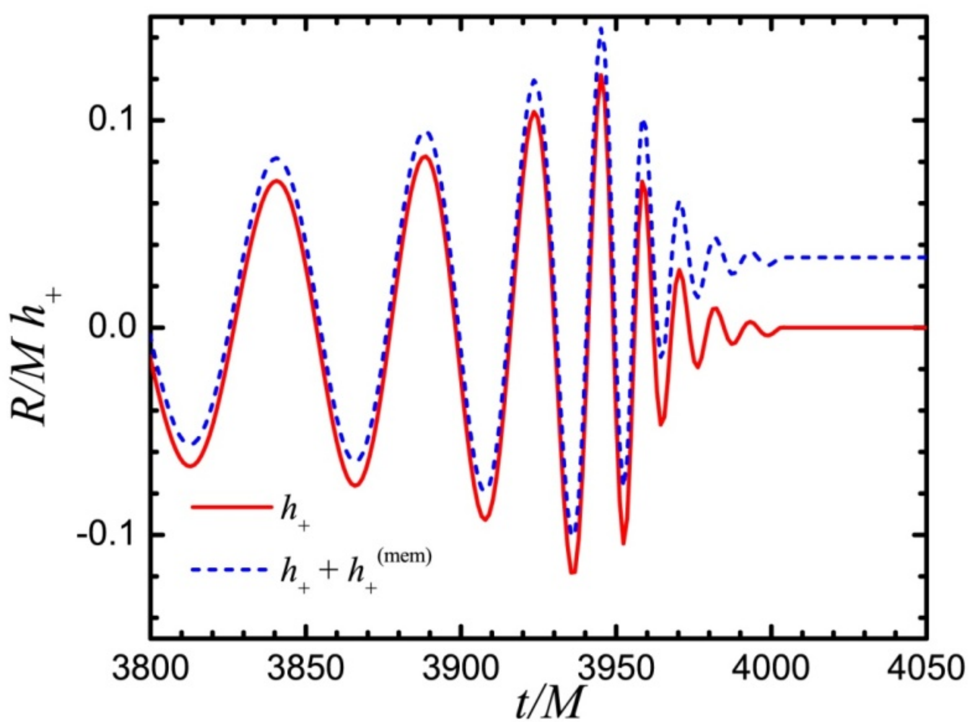
Modeling & detectability of the nonlinear gravitational-wave memory



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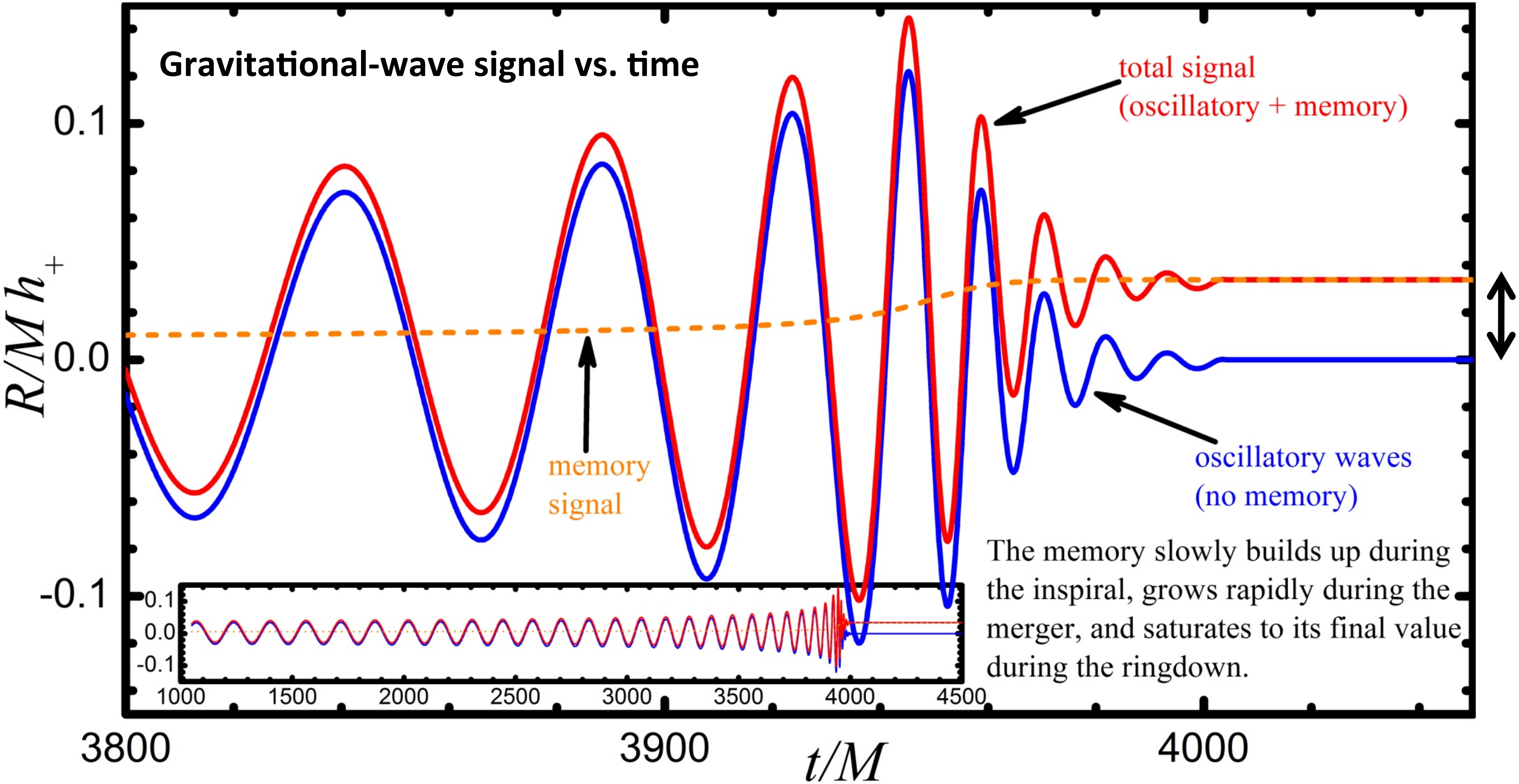
Xinyi Guo (Harvard)

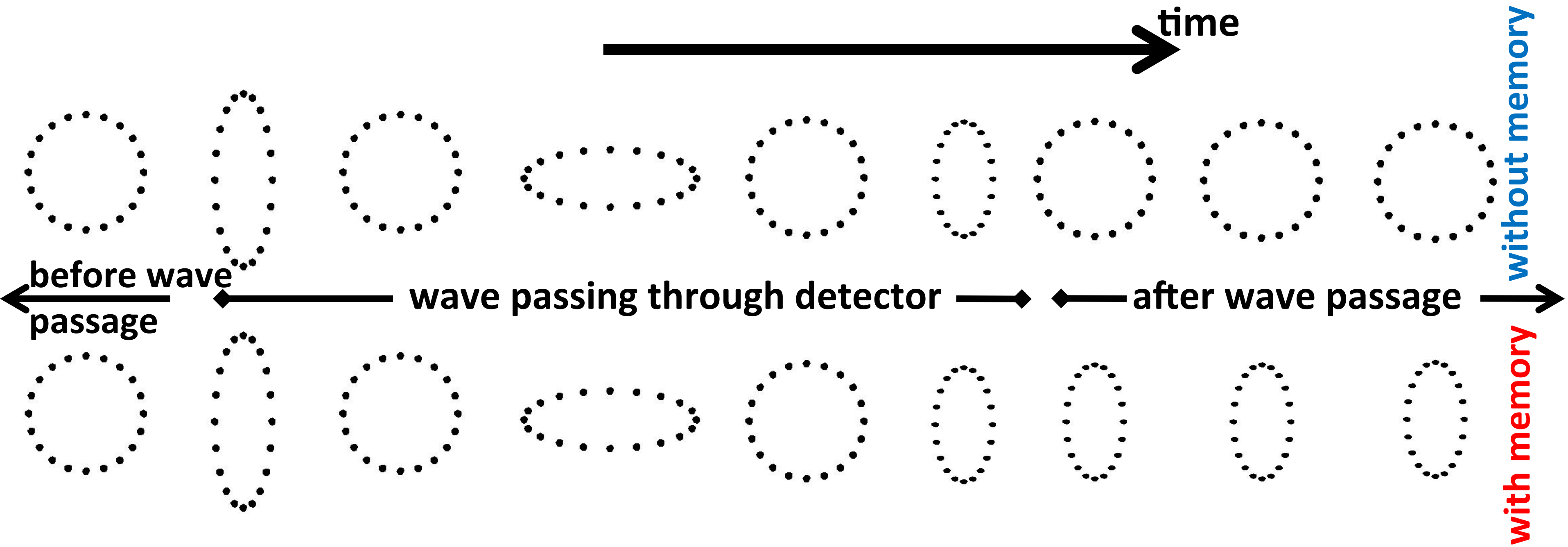
What is the gravitational-wave memory?

example: nonlinear memory from binary black-hole mergers



The wave no longer returns to the zero-point of its oscillation.
This growing-offset is called the **memory**.

Why is this called "memory"?

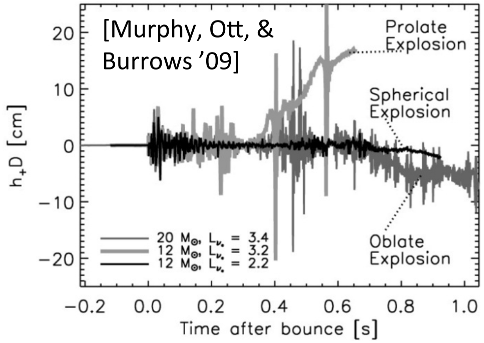


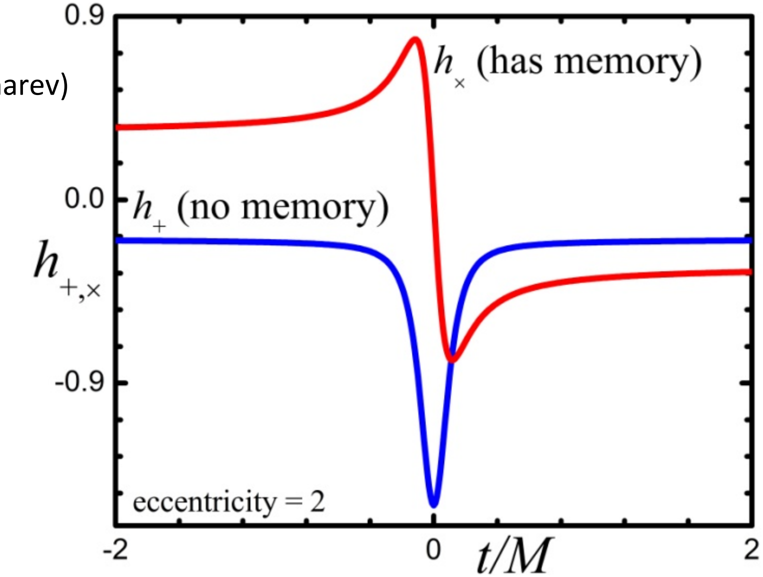
(gravitational-waves propagating into the screen)

The linear and nonlinear memory:

Linear memory: (Braginskii, Grishchuck, Thorne, Zeldovich, Polnarev)

- Arises from the non-oscillatory motion of a source, especially due to unbound masses.
- Ex: hyperbolic orbits, mass/neutrino ejection in supernovas/GRBs





Nonlinear memory: (Christodoulou, Blanchet, Damour)

- Arises from the GWs produced by GWs:

$$T^{jk}_{\rm GW} = \frac{1}{R^2}\frac{dE_{\rm GW}}{dt d\Omega} n_j n_k$$

$$\Box \bar{h}^{jk} = -16\pi(-g)(T^{jk} + T^{jk}_{\rm GW}[\bar{h},\bar{h}]) + O(\bar{h}^2)$$

- "Unbound particles" are the individual "radiated gravitons". [Thorne '92]
- Produced by all sources of GWs.
- Allows us to probe one of the most nonlinear features of GR.

Previous work:

Previous nonlinear memory calculations (inspiralling binaries):

✓ 0PN inspiral, circular, nonspinning: Wiseman & Will ’91

✓ 3PN inspiral, circular, nonspinning: MF ’09a

✓ 0PN inspiral, eccentric, nonspinning: MF ’11

✓ merger/ringdown, nonspinning, equal-mass: MF ’09b, ‘10

✓ merger/ringdown, aligned-spins, equal-masses: Pollney & Reisswig ‘11

✓ crude detectability estimates: MF ‘09b, MF’11, Pollney & Reisswig ’11

✓ estimates of recoil-induced memory and QNM Doppler shift: MF ’09c

✓ pulsar timing studies/searches: Seto ‘09, van Haasteren & Levin ‘10, Pshrikov et al’10, Cordes & Jenet ‘12, Madison, Cordes, Chatterjee ‘14, Wang et al ’15, Arzoumanian et al ’15

[See also mathematical aspects of memory addressed at this conference: Garfinkle, Tolish.]

Motivation for this work:

Part I: *spin* corrections to inspiral memory waveform

- Memory builds up slowly through the inspiral + merger/ringdown. Correct inspiral description is needed as initial condition to the NR piece of the memory. (Extend previous work to spinning case.)
- *Complete* spinning PN waveform amplitude to 1.5PN order.

Part II: memory from merger/ringdown of *nonspinning* black holes

- Need accurate model of entire coalescence to model/detect memory.
- Nonlinear memory difficult to compute with NR simulations.
- Use non-memory modes from SXS waveform catalog + analytic formula to generate the memory.

Part III: detectability estimates

- Use simple model of nonlinear memory to estimate signal-to-noise ratios for ground-based detectors.

Summary of calculations:

1. Waveform can be expanded in spin-weighted spherical harmonic modes:

$$h_+ - ih_\times = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h_{lm}(T_R, R)\, {}_{-2}Y_{lm}(\Theta, \Phi)$$

2. The nonlinear memory modes are related to the GW energy flux [MF '09a]:

$$h_{lm}^{(\mathrm{mem})} = \frac{16\pi}{R}\sqrt{\frac{(l-2)!}{(l+2)!}} \int_{-\infty}^{T_R} dt \int d\Omega \frac{dE_{\mathrm{GW}}}{dtd\Omega}(\Omega) Y_{lm}^*(\Omega)$$

3. The energy flux is related to the oscillating (non-memory) h_{lm} modes:

$$\frac{dE_{\mathrm{GW}}}{dtd\Omega} = \frac{R^2}{16\pi}\langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle = \frac{R^2}{16\pi} \sum_{l_1,l_2,m_1,m_2} \langle \dot{h}_{l_1 m_1} \dot{h}_{l_2 m_2}^* \rangle\, {}_{-2}Y_{l_1 m_1}\, {}_{-2}Y_{l_2 m_2}^*$$

4. Compute time-derivative of $h_{lm}(v, \boldsymbol{L}, \boldsymbol{S}_1, \boldsymbol{S}_2)$ [Arun et al. '09], substitute eq. of motion & solutions at leading SO order [Blanchet et.al'11], simplify and integrate.
   [For merger/ringdown, substitute non-memory modes from NR simulation, numerically integrate, and match to analytic inspiral.]

Spin-orbit corrections to inspiral memory:

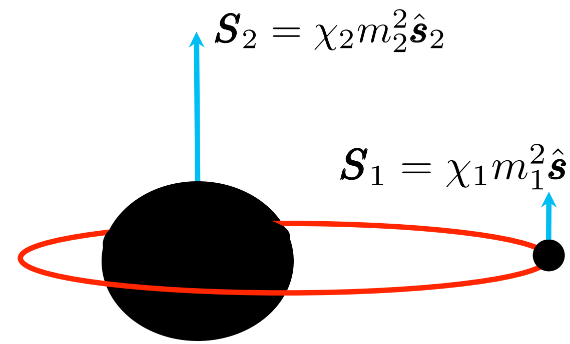
Aligned-spin case: [w/ Xinyi Guo]

$$h_+^{(\mathrm{mem})} = -\frac{2\eta M}{R} v^2 \sin^2\Theta \left[H_+^{0\mathrm{PN,nonspin}} + v^2 H_+^{1\mathrm{PN,nonspin}} + v^3 H_+^{1.5\mathrm{PN,spin}} + \cdots v^6 H_+^{3\mathrm{PN,nonspin}} \right]$$

$$H_+^{0\mathrm{PN,nonspin}} = \frac{17+\cos^2\Theta}{96}$$

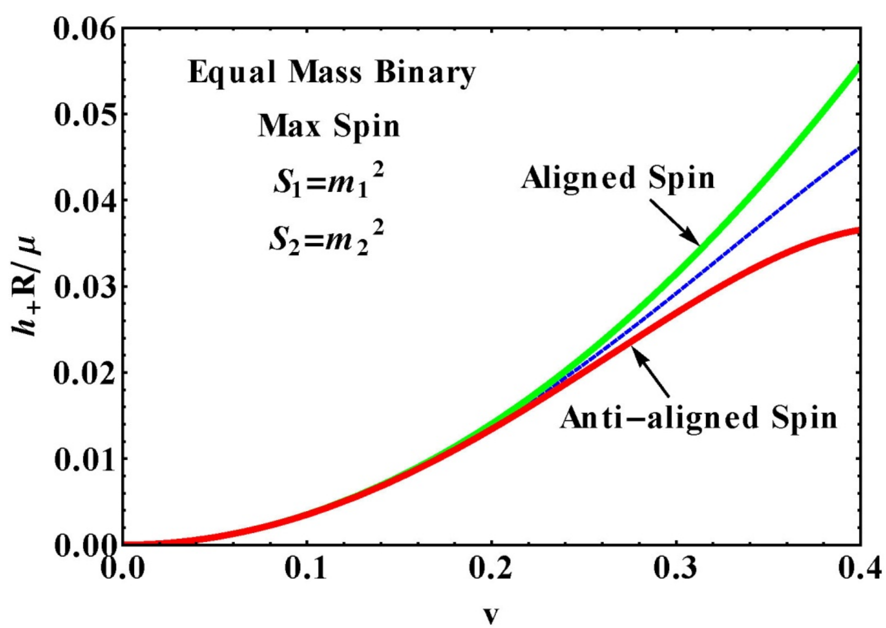
$$H_+^{1\mathrm{PN,nonspin}} = F(\cos^2\Theta, \eta)$$

$$H_+^{1.5\mathrm{PN,spin}} = \frac{1}{768} \sum_{i=1,2} \chi_i \kappa_i \left[369 \frac{m_i^2}{M^2} + 351\eta + \cos^2\Theta \left(23 \frac{m_i^2}{M^2} + 57\eta \right) \right]$$



$$\kappa_i = \hat{\boldsymbol{L}}_N \cdot \hat{\boldsymbol{s}}_i$$

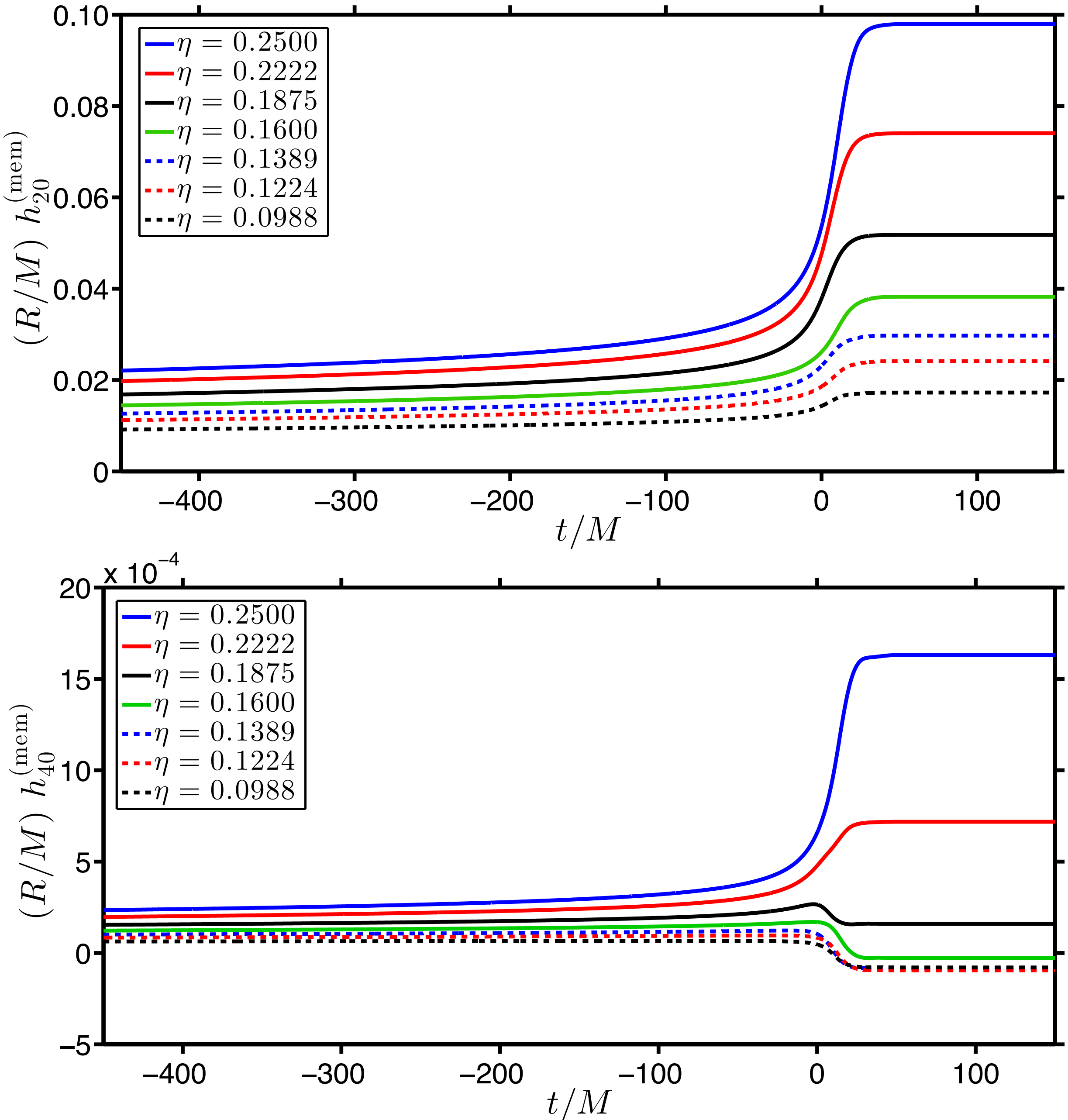
- Spin correction maximized for maximally spinning, aligned binaries.
- Spin terms produce ∼20% maximum correction at Schwarzchild ISCO.
- Small-inclination angle case also computed analytically. (Depends on perpendicular spin components.)
- Generic precessing case computed numerically.



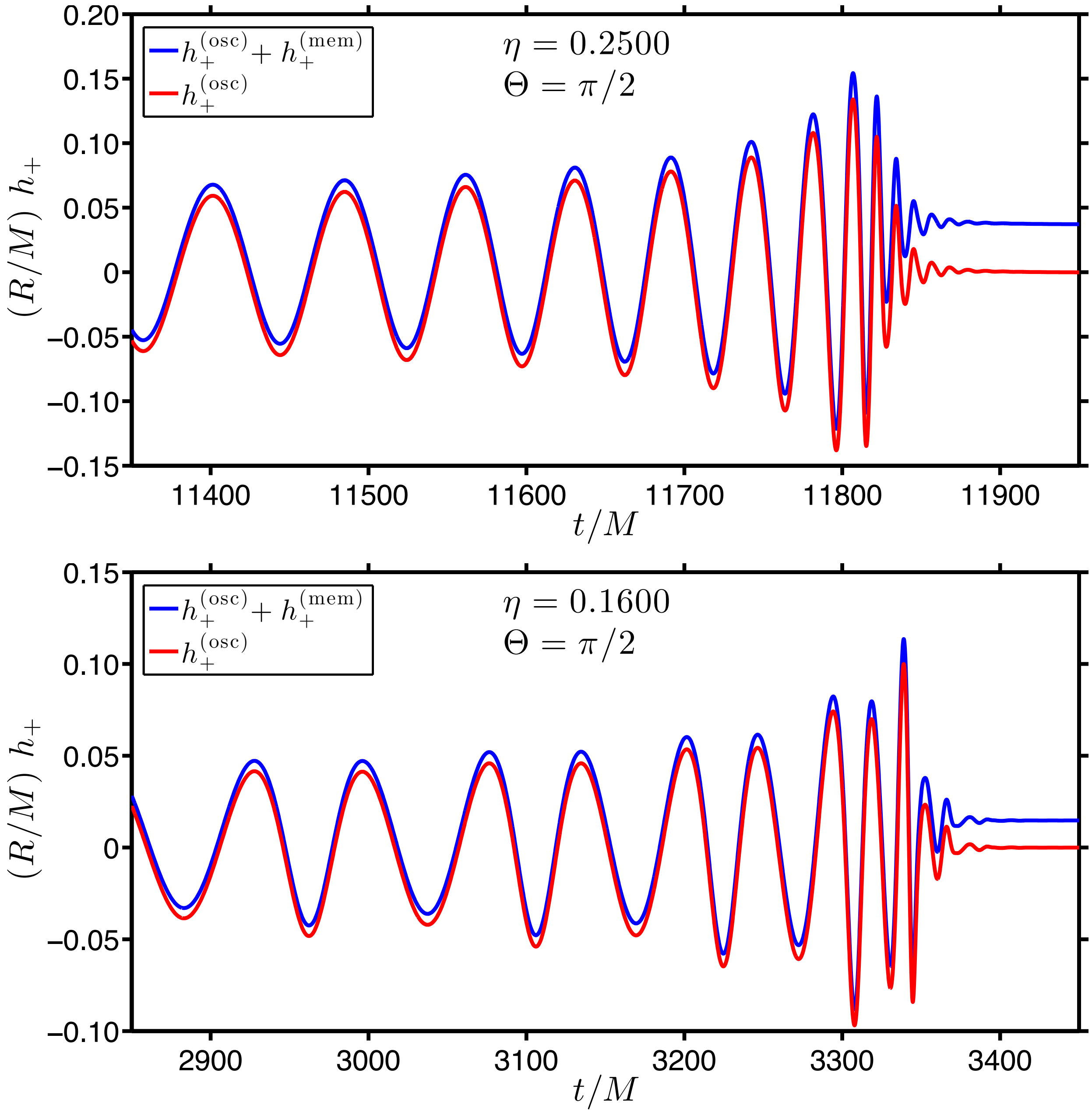
Merger/ringdown memory (nonspinning):

[w/ Goran Dojcinoski]

- Express $m=0$ memory modes in terms of oscillatory modes.
- Use h_{lm} from SXS catalog.
- Match to inspiral memory.



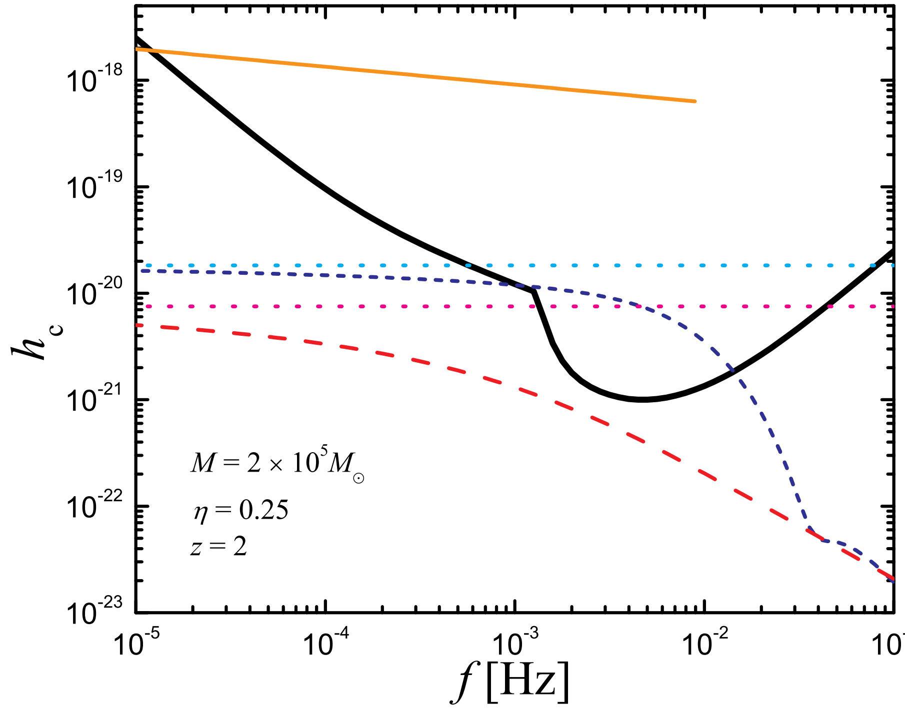
Merger/ringdown memory (nonspinning):

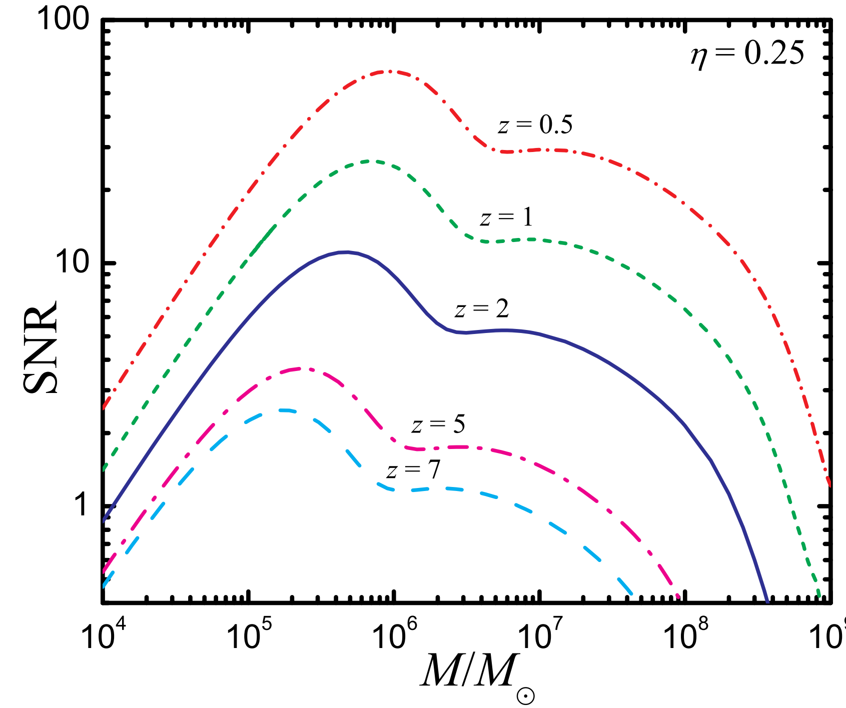


Detectability of memory:

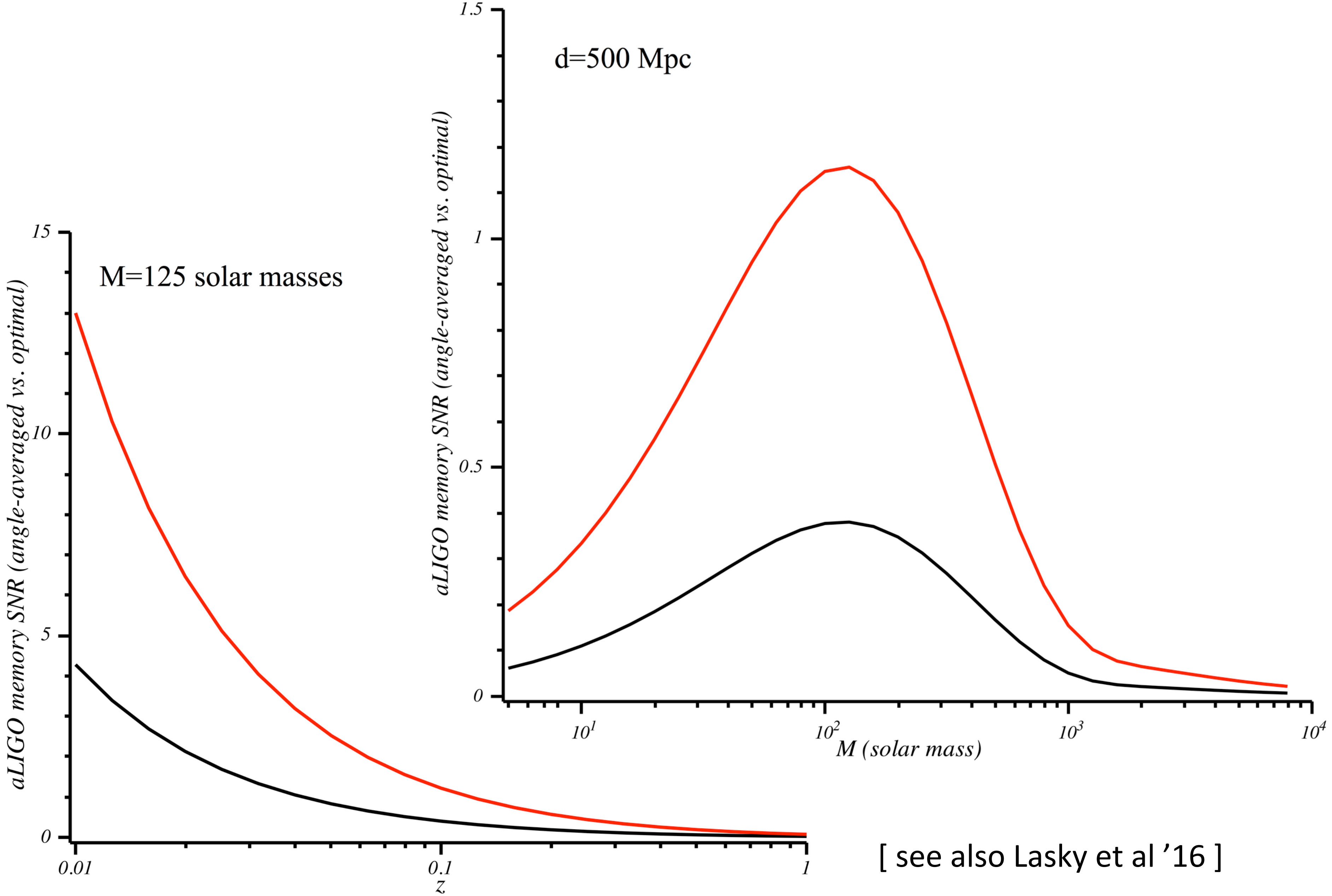
[w/ Emanuele Berti]

- Use analytic model from MF ApJL '09 to compute SNR for equal-mass case (extension to other mass ratios via new waveforms in progress).
- MF ApJL '09 focused on detectability by LISA. (SMBH memory easily seen to z=2.)
- Also estimated aLIGO SNR of 8 for 100 $M_\odot$ binary at 20 Mpc.
- Here we extend the analysis to ground-based detectors...



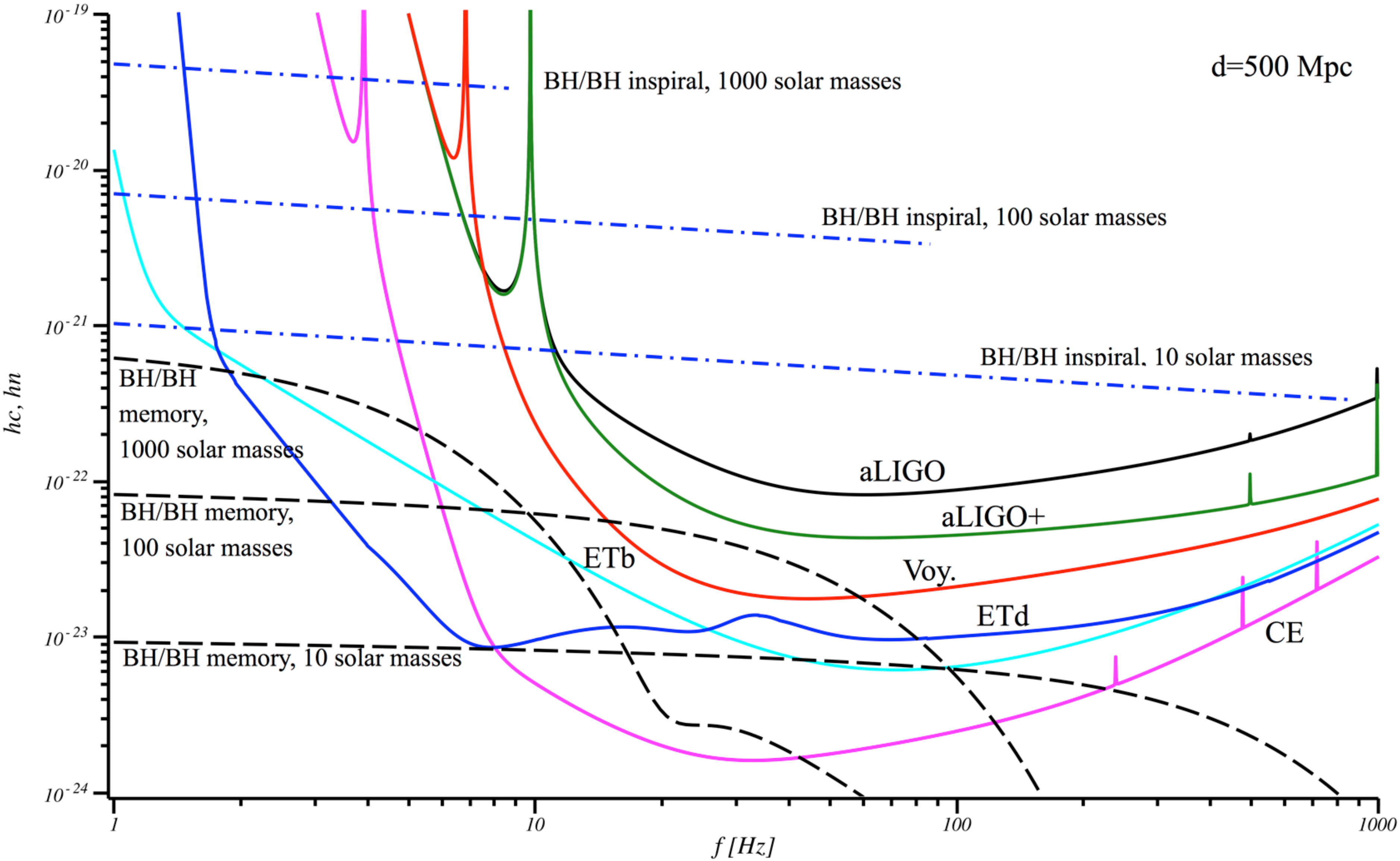


Detectability: aLIGO (preliminary)

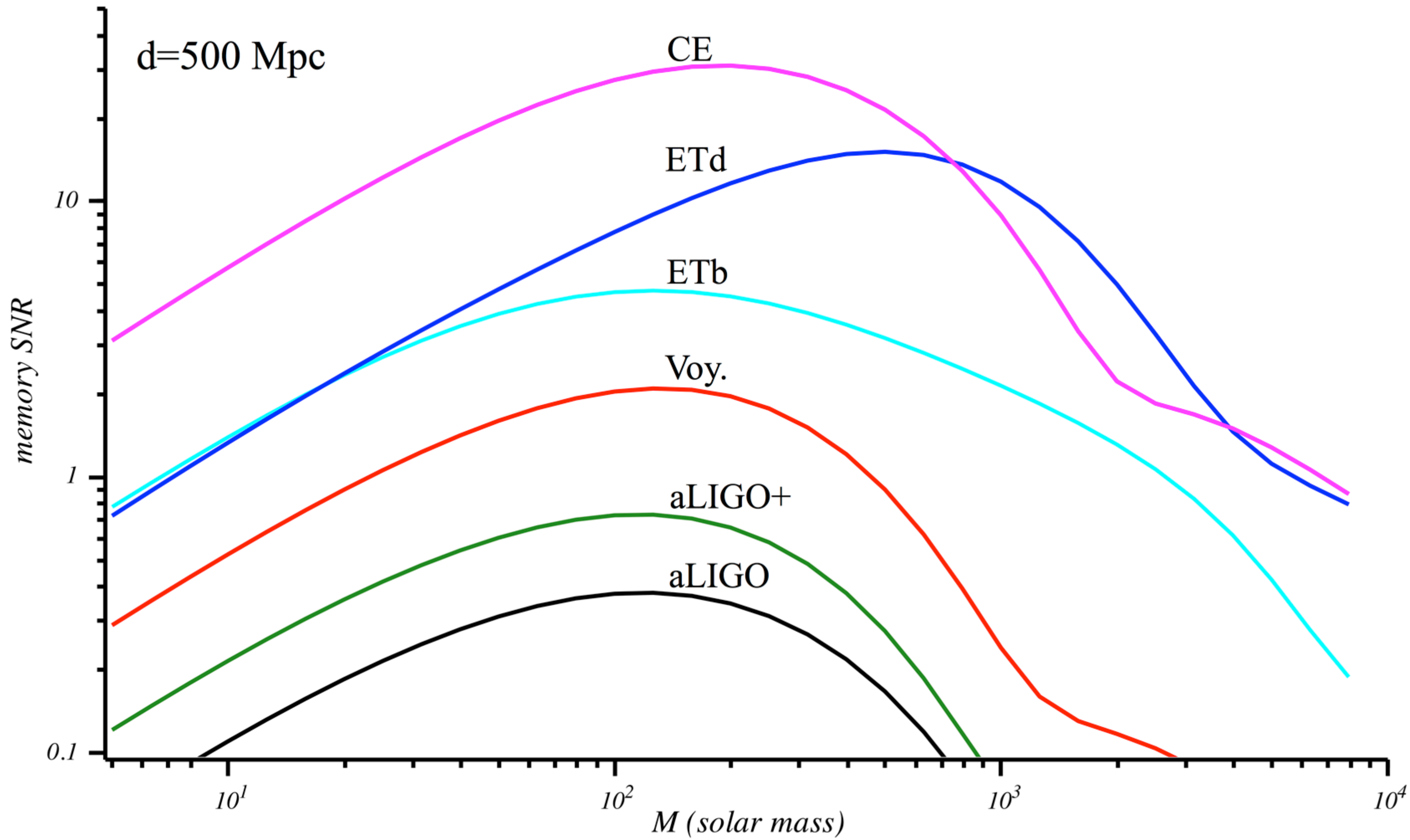


[see also Lasky et al ’16]

Detectability: future ground-based (preliminary)



Detectability: future ground-based (preliminary)



- Good prospects for most sensitive 3rd generation detectors.
- For masses ∼5 to 4000 solar masses, memory SNR ∼O(1%) of inspiral SNR.

Summary & Conclusions:

- The nonlinear memory is a non-oscillatory correction to the GW amplitude that arises from the GWs produced by GWs.

- We computed the effect of the spin-orbit interaction on the inspiral portion of the memory. Spin contributes a maximum correction of ∼20% during the inspiral.

- These analytic inspiral corrections are needed as input for numerical relativity calculations of the memory effect.

- Also computed full memory signal (inspiral+merger+ringdown) for *nonspinning* BH systems (needed for accurate detection estimates).

- Detectability estimates: not great for aLIGO; good for 3rd gen. ground-based detectors and LISA.

[This work supported by NSF Grant No. PHY-1308527.]