcal{S}_{UV}$ = # relevant directions

= # free parameters in the a.s. quantum field theory

UV → IR

Θ : critical exponent (neg. eigenvalue of lin. flow)

The Einstein-Hilbert Truncation

MR, 1996

Ansatz:

$$\Gamma_k = -\frac{1}{16\pi G_k} \int d^dx \sqrt{g} \left(R - 2\Lambda_k\right)$$

+ classical gauge fixing and ghost terms

Running coupling constants:

Newton constant G_k, dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant Λ_k, dimensionless: $\lambda(k) = k^{-2} \Lambda_k$

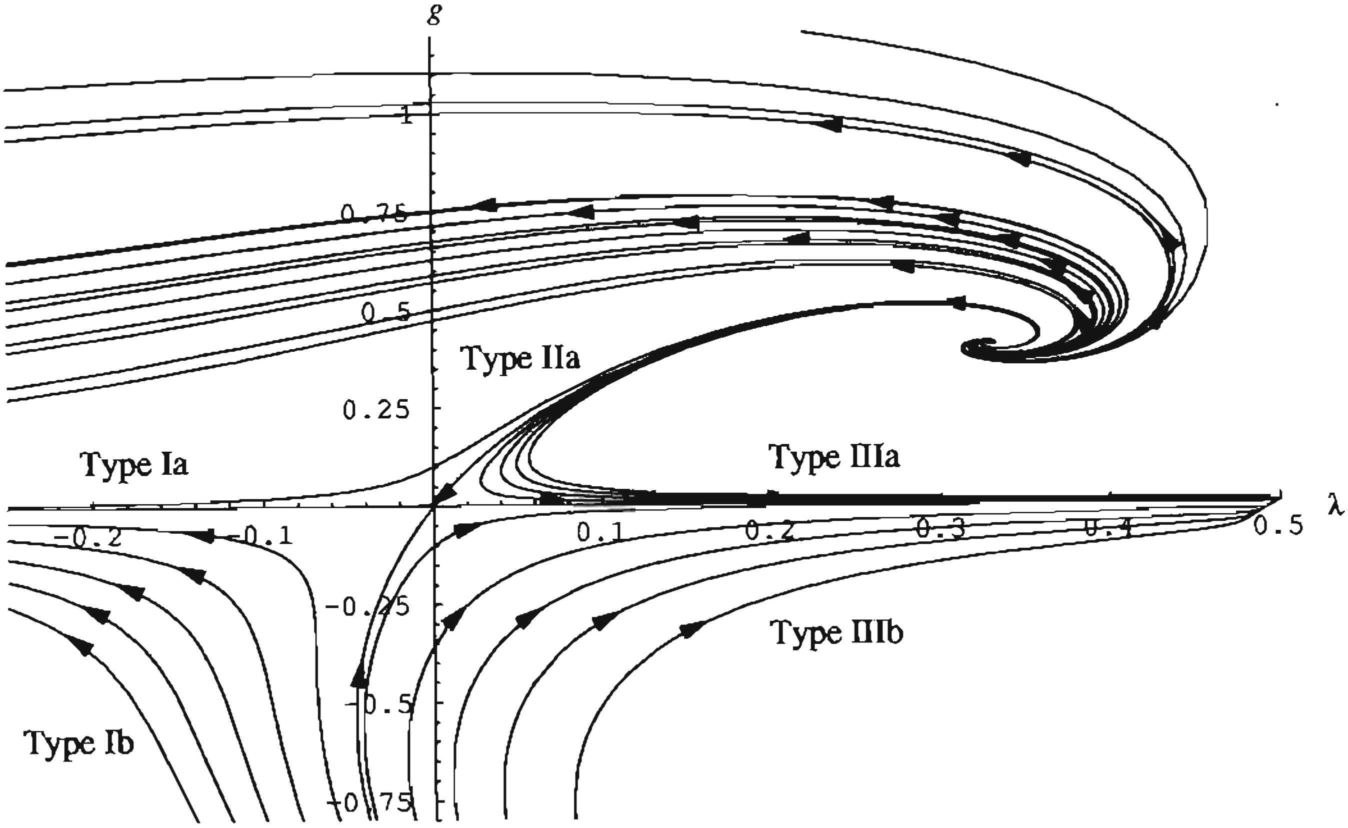
Insert ansatz into FRGE, "project out" monomials retained:

$$\mathrm{Tr}[\cdots] = (\cdots) \int \sqrt{g} + (\cdots) \int \sqrt{g} R + \cdots \quad \Rightarrow$$

$$\boxed{\begin{aligned} k\,\partial_k g(k) &= \beta_g(g,\lambda) \\ k\,\partial_k \lambda(k) &= \beta_\lambda(g,\lambda) \end{aligned}}$$

Einstein - Hilbert Truncation :

RG Flow on the $g-\lambda$ plane



M.R., F. Saueressig, hep-th/0110054

Agrees with the $g - \lambda$ projection from all generalized truncations investigated:

Quantum Einstein Gravity is

most likely asymptotically safe,

i. e. non-perturbatively renormalizable!

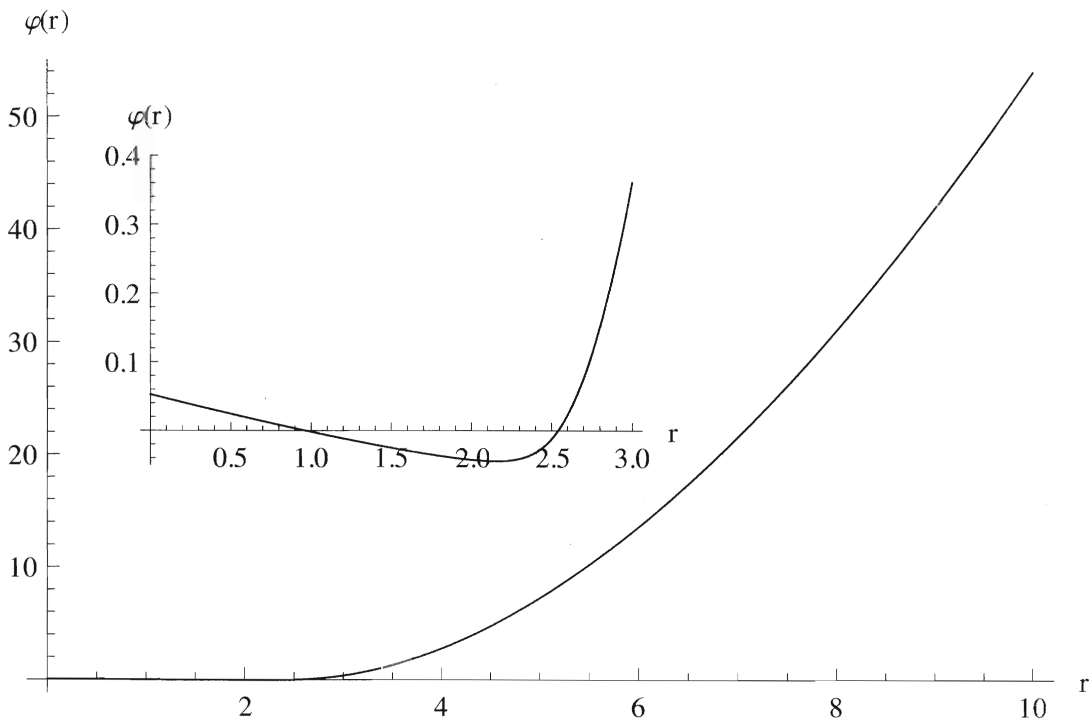
An ∞-dimensional truncation :

$$\Gamma_k = \int d^4x \sqrt{g}\, F_k(R) + \cdots$$

$$F_k(R) =: k^4 \varphi_k(r)$$

$$r := R/k^2$$

Has NGFP with fixed function $\varphi_*(r)$:



Demmel, Saueressig, Zanusso (2015)

$r \ll 1$: $\varphi_*(r) = c_1 - c_2\, r + \cdots$ $\approx$ Einstein-Hilbert

$r \gg 1$: $\varphi_*(r) \sim r^2$

The Key requirement taken over from classical GR:

Background Independence

No metric should be singled out and play a special role in setting up the quantum theory.

"Vacuum" must arise dynamically:

$$g_{\mu\nu} = \langle \Psi | \hat{g}_{\mu\nu} | \Psi \rangle$$

Derive, rather than postulate, the "arena" of all non-gravitational physics (Minkowski space, etc.).

Background independence via background fields

De Witt, 1963

Background – quantum field split:

$$\boxed{\hat{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \hat{h}_{\mu\nu}}$$

$\hat{g}_{\mu\nu}$: integration variable (operator)

$\bar{g}_{\mu\nu}$: backrd. metric (left completely arbitrary)

$\hat{h}_{\mu\nu}$: fluctuation, quantized on classical space time with metric $\bar{g}_{\mu\nu}$

background independence $\widehat{=}$ Quantization of $\hat{h}_{\mu\nu}$ on all possible backgrounds "at a time"

- Expectation values functionally $\bar{g}$-dependent:

$$\langle \hat{g}_{\mu\nu} \rangle_{\bar{g}} = \bar{g}_{\mu\nu} + \langle \hat{h}_{\mu\nu} \rangle_{\bar{g}} \equiv \bar{g}_{\mu\nu} + h_{\mu\nu}[\bar{g}]$$

- Self-consistent backgrounds:

$$h_{\mu\nu}[\bar{g}^{\,\text{self-con}}] = 0 \,, \quad \langle \hat{g}_{\mu\nu} \rangle_{\bar{g}^{\,\text{self-con}}} = \bar{g}^{\,\text{self-con}}_{\mu\nu}$$

Split symmetry: $\delta \bar{g}_{\mu\nu} = \varepsilon_{\mu\nu} \,, \quad \delta \hat{h}_{\mu\nu} = - \varepsilon_{\mu\nu}$

broken at intermediate steps of the quantization

$$\int \mathcal{D}\hat{g}\; e^{-S[\hat{g}]} \longrightarrow$$

$$\int \mathcal{D}\hat{h}\; e^{-S[\bar{g}+\hat{h}]} \cdot \int \mathcal{D}(\text{ghosts})\; e^{-S_{g.f.} - S_{ghost}} \cdot e^{-\Delta S_k}$$

<u>Analyze by means of the corresponding Eff. Av. Action</u> :

$$g_{\mu\nu} \equiv \langle \hat{g}_{\mu\nu} \rangle = \bar{g}_{\mu\nu} + \langle \hat{h}_{\mu\nu} \rangle \equiv \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\boxed{\Gamma_k[h, \text{ghosts}; \bar{g}] \equiv \Gamma_k[g, \bar{g}, \text{gh.}]\Big|_{g=\bar{g}+h}}$$

$\Delta S_k \sim \int \hat{h}_{\mu\nu}\, \mathcal{R}_k(-\bar{D}^2)^{\mu\nu\rho\sigma}\, \hat{h}_{\rho\sigma}$, and

$S_{gf} + S_{gh.}$ spoil invariance under split transf. !

$\Longrightarrow$ Control split symmetry violation / restoration by Ward identities:

$$\frac{\delta}{\delta \bar{g}_{\mu\nu}} \Gamma_k[g, \bar{g}, \cdots] = \mathrm{Tr}[\cdots]$$

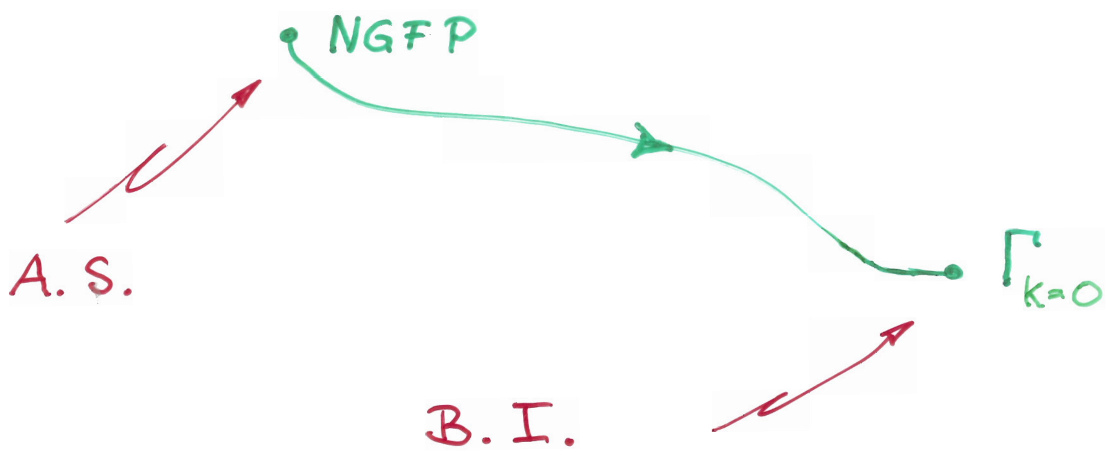
"extra" backgrd. dependence

Can Background Independence coexist with Asymptotic Safety ?

Are there RG trajectories which

(i) emanate from a NGFP in the UV

(ii) restore split symmetry in the physical sector for $k \to 0$?



Truncations of theory space

$$\mathcal{T} = \left\{ A[g, \bar{g}, C, \bar{C}] \;\middle|\; \begin{array}{l} \text{backgrd. gauge invariant,} \\ \text{satisfy Ward identities of} \\ \text{BRST and split symmetry} \end{array} \right\}$$

"single metric truncations": ~ 1996 — 2009

$$\Gamma_k = \Gamma_k[g] + \int \left(F(\bar{g})(g-\bar{g}) \right)^2 + \int \bar{C} M(\bar{g}) C$$

"bi-metric" truncations include e.g.

$$\Gamma_k = \underbrace{\Gamma_k[g, \bar{g}]}_{\equiv \Gamma_k[h; \bar{g}]} + \int \left(F(\bar{g})(g-\bar{g}) \right)^2 + \int \bar{C} M(\bar{g}) C$$

$$\equiv \Gamma_k[h; \bar{g}]$$

$$= \sum_{p=0}^{\infty} \frac{1}{p!} \int \gamma_k[\bar{g}]^{(p)} \cdot h \cdot h \cdot \dots \cdot h$$

Einstein-Hilbert type bimetric truncation

Manrique, MR, Saueressig, 2010

D. Becker, MR 2014

$$\Gamma_k[g,\bar{g}] = -\frac{1}{16\pi G_k^{Dyn}} \int \sqrt{g}\, \{ R(g) - 2\Lambda_k^{Dyn} \}$$

$$- \frac{1}{16\pi G_k^{B}} \int \sqrt{\bar{g}}\, \{ R(\bar{g}) - 2\Lambda_k^{B} \}$$

expand

$$\equiv -\frac{1}{16\pi G_k^{(0)}} \int \sqrt{\bar{g}}\, \{ R(\bar{g}) - 2\Lambda_k^{(0)} \}$$

$$+ \frac{1}{16\pi G_k^{(1)}} \int \sqrt{\bar{g}}\, [\bar{G}^{\mu\nu} + \Lambda_k^{(1)} \bar{g}^{\mu\nu}]\, h_{\mu\nu}$$

$$+ \frac{1}{16\pi G_k^{(2)}} \int \sqrt{\bar{g}}\, h [\cdots \bar{D}^2 \cdots] h \quad + O(h_{\mu\nu}^3)$$

4-dim. theory space $\ni (g^{Dyn}, \lambda^{Dyn}, g^B, \lambda^B)$

split-symmetry (WISS, leading order) $\Longleftrightarrow \left(\frac{1}{G_k^B} = 0 \, , \, \frac{\Lambda_k^B}{G_k^B} = 0 \right)$

$$\Longleftrightarrow \left(G_k^{(0)} = G_k^{(1)} = G_k^{(2)} = \cdots \, , \quad \Lambda_k^{(0)} = \Lambda_k^{(1)} = \cdots \right)$$

Broken during RG evolution; try to restore in the physical limit $(k \to 0)$.

Result :

Becker, 2015

- There exists a NGFP $(g_*^{Dyn}, \lambda_*^{Dyn}, g_*^{B}, \lambda_*^{B})$ which is suitable for the Asymptotic Safety construction.

- The <u>4-parameter</u> family of trajectories leaving the NGFP has a <u>2-parameter</u> subset of trajectories which restore split symmetry in the IR.

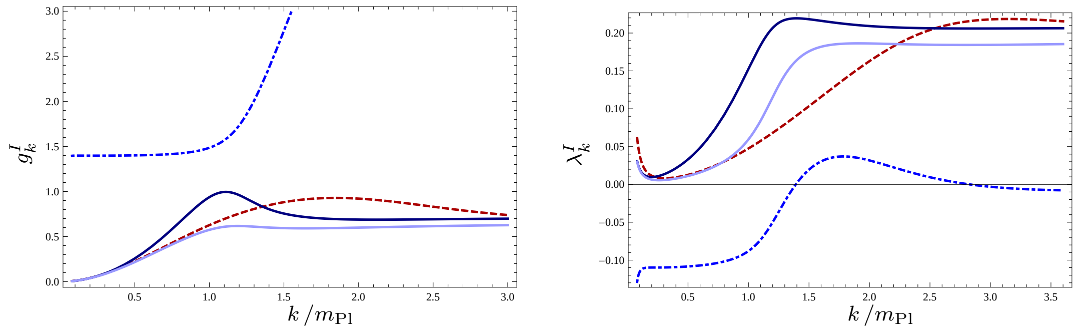


Figure 12. Type (IIIa)$^{\mathrm{Dyn}}$-**(Attr)**$^{\mathbf{B}}$trajectory: dimensionless couplings.

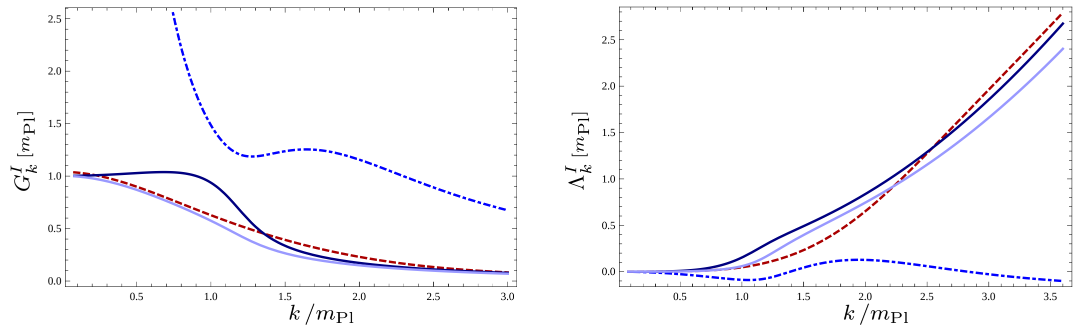


Figure 13. Type (IIIa)$^{\mathrm{Dyn}}$-**(Attr)**$^{\mathbf{B}}$trajectory: dimensionful couplings.

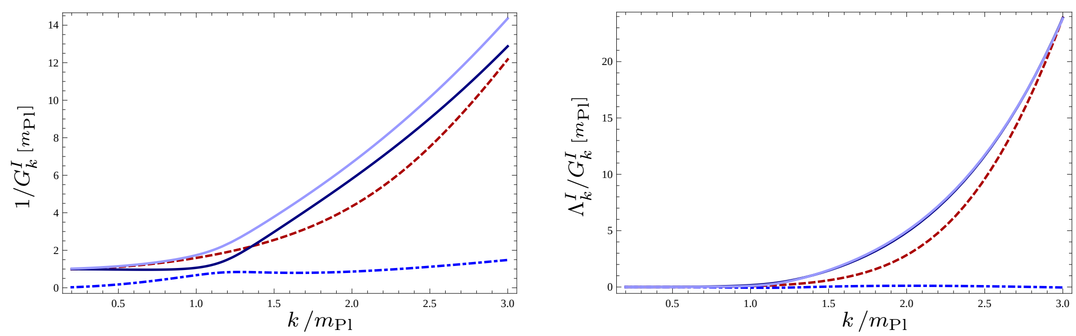


Figure 14. Type (IIIa)$^{\mathrm{Dyn}}$-**(Attr)**$^{\mathbf{B}}$trajectory: the coefficients as they appear in the EAA. Note again the vanishing $1/G_k^{\mathrm{B}}$ and $\Lambda_k^{\mathrm{B}}/G_k^{\mathrm{B}}$, indicative of split-symmetry restoration in the limit $k \to 0$.

dashed (red):	$I = \mathrm{sm}$ (single-metric)
solid (dark-blue):	$I = \mathrm{Dyn} \equiv (p)$ for $p \geq 1$
solid (light-blue):	$I = (0)$
dot-dashed (blue):	$I = \mathrm{B}$

Summary

By now we have highly significant evidence indicating that there does exist an asymptotically safe theory of *Quantum Einstein Gravity* **which is well defined and predictive on the shortest possible length scales, and also complies with the other basic physical principles (background independence, ...).**

While non-perturbative concepts play a crucial role, there seems to be no reason for a radical departure from the framework of Quantum Field Theory —which has been so successful in the case of the other 3 fundamental forces of Nature!

Frequent objections: Too conservative,

Not radical enough, …

Recall however how modern QED took shape at the Shelter Island conference:

"*…the conference brought together theorists who had in their own individual ways been thinking about renormalization…*

When the revolution came in the late 1940s, it was made by physicists who, though mostly young, were playing a <u>conservative</u> role, turning away from the search by their predecessors for a <u>radical</u> solution."

(S. Weinberg, QFT, Vol.1)