Surprises in (strong/non linear) gravity

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prologue

- 100 yrs of General Relativity so far. A very successful gravitational theory built from fundamental principles and described in geometrical terms & governed by a complex PDE system
- Despite its complicated character; powerful geometrical analysis led to extracting deep results like establishing singularity theorems, deriving geometric inequalities, assessing asymptotic structures, etc.
- Specific solutions found in very special situations (under various assumptions) & much progress on linearized solutions off these.
- The above (while in a sense incomplete) has played a key role in many applications (e.g. astrophysics & cosmology)

- Exciting opportunities to test a still enigmatic theory and hopefully lead the way to its replacement. What/where/how test it?
  - It should break but where & how? (what is DE implying?).
  - Still much to understand wrt to stability of relevant (?) solutions
  - Cosmic censorship? BH formation & approach to singularities
  - Observational surprises need not hint of physics beyond GR
  - *Need to 'stress-test' the theory & understand possible outcomes*

Efforts along these fronts will be fundamentally influenced by the understanding of dynamical solns; which in turn requires (in part or whole) numerical studies

- E.g. recent gains:
  - Critical phenomena in GR (and in arbitrary dims)
  - 2-body problem
  - (in)stability & final fate of different solutions
  - Explorations in cosmology

Stress-testing the theory, where?

- *Some we know*: linear analysis reveals growing modes (e.g. superradiance, Gregory-Laflamme instability) or infinite blueshifting of perturbations (CH instability)
- *Some we suspect*: no decay (or too slow decay) at linear order: e.g. instability of pure AdS (weak turbulence)
- *Some are more obscure*: 'good' decay at linear order but there might be reasons to suspect possible departures at nonlinear levels (turbulence)

Outline *(some examples)*

- Superradiance
- Black holes & cosmic censorship

- AdS (in)stability?

- Turbulence in gravity

In the above list, motivations and intuition come from (and goes to) several different directions: [holography, dualities, analogies, etc]

Also, even when the 'final destination' might be known (or guessed), it is interesting to uncover the path to get there

Background & perturbations

- (standard) Perturbations ($g = g_B + h + \cdots$) described by a 'quasi-normal' spectrum $h{\sim}e^{i\omega t}f_{lm}(\mathrm{r})Y_{lm}$ with $\omega_{lm}{\sim}\omega_R + i\ \omega_I$ [Price 70's/Teukolsky 80's, Bardeen 80's, GL 90s, 'blackfold' program by Emparan+] [see Holzegel's talk Monday]

- Decay reflects loss of energy through the horizon and to infinity → scalar/gauge/gravitational waves. Growth: energy extraction/redistribution.
  - Instability & final state sometimes understood in terms of thermodynamical arguments [Gregory-Laflamme][Gubser-Mitra], via Penrose inequalities [Figueras-Murata-Reall] also [Hollands-Wald] & mappings to unstable solutions [e.g. Emparan-Myers, ...]

Superradiance

- Scattering bosonic fields can extract energy from rotating or charged black holes thus increasing their amplitude. If reflected, the process repeats → exponential growth.
- Process called for in astrophysical and holographic applications. And can play a significant role in equilibrium questions.
- For gravitational perturbations, linear analysis implies a very slow growth rate and non restrictive symmetries. Charged scalar fields however grow faster and can be studied in spherical symmetry. Instability if : $\omega R < q Q$

$$16\pi G_N \mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F_{ab}F^{ab} - |D_a\psi|^2, \qquad (1)$$

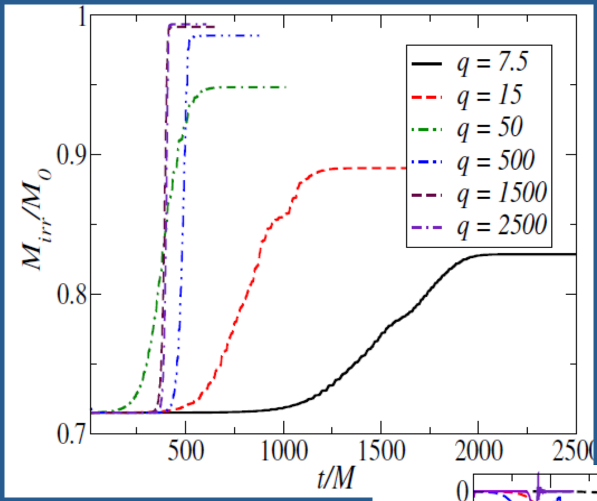
where $D_a \equiv \nabla_a - iqA_a$ is the gauge covariant derivative. This gives rise to the Einstein equation

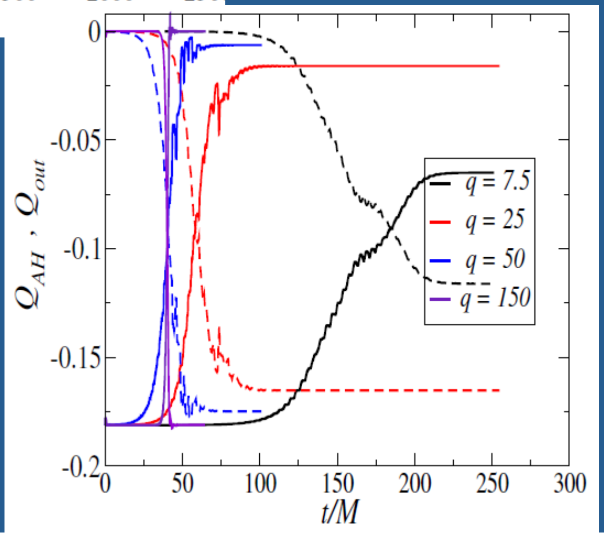
$$G_{ab} - \frac{3}{L^2}g_{ab} = 8\pi T^{\psi}_{ab} + 8\pi T^{\text{EM}}_{ab}, \qquad (2)$$

[Sanchis-Gual,Degollado + '16 Bosch-Gomez,Green,LL '16]

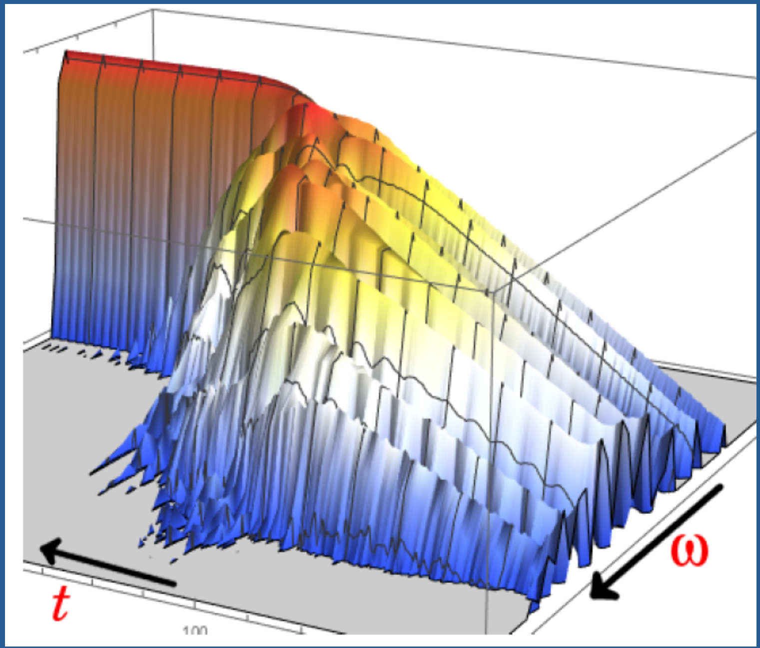
Results (in AdS)

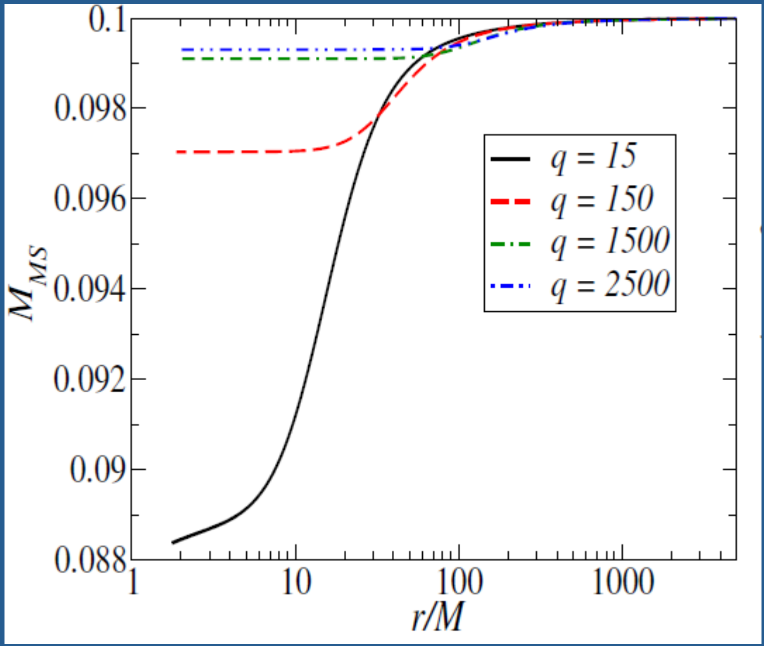
- For small enough q, below superradiance regime, perturbation decays with QNM spectra (as computed by Uchikata-Yoshida ’11.)
- As q increases, super-radiance ‘kicks in’ and, and initial growth rate agrees with QNM calculation. Then non-linear behavior sets in.

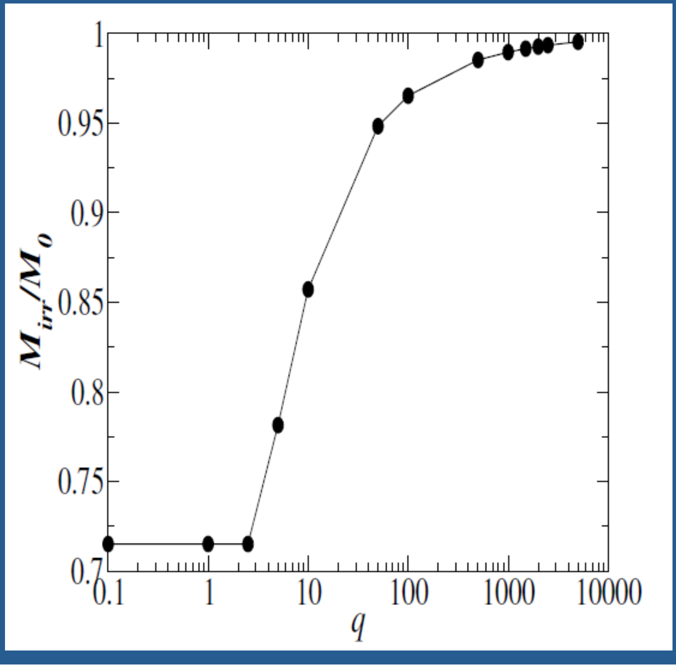




Mode picture helps to understand what is going on







- Final solution is a BH with scalar wig (that might be quite far out). Such solutions were known & speculated to be the end state [Basu +'10, Dias+ '12] but could not say how the system would get to a particular one

- How much can this say about the general case?

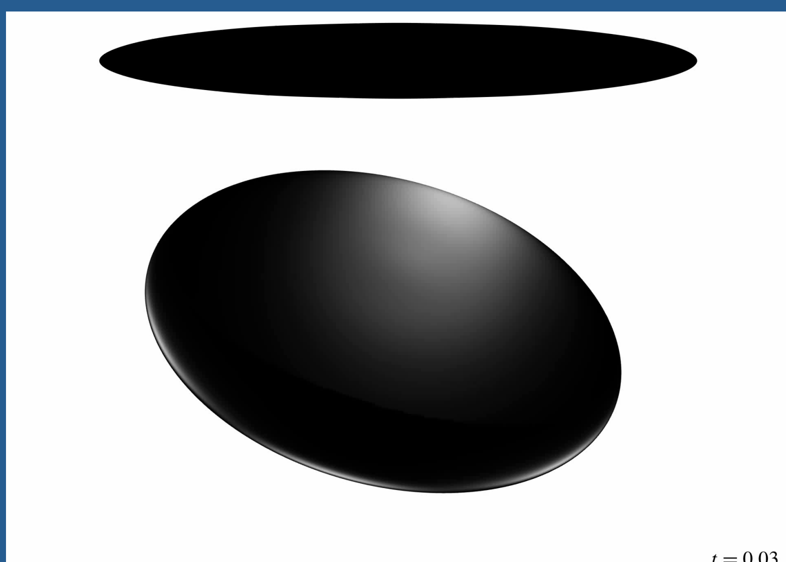
  – Mode behaviour picture would still apply, however a key difference is: instability in charged case:
    - $\omega R < q Q$ .vs. $\omega R < m \Omega$.

  – If a given *m* drops from unstable branch, coupling to higher m's will render others unstable.
    - *End state in AF? There is no natural reflecting scenario*
    - *End state in AdS? Perhaps there is not a stationary one [Dias-Horowitz-Santos '12].* Though… what if there is a *non-linear* process by which energy can not be or is only selectively transferred to higher m's ?

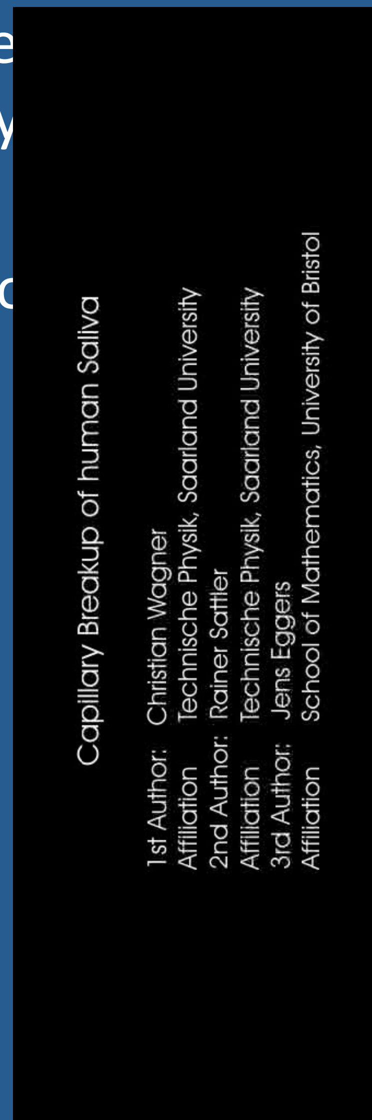
  – Incipient efforts to study superradiant systems [East-Ramazanoglu-Pretorius '14]; though not yet within a reflecting cavity (in AF or AdS)

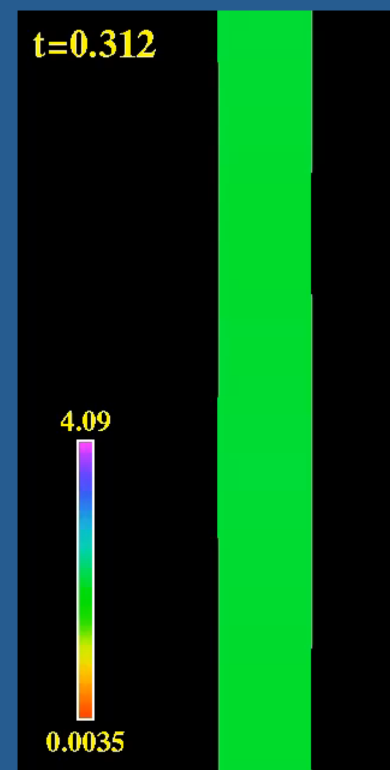
Cosmic censorship?

- Critical solution (all d’s). Spherical symmetry & finely tuned
- Not ‘that special’ behavior. Collapse [redacted] momentum → ang. momentum ‘shed more rapidly [redacted] & BH forms non-spinning [Gundlach-Baumgarte ‘16]
- Black string in d=5 dimension with [redacted] ed [LL-Pretorius ‘10]
- AF ultra-spinning black holes!



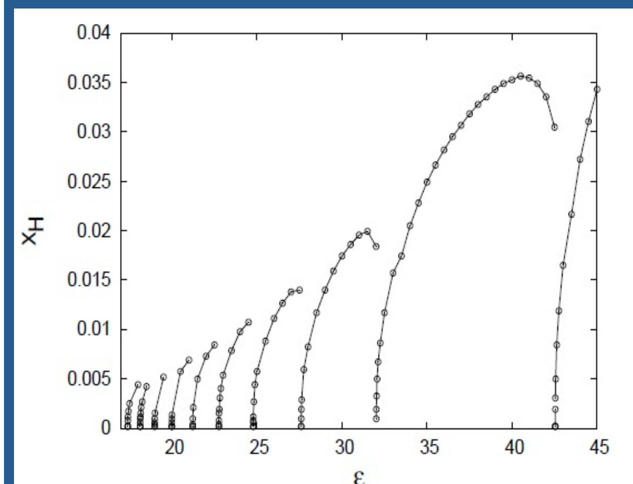
[Figueras,Kunesh,Tunyasuvunakool in prep]

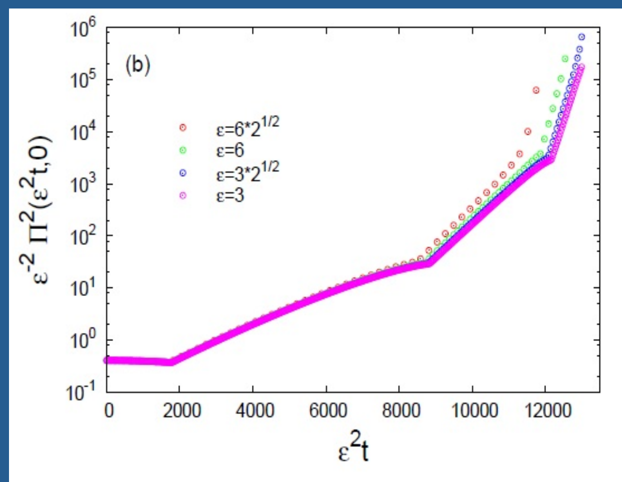




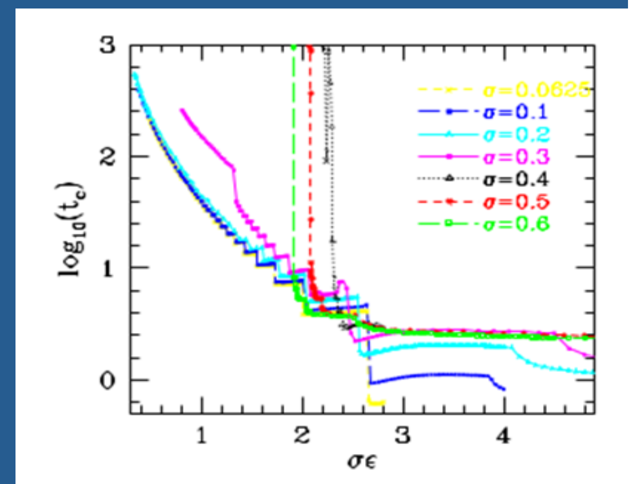
Ads (in) stability

- Conjecture: AdS is unstable [Dafermos, Anderson ‘06]
- For simplicity, Bizon-Rostworowski considered a scalar field minimally coupled to gravity (also [Dias-Horowitz-Santos]). Linearized perturbations of AdS: normal modes & conmensurate spectra:
  - ‘borderline stable’ , i.e. non-linearities could push it over the edge
  - multiple resonances →KAM theory -> energy cascade & ergodic behavior





[Bizon-Rostworowski]



[Buchel,LL,Liebling]

- Different ID has quite different behavior, even some stable solutions
- Mathematically: it’s (very likely) unstable. Holographically: thermalization?

Further tensions & partial resolution (I)

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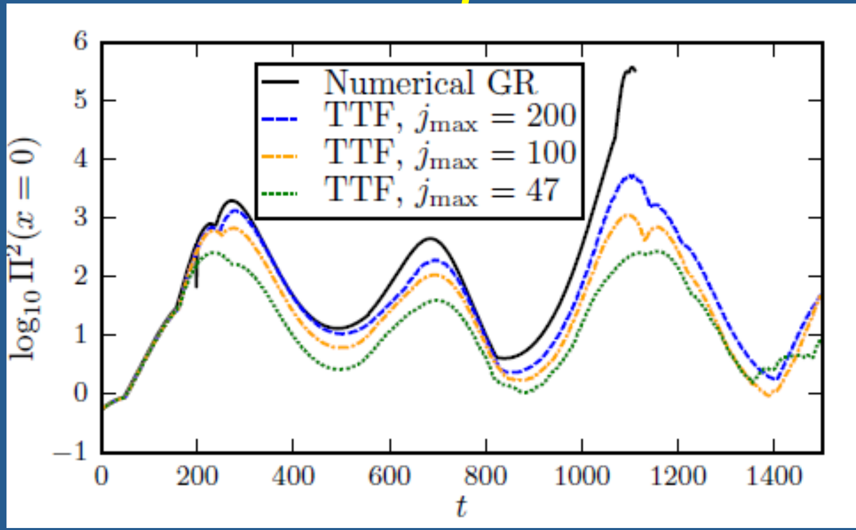


- Introduced perturbative analysis in two-time scale expansion to capture energy exchange between modes
  - $\varphi = \sum A_j(\tau) e_j(r) e^{\omega_j t}$
- → coupled ODE eqns for $A_j(\tau)$ (at order 3): $\dot{A}_j = \sum k_{jklm} A_k^* A_l^* A_m$
  [Buchel,Green,LL,Liebling]+[Craps,Evnin] also [Deppe;Bizon-Rostworowski]

- Analysis of system reveals:
  - New conserved quantities $N = \sum A_j^2 \omega_j$; E= $\sum A_j^2 \omega_j^2$ → which imply inverse energy cascade is also present

Tensions and partial resolutions (II)

- Further, quasi-periodic solutions can be found [Maliborksi+, Buchel+]
- & a Floquet-type analysis reveals such solutions are stable. Furthermore, oscillations around such solutions give a consistent picture of recurrence times [Green,Maillard,LL,Liebling '15]

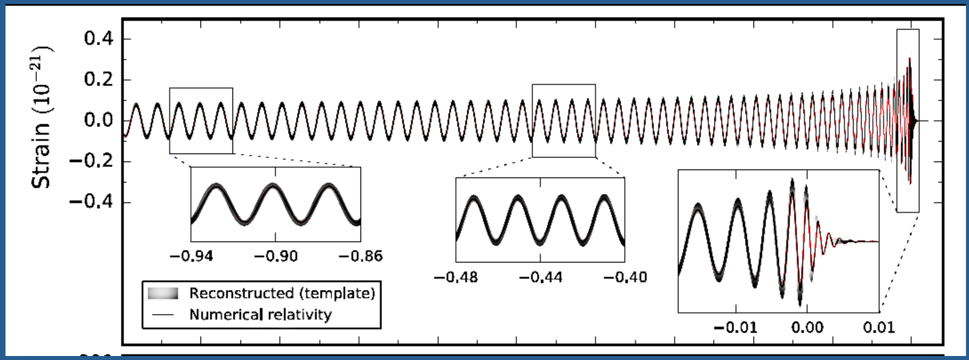
Open qns:

- Academic: how about a theorem?
  - Weak turbulence arguments? [Freivogel-Yang], Curvature role? [Dias-Santos-Marolf]
- Practical: Holographic tension?
  - small black holes require corrections, but Gauss-Bonnet exhibits similar 'stability'/instability behavior [Kunstatter-Deppe,Buchel+]. However GB has a mass-gap and a 'stringy' ball would likely be encountered

Math->perturbations->NR->perturbations->….

But is this all?

- *I.e. is any suprising behavior restricted to what linearly 'unstable' scenarios might sugest?*
- Is gravity (at least in d=3+1) as described in GR 'so tame' in spite of its highly complex underlying model?

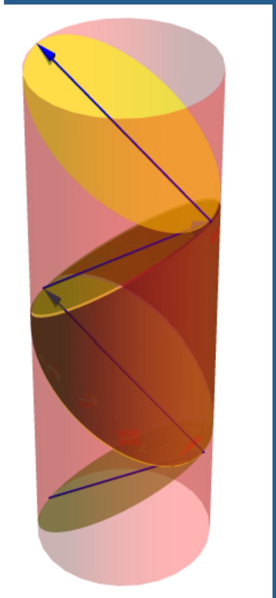


[GW151226]

- Still much intuition lacking… What lamp posts can we think of that might help as guide?

AdS/CFT…

- The AdS/CFT correspondence relates a d-dimensional QFT with a (d+1)–dimensional gravity.



[Image: J. Santos]

  – Any gravitational phenomena should have an equivalent CFT analog, *and vice-versa*.
  – A natural arena to study field theory open questions: transport properties in strongly coupled field theories, quantum quenches, thermalization, etc.
  – Plenty of applications. Most of which in equilibrium situations and in the probe limit (phase space analysis) (e.g. CMT applications)
  – From gravity standpoint: 'Hydrodynamization'/equilibration often inferred through quasi-normal decay
  – A route to quantum gravity

  – BUT CFT→ hydrodynamics (→ AdS/hydro)… and the latter can go turbulent, can gravity do that?!

‘Turbulence’ in gravity?

- Perhaps it doesn’t … (arguments against it, mainly in 4d)
  - Perturbation theory (e.g. QNMs, no tail followed by QNM)
  - Numerical simulations (e.g. ‘scale’ bounded)
  - (hydro develops shocks/turbulence, GR does not develop shocks)
- Perhaps it does…
  - AdS/CFT <-> AdS/Hydro (→ turbulence?! [Van Raamsdonk 08])
  - Applicable if LT >> 1 → $L\,(\rho/\nu) \gg 1$ → $L\,(\rho/\nu)\;\; v$ = Re >> 1
  - (membrane paradigm?)
- → (partial) List of questions…
  - Tension in the correspondence or gravity?
  - Reconcile with QNMs expectation? (and perturbation theory?)
  - Does it have similar properties?
  - What’s the analogue `gravitational’ Reynolds number?

Turbulence (in hydrodynamics)

some would say: "that phenomena you know is there when you see it"

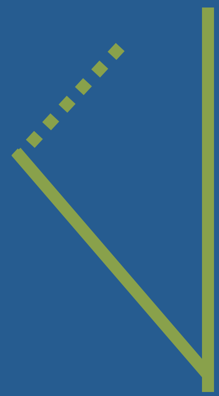
For Navier-Stokes (incompressible case):

- Breaks symmetry (recovered only in a 'statistical sense')
- Exponential growth of (some) modes [not linearly-stable]
- Global norm (non-driven system): Exponential decay possibly followed by power law, then another exponential
- Energy cascade (direct d>3, *inverse/direct d=2*)
- Occurring if Reynolds number is sufficiently high
- $E(k) \sim k^{-p}$ (5/3 and 3 for 2+1)
- Correlations: $< v(r)^3 > \sim r$ (but $\{-r, r^3\}$ in 2+1)

- AdS/CFT → gravity/fluid correspondence *[more than a dictionary!]*
  [Bhattacharya,Hubeny,Minwalla,Rangamani; VanRaamsdonk; Baier,Romatschke,Son,Starinets,Stephanov]

- Take EEs but cast perturbation in a gradient expansion
  - $g = g\big(M(x), a(x)\big)\ s.t.\ \partial^n F < \partial^{n-1} F \ \ (F = \{M, a\})$

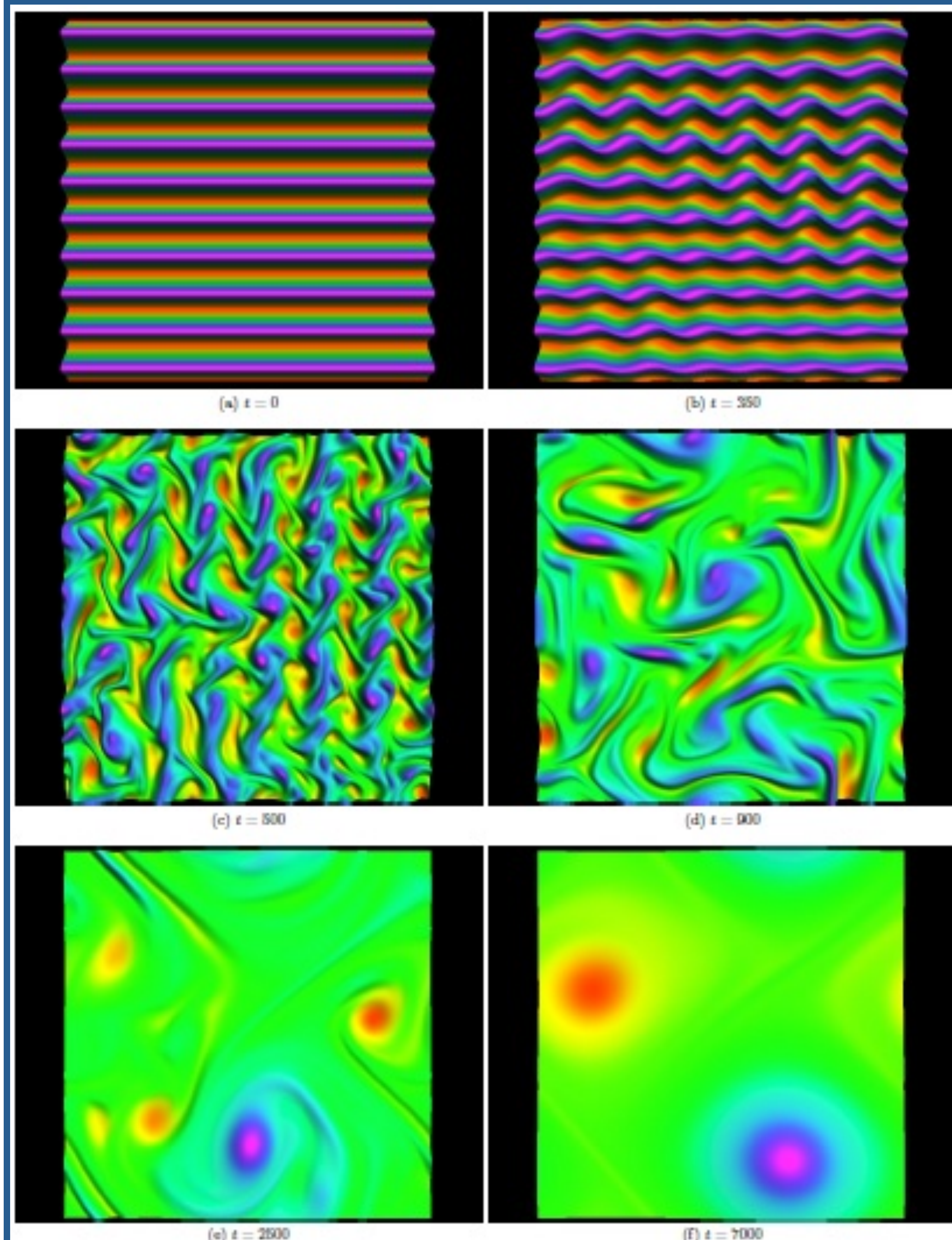
→ Hierarchy of eqns: *(d+1 decomposition)*

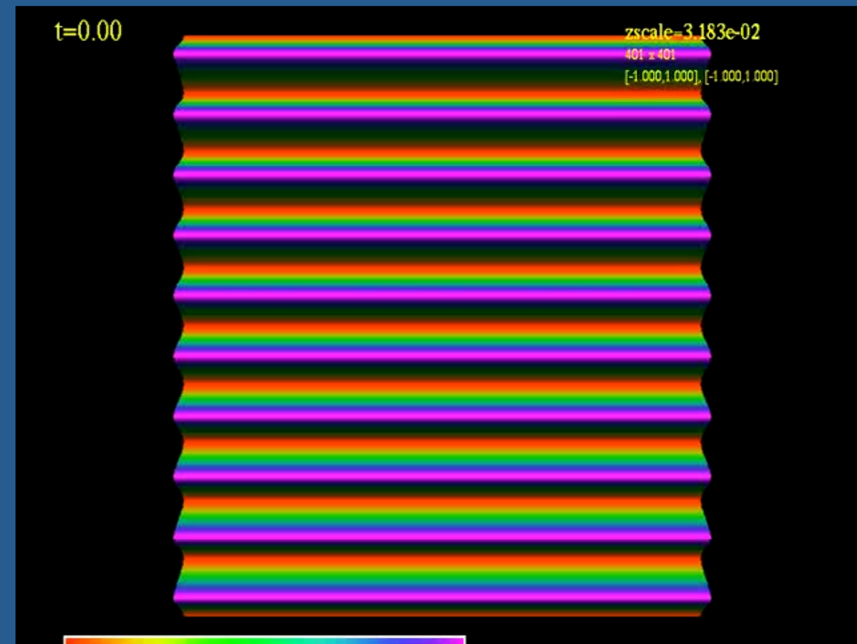


- Project to AdS bdry :-> $\nabla_a (T)^{ab} = 0, T_a^a = 0$
- Off the AdS bdry (into de bulk) simple 'radial' eqns
- $T_{ab}=(\rho+p)u_a\, u_b + p\, g_{ab} + (DISS)_{ab}$
- ρ <–> BH temperature; u_a <-> BH rotation/boost ; viscosity <-> grav wave loss

- Furthemore: quasi conserved enstrophy (~quasi-conserved vorticity^2 in Navier-Stokes) analog exists → *inverse energy cascade*

[Carrasco,LL,Myers,Reula,Singh '13]





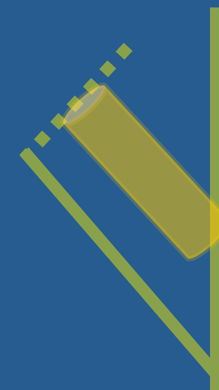
Late stage → Oseen's vortex : [Gallay-Wayne: stable, attractor for NS solns in 2+1 dims]

$$u_\theta = \frac{A}{r}\left(1 - e^{-r^2/4\eta t}\right)$$

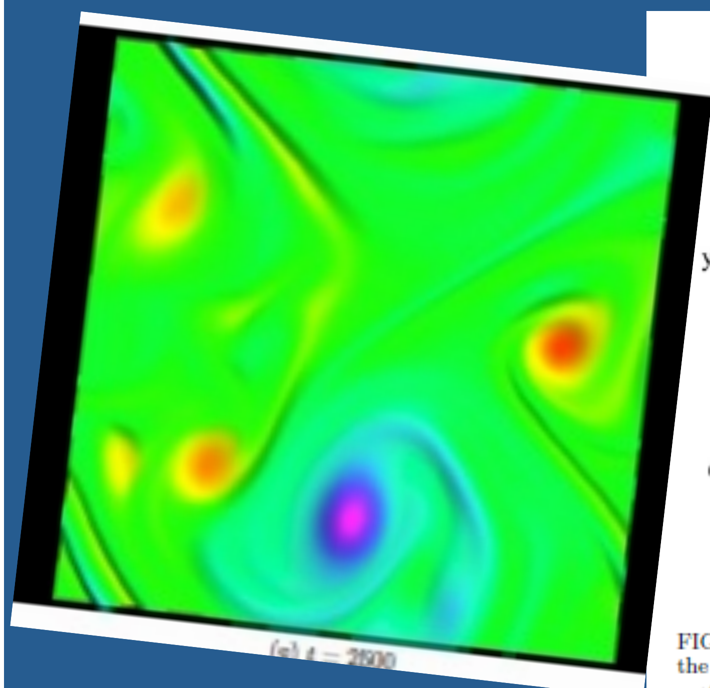
Bulk & boundary

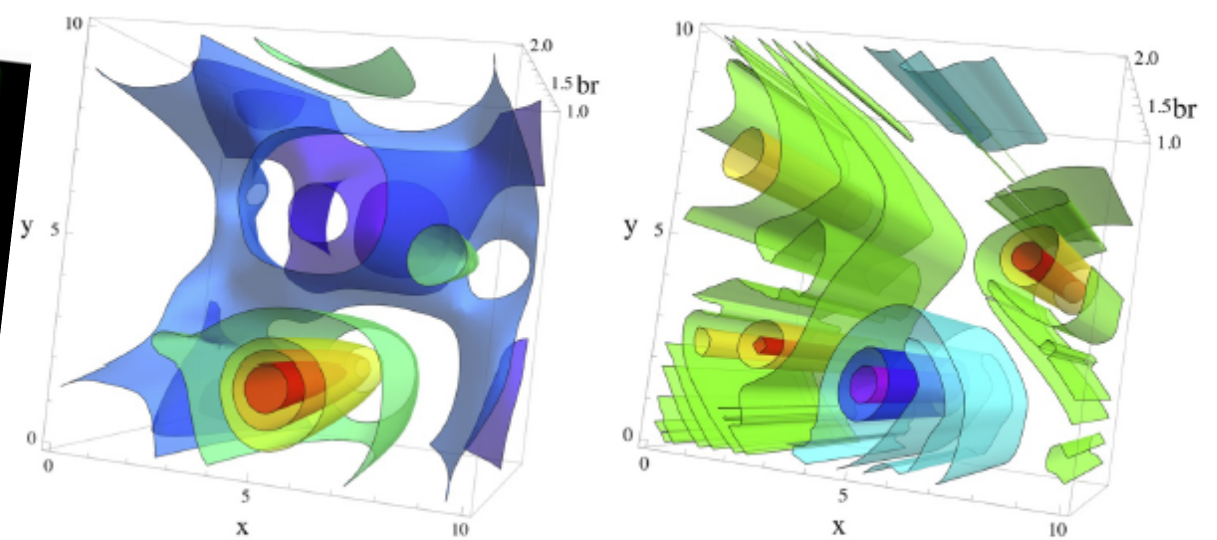
GW ‘vorticity’ plays a key role. It is encoded everywhere!

- (Adams-Chesler-Liu): Pontryagin density: $R_{abcd}\ {}^{*}R^{abcd} \sim \omega^2$
- (Eling-Oz): $\mathrm{Im}(\Psi_2) \sim T\,\omega$
- (Green,Carrasco,LL): $\Psi_1 \sim T^3\,\omega\ ;\ \Psi_3 \sim T\,\omega\ ;\ \Psi_4 \sim i\,\omega/T$



- Structure: (geon-like) gravitational wave ‘tornadoes’

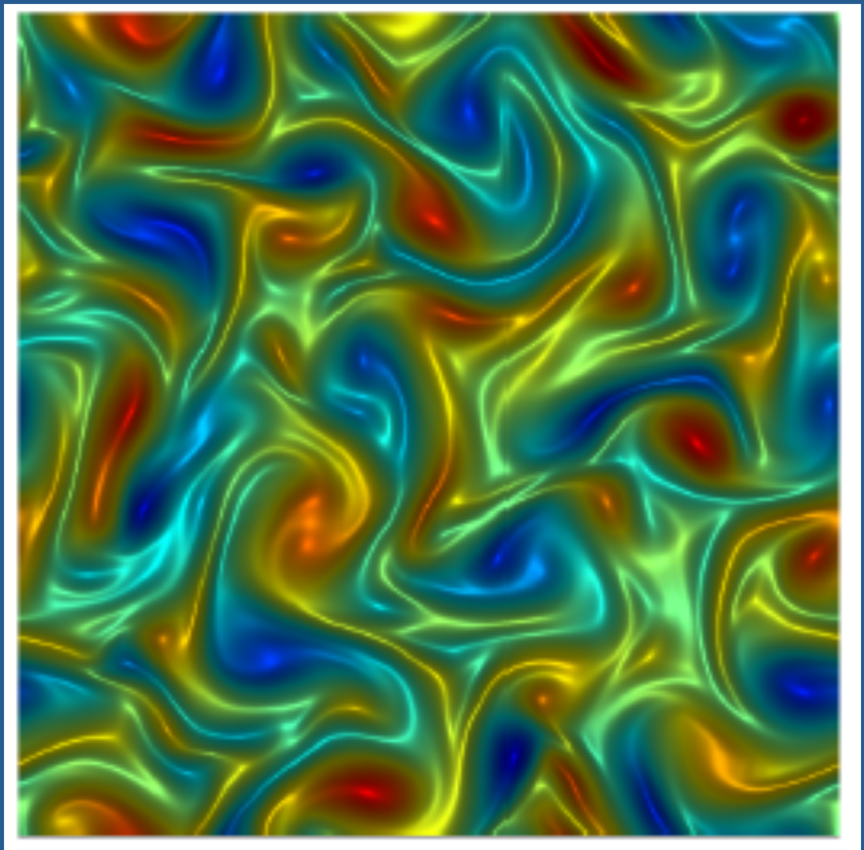




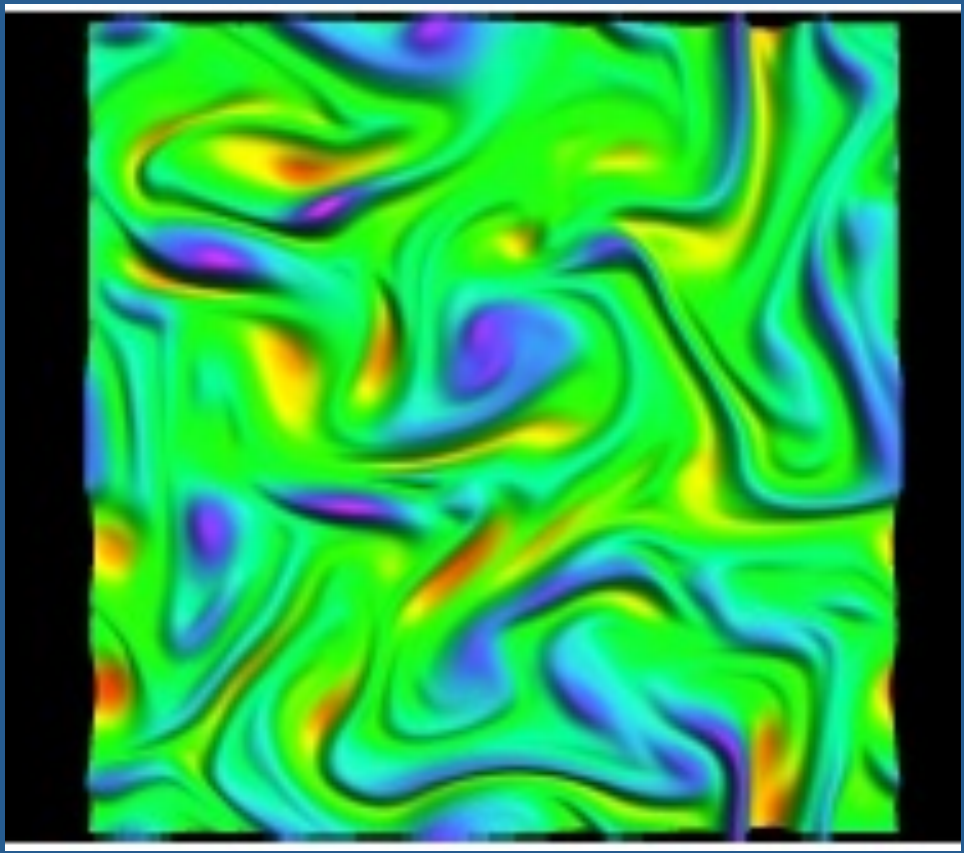
(a) $r^6 C_{ABCD}C^{ABCD}$ (b) $r^7\, {}^{*}C_{ABCD}C^{ABCD}$

FIG. 8. Contour plots of principal invariants of the Weyl tensor in the bulk, computed from the zeroth order metric (2.1), from the simulation snapshot in Fig. 1e. Notice that (a) is representative of the energy density ρ, while (b) is representative of the vorticity, as expected from Eq. (B6).

Bulk & holographic calculation

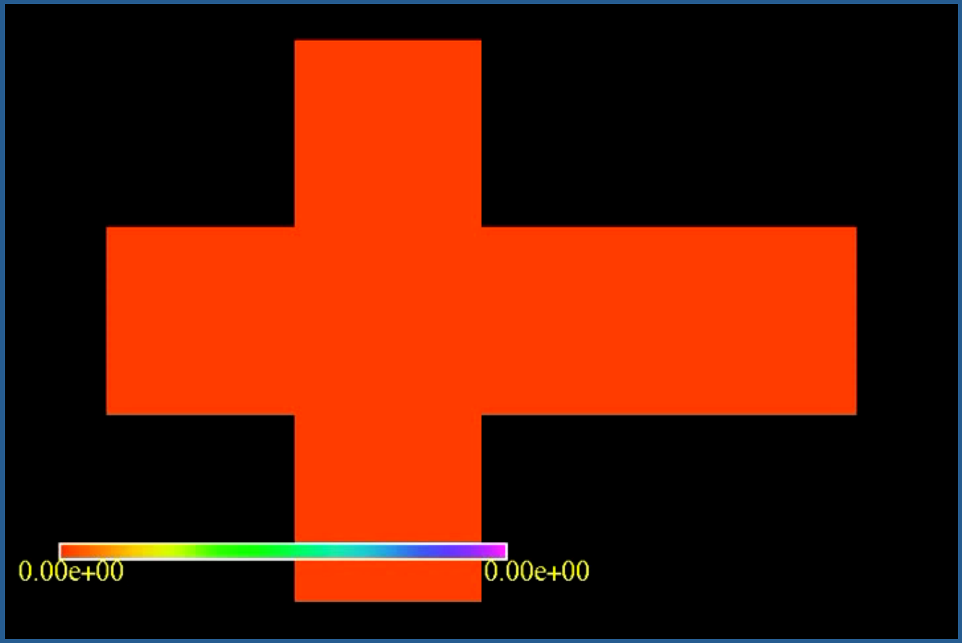


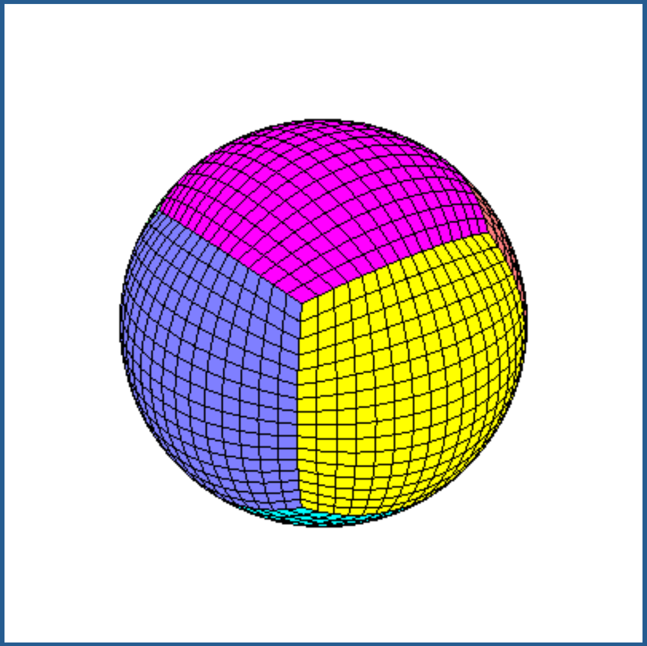
[Adams,Chesler,Liu. '14]



[Green,Carrasco,LL, '13]

Global AdS





[we'll come back to this]

Some observations

- Inverse cascade carries over to relativistic hydro and so, gravity turbulence in 4+1 and {3+1} move in opposite directions. $< T_{ti} T'_{ij} > \sim r, \{-r, r^3\} \rightarrow < g_{ti} g'_{ij} >$
  [Westernacher-Schneider,LL,Oz ‘15]

- 4+1 gravity equilibrates more rapidly (→ direct cascade dissipation at viscous scales the opposite in 3+1 gravity) [regardless of QNM differences]
  - 2+1 hydro → if initially in, the correspondence stays ok
  - 3+1 hydro → stays ‘temporarily’ within the correspondence (viscous scale!)

[Also…warning for GR-sims!, (the necessary) imposition of symmetries can eliminate relevant phenomena]

- From a hydro standpoint: geometrization of hydrodynamics in general and turbulence in particular:
  - Provides a new angle to the problem, it could be exploited to define scalings/Reynolds number in relativistic hydro case, etc. Answer long standing questions from a different direction.
  - *What mediates vortices merging/splitting in 2 vs 3 spatial dims?*
  - *Can we predict global solns on hydro from geometry considerations? (e.g. Oz-Rabinovich '11)*
  - *However, to actually do this we need to understand things from a purely gravitational standpoint.*
    - *Can we interpret how turbulence arises within GR?*
    - *How 'generic' is this behavior?*
    - *What else do we get? .e.g. correlations in the spacetime?*

On to the 'real world'

- Ultimately what triggered turbulence?
  - AdS 'trapping energy' → slowly decaying QNMs & turbulence
  - Or slowly decaying QNMs → time for non-linearities to ``do something''?

- In AF spacetimes, membrane paradigm! **However** this is delicate. Let's try something else, taking though a page from what we learnt from fluids.

- First, recall the behavior of parametric oscillators:
  - $q_{,tt} + \omega^2 (1 + f(t)) q + \gamma q_{,t} = 0$
  - Soln is generically bounded in time *except* when f(t) oscillates approximately with $\omega' \sim 2\omega$. [e.g. $f(t) = f_o \cos(\omega' t)$] . If so, an unbounded solution is triggered behaving as $e^{\alpha t}$ with $\alpha = (f_o^2 \omega^2/16 - (\omega'-\omega)^2)^{1/2} - \gamma$

[Yang-Zimmerman,LL '14]

Take a Kerr BH

- Let's consider now a BH with a mode that perturbs it with *(l,m)*
- Now, to linear order $g_{full} = g_{kerr} + h_1$ (h_1 -> $h_o(t) = \varepsilon\, e^{i\omega t}\, Y_{lm}$)
- QNMs → $\omega_{lmn} = {}^{m}/_{2} - {}^{\delta\sqrt{\kappa}}/_{\sqrt{2}} - i\,(n + {}^{1}/_{2})\ {}^{\sqrt{\kappa}}/_{\sqrt{2}}$
- with $\kappa = [1\text{-}a/m]^{1/2}$: thus,if sufficiently highly spinning, QNMs decay → 0.
- Consider the next order as determined by this –time dependent– background → parametric oscillator analogue!

- As a simplification, consider a single mode for h_1 and we'll take only a scalar perturbation (the general case is similar). One obtains:

$$[\, Box_{kerr} + O(h_1) \,]\, \Phi = 0.$$

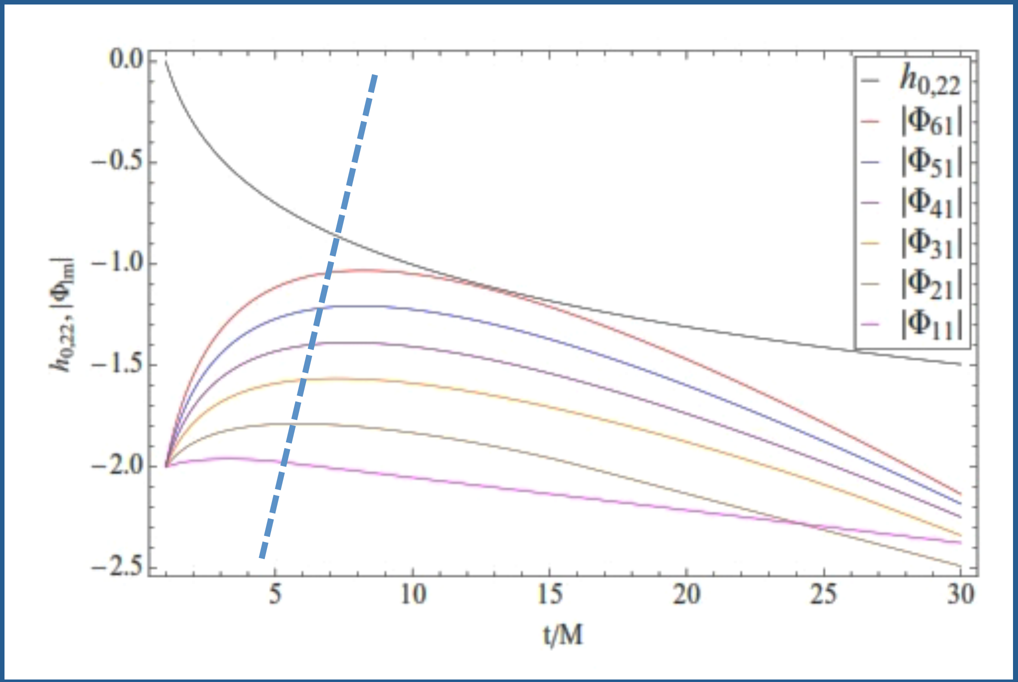
- → if Φ has l, m/2 → a parametric instability can turn on; i.e. inverse cascade. Subject to 'critical values' for growth onset.

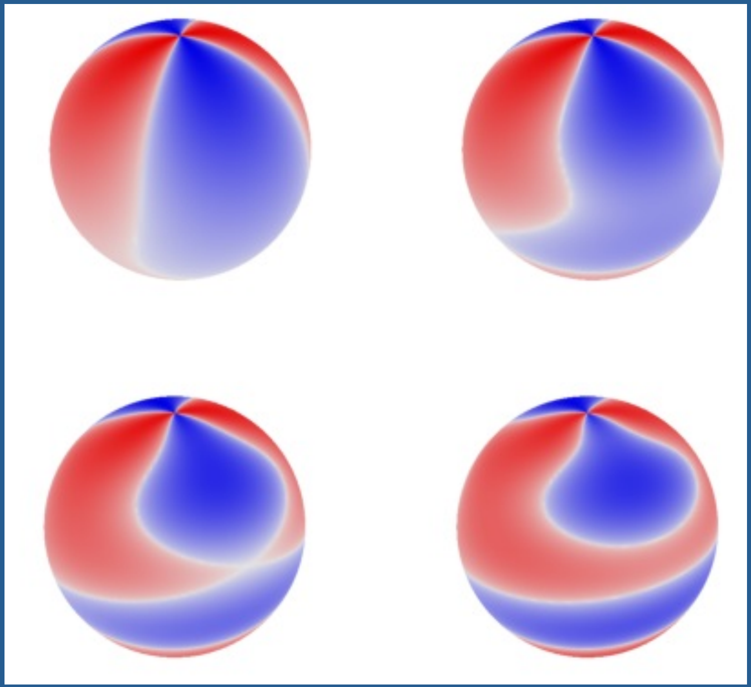
$$Re_g = h_o/(m\, \omega_\nu)$$

- identify λ<–> $1/m$; v <-> h_o ; η/ρ <-> ω_ν

$$\Diamond\ Re_g = Re$$

Critical ``Reynolds'' number & instability





a = 0.998, perturbation ~ 0.02%, initial mode l=2,m2

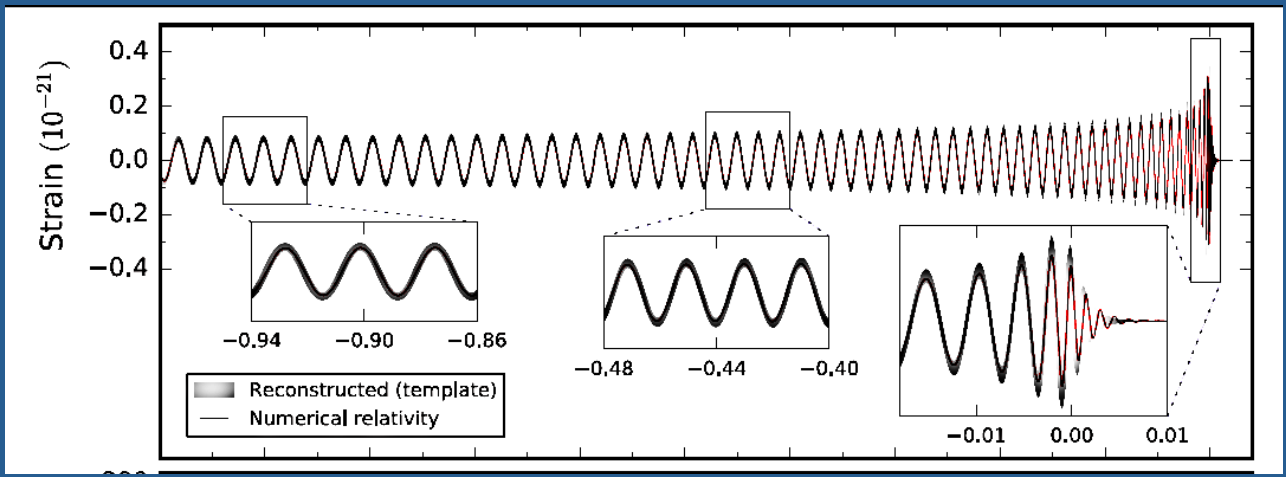
Could 'potentially' have observational consequences (especially if 'gargantua' exists beyond Hollywood). Signal is different from that expected at the linear level

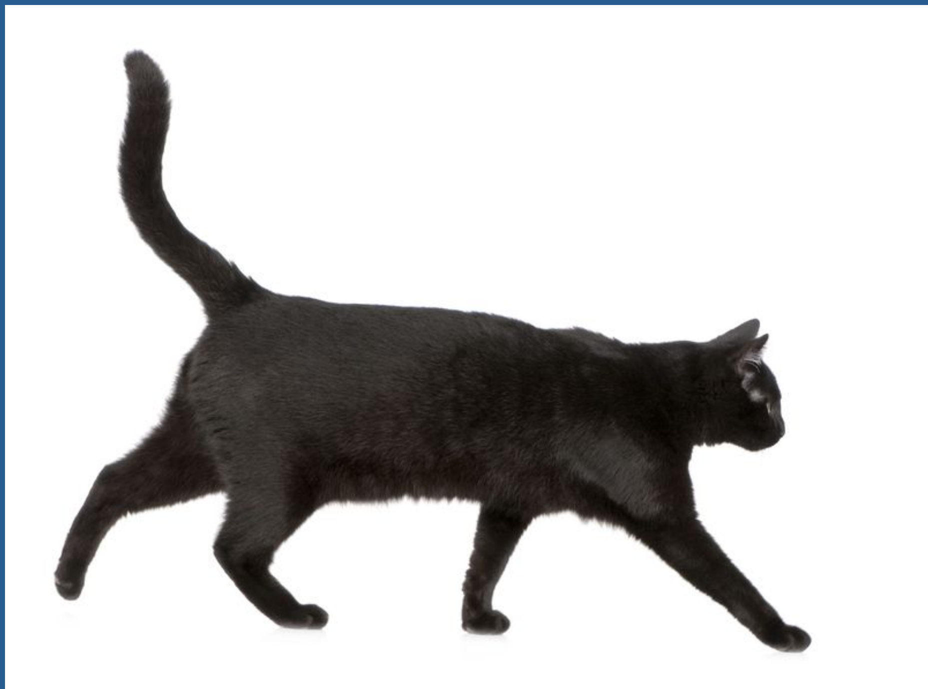
more general?

Tantalizingly…. $h_o \sim \kappa^p$ [Hadar,Porfyaridis,Strominger], but also ω_ν ◊ instability still possible beyond high spin case!

- $Re_g \sim \# \kappa^p / (m \# \kappa^p) \sim \# 1/m$ + *corrections*

Carrying this further, perhaps this gives an 'alternative explanation' to:






observations

Summary:

- Gravity does go turbulent in the right regime, and a gravitational analog of the Reynolds number can be defined
- AdS is 'convenient' but not necessary
- Some possible observable consequences
- 'geometrization' of turbulence is exciting/intriguing, what else lies ahead?
- Role as a cosmic censor?
- Fractal behavior of horizon?
- *Where/What else?*

Final words

- Only covered a (small) subset of efforts. In addition, (a still very partial list of efforts in) non-linear & dynamical studies of gravity are reaching into:
  - Randall-Sundrum scenarios [Wang-Choptuik '16]
  - Role of non-linear gravity in cosmology [Wainright +'13-14, East + '15,Bentivegna + '15, …]
  - Black hole evaporation [Ashtekar +10, Chesler + '11]
  - Self-force as cosmic censorship in d=4 [Colleoni + '15]
  - Holographic applications [Chesler+,Heller+,Van der Schee+,Murata+,Rozalli+,Mas+, ….]
  - Incipient efforts in alternative theories of gravity

Incipiently, non-linear analysis (analytical PDE, higher order pert theory and numerical simulations) is giving us an inside/detailed look at the behavior of GR in nonlinear/dynamical scenarios, and at 100 y.o. GR is still very young at its core!