Self Gravitating Media and Modified Gravity

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DSFC

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in
collaboration with G. Ballesteros and D. Comelli

Diff inv. + the metric is the only field $\longrightarrow$ GR

What if Dark Energy is not a cosmological constant ?

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almost always more fields and propagating
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Dark Energy models based on symmetries
which determine its mechanical and
thermodynamical properties

Four scalars fields coupled with gravity

$$S = \int d^4x \sqrt{-g} \left[M_{pl}^2 \, R + U(\partial \Phi) \right]$$

Shift symmetry $\qquad \Phi^A \to \Phi^A + c^A \,, \qquad \partial_\mu c^A = 0$ $\qquad A = 0, 1, 1, 2$

$a = 1, 1, 2$

EMT conservation <=> scalar field equations

Diff invariance of the action for the scalars is encoded in their equations of motion

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Carter 1973, Carter, Langlois 1993

Andersson, Comer 06

many recent applications:

Endlich, Nicolis, Rattazzi, Wang

Dubovsky, Gregoire, Nicolis, Rattazzi

...

Geometrical Interpretation

Φ^a Spatial coordinates of the medium space

Φ^0 Temporal coordinate of the medium space

operators invariant under internal rotations

$$b \equiv \sqrt{\det \boldsymbol{B}} \qquad \tau_n = \mathrm{Tr}(\boldsymbol{B}^n) \qquad n = 1,2,3 \qquad B^{ab} = g^{\mu\nu}\partial_\mu \Phi^a \partial_\nu \Phi^b$$

$$w_n = \mathrm{Tr}(\boldsymbol{W}^n) \qquad n = 1,2,3 \qquad W^{ab} = B^{ab} - \frac{g^{\mu\nu}\partial_\mu \Phi^a \partial_\nu \Phi^0 \, g^{\alpha\beta}\partial_\alpha \Phi^b \partial_\beta \Phi^0}{X}$$

$$X = g^{\mu\nu}\partial_\mu \Phi^0 \, \partial_\nu \Phi^0$$

Lagrangian coordinates

$$u^\mu = -\frac{\epsilon^{\mu\nu\alpha\beta}}{6\, b\sqrt{-g}} \epsilon_{abc} \, \partial_\nu \Phi^a \; \partial_\alpha \Phi^b \, \partial_\beta \Phi^c \qquad u^\mu \, \partial_\mu \Phi^a = 0$$

$$\mathcal{V}^\mu = -\frac{\partial^\mu \Phi^0}{\sqrt{-X}} \qquad Y = u^\mu \partial_\mu \Phi^0$$

Shift symmetry + rotational invariance

$$U(\partial\Phi) \to U(b, X, Y, \tau_n, w_n)$$

Solids

$$S = \int d^4\sqrt{-g}\left[M_{pl}^2\,R + U(\tau_1,\tau_2,\tau_3)\right]$$

$$T_{\mu\nu} = U\,g_{\mu\nu} - 2\left(U_{\tau_1}\,\delta^{ab} + 2\,U_{\tau_2}\,B^{ab} + 3\,U_{\tau_3}\,B^{ac}\,B^{cb}\right)\partial_\mu\Phi^a\partial_\nu\Phi^b$$

Deviation of B^{ab} from δ^{ab} measures solid's strain status

Anisotropic stress is present

$$B^{ab} = g^{\mu\nu}\,\partial_\mu\Phi^a\partial\Phi^b$$

More internal symmetries

Four-dimensional media		
Symmetries of the action	**LO scalar operators**	**Type of medium**
$SO(3)_s \quad \& \quad \Phi^A \to \Phi^A + f^A, \quad \partial_\mu f^A = 0$	X, Y, τ_n, y_n	supersolids
$\Phi^a \to \Phi^a + f^a(\Phi^0)$	X, w_n	
$\Phi^0 \to \Phi^0 + f(\Phi^a)$	Y, τ_n	
$\Phi^0 \to \Phi^0 + f(\Phi^0)$	τ_n, w_n, $\mathcal{O}_{\alpha\beta n}$	
$\Phi^a \to \Phi^a + f^a(\Phi^0)$ & $\Phi^0 \to \Phi^0 + f(\Phi^0)$	w_n	
V_sDiff: $\Phi^a \to \Psi^a(\Phi^b), \quad \det \lvert\partial\Psi^a/\partial\Phi^b\rvert = 1$	b, Y, X	superfluids
$\Phi^0 \to \Phi^0 + f(\Phi^0) \quad \& \quad V_s$Diff	b, $\mathcal{O}_\alpha$	
$\Phi^0 \to \Phi^0 + f(\Phi^a) \quad \& \quad V_s$Diff	b, Y	perfect fluid
$\Phi^A \to \Psi^A(\Phi^B), \quad \det \lvert\partial\Psi^A/\partial\Phi^B\rvert = 1$	bY	perfect fluid with $\rho + p = 0$

Perfect Fluids

v-diff + fluid clock reparametrization

$$\Phi^a \to \Psi^a(\Phi^b)\,, \quad \det\left(\frac{\partial \Psi^a}{\partial \Phi^b}\right) = 1\,, \quad a, b = 1, 2, 3$$

$$\Phi^0 \to \Phi^0 + f(\Phi^a)$$

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select perfect fluids: b, Y

$$T_{\mu\nu} = (U - b\,U_b)\; g_{\mu\nu} + (Y\,U_Y - b\,U_b)\; u_\mu\, u_\nu$$

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Fluid Lagrangians

Lagrangian	ρ	p	Currents
$U(b)$	$-U$	$U - b\,U_b$	$J^\mu = b\,u^\mu$
$U(Y)$	$-U + Y\,U_Y$	U	$J^\mu_{(1)} = U_Y\,u^\mu$
$U(X)$	$-U + 2\,X\,U_X$	U	$J^\mu_{(2)} = -2\,(-X)^{1/2}\,U_X\,\mathcal{V}^\mu$
$U(b, Y)$	$-U + Y\,U_Y$	$U - b\,U_b$	$J^\mu = b\,u^\mu,\; J^\mu_{(1)} = U_Y\,u^\mu$

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Restricting to just v-diff gives a superfluid $\qquad U(b, X, Y)$

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Example: $U(b, Y)$

$$\rho = Y\,U_Y - U \qquad p = U - b\,U_b \qquad \mathcal{J}_\mu = b\,u_\mu \qquad \mathcal{Y}_\mu = Y\,u_\mu$$

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first possibility

$$s = b \qquad \mu = Y$$

$$n = U_Y \qquad T = -U_b$$

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Stability

In flat space dynamical stability is equivalent to thermodynamical stability + NEC:

$$\rho + p \geq 0$$

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$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N^j \gamma_{ij} & \gamma_{ij} N^j \\ \gamma_{ij} N^j & \gamma_{ij} \end{pmatrix}$$

ADM Form

$$U(b, X, Y, \tau_n, w_n) \to U(N, N^i, \gamma_{ij})$$

Rotational invariant function of ADM variables

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Diff invariance is "restored" by the Stuckelberg fields Φ^A

GR has 2 DoF realized in a minimal way (just the metric)
other theories have more DoF and more fields

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Massive gravity as medium

$$g_{\mu\nu} = a^2\left(\eta_{\mu\nu} + h_{\mu\nu}\right)$$

expanding around flat space or FRW the medium's action produces rotational invariant mass terms for graviton

$$\sqrt{-g}\,U = t^{\mu\nu}\,h_{\mu\nu} + \frac{M_{pl}^2}{4}\left(m_0^2\,h_{00}^2 + 2\,m_1^2\,h_{0i}h_{0i} - 2\,m_4^2\,h_{00}\,h_{ii} + m_3^2\,h_{ii}^2 - m_2^2\,h_{ij}\,h_{ij}\right)$$

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Media	Operators	m_0^2	m_1^2	m_2^2	DoF
perfect fluids	b	0	$\neq 0$	0	3
	X	$\neq 0$	0	0	3
	Y	$\neq 0$	$\neq 0$	0	3
	$b,\ Y$	$\neq 0$	$\neq 0$	0	3
superfluids	$b,\ X$	$\neq 0$	$\neq 0$	0	4
	$b,\ X,\ Y$	$\neq 0$	$\neq 0$	0	4
solid	τ_n	0	$\neq 0$	$\neq 0$	5
special supersolid	$\tau_n,\ Y$	$\neq 0$	$\neq 0$	$\neq 0$	6
special supersolid	$w_n,\ X$	$\neq 0$	0	$\neq 0$	2

LO self-gravitating media	Map	Massive gravity
$\mathcal{L}(C^{AB}, g_{\mu\nu})$	Unitary gauge $\longrightarrow$	$\mathcal{L}(h_{\mu\nu}, g^{\mu\nu})$
$\mathcal{O}_{LO}$: X, Y, τ_n, y_n	$\longleftarrow$ Stückelberg "trick"	$SO(3)$ invariants of ADM's N, N^i, γ_{ij}

$\Phi^A(x)$ Stuckelberg fields

Similar treatment

Inflation EFT

Cheung-Creminelli-Fizpatrick-Kaplan-Senatore 2008

Einstein aether theory (low energy limit of Horava gravity)

Blas-Pujolas-Sibryakov 2010
Jacobson 2010

Conclusions

- deviation from GR+CC   new DoFs
- new DoFs interpreted as medium's excitations
- DE vs modified gravity: often a matter gauge choice
- DE mechanical and thermodynamical properties determined by symmetries
- the presence of a medium $\longrightarrow$ preferred frame Lorentz violation is natural
- In progress:

- observable signatures vs mechanical thermodynamical properties of DE
- stability around FLRW

Volume preserving diffs and time diffs selects fluids

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$$T_{\mu\nu} = (U - b\, U_b)\ g_{\mu\nu} + (Y\, U_Y - b\, U_b)\ u_\mu\, u_\nu + 2X\, U_X\, \mathcal{V}_\mu\, \mathcal{V}_\nu$$

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$$\left(\delta^\alpha_\mu + v_\mu\, v^\alpha\right) \nabla^\nu T_{\alpha\nu} = 0 \quad \Rightarrow \quad (p+\rho)\, a_\mu + \left(\delta^\alpha_\mu + v_\mu\, v^\alpha\right) \nabla_\alpha\, p = 0$$

is even possible to have mGR theories with 2 DoF

$$U = \sqrt{-X}\,\mathcal{E}(w_n) + \lambda$$

$$U = \mathcal{U}(\tau_n) + \sqrt{-X}\,\mathcal{E}(w_n)$$

special super solid

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special super solid

in the unitary gauge it appears as a set of massive gravity theories with 5 DoF, ghost free, no vDVZ discontinuity

is even possible to have mGR theories with 2 DoF

$$U = \sqrt{-X}\,\mathcal{E}(w_n) + \lambda$$

	$U(b)$	$U(Y)$	$U(X)$	$U(b, Y)$
	$\rho = -U$ $p = U - b\,U_b$ $\mathcal{J}_\mu = b\,u_\mu$	$\rho = -U + Y\,U_Y$ $p = U$ $\mathcal{Y}_\mu = U_Y\,u_\mu$	$\rho = -U + 2\,X\,U_X$ $p = U$ $\mathcal{X}_\mu = -2\,(-X)^{1/2}\,U_X\,\mathcal{V}_\mu$	$\rho = -U + Y\,U_Y$ $p = U - b\,U_b$ $\mathcal{J}_\mu = b\,u_\mu,\ \mathcal{Y}_\mu = U_Y\,u_\mu$
(μ, s) $\mathcal{I} = -U$	$b = s$ $n = 0$ $T = -U_b$ $\mathcal{J}_\mu = s\,u_\mu$	$Y = \mu$ $n = U_Y$ $T = 0$ $\mathcal{Y}_\mu = n\,u_\mu$	$X = -\mu^2$ $n = -2\,U_X\,\sqrt{-X}$ $T = 0\,,\quad \mathcal{X}_\mu = -\,n/2\,\mathcal{V}_\mu$ $\mathcal{X}_\mu = n\,\mathcal{V}_\mu$	$b = s,\ Y = \mu$ $n = U_Y$ $T = -U_b$ $\mathcal{J}_\mu = s\,u_\mu\,,\ \mathcal{Y}_\mu = n\,u_\mu$
(n, T) $\mathcal{F} = -U$	$b = n$ $s = 0$ $\mu = -U_b$ $\mathcal{J}_\mu = n\,u_\mu$	$Y = T$ $s = U_Y$ $\mu = 0$ $\mathcal{Y}_\mu = s\,u_\mu$	$X = -T^2$ $s = -2\,U_X\,\sqrt{-X}$ $\mu = 0$ $\mathcal{X}_\mu = s\,\mathcal{V}_\mu$	$b = n,\ Y = T$ $s = U_Y$ $\mu = -U_b$ $\mathcal{J}_\mu = n\,u_\mu\,,\ \mathcal{Y}_\mu = s\,u_\mu$
(μ, T) $\omega = -U$ $z = \mu/T$		$Y = T\,f\,(z)$ $s = U_Y\,(f - z\,f')$ $n = f'\,U_Y$ $\mathrm{T\ f}\,\mathcal{Y}_\mu = (\rho + p)\,u_\mu$	$X = -T^2\,f\,(z)$ $s = U_X\,(\mu\,f' - 2\,T\,f)$ $n = -U_X\,T\,f'$ $T\,f^{1/2}\mathcal{X}_\mu = (\rho + p)\mathcal{V}_\mu$	
(n, s) $\rho = -U$ $\sigma = s/n$	$b = s\,f\,(\sigma^{-1})$ $\mu = -U_b\,f'$ $T = -U_b\,(f - f'/\sigma)$ $\mathcal{J}_\mu = s\,f\,u_\mu$			

Thermodynamical Stability is equivalent to dynamical stability + null energy condition

For instance take U(b) Increase of entropy gives: $U_{bb} = U'' \leq 0$

Quadratic action around flat space

$$\Phi^a = \delta^a_i \, x^i + \pi^a(x) \qquad g_{\mu\nu} = \eta_{\mu\nu}$$

$$S^{(2)}[b] = \frac{1}{2}(\rho + p) \int d^4x \left[\dot{\pi}^i_T \dot{\pi}^i_T - \dot{\pi}_L \Delta \, \dot{\pi}_L - c_s^2 \, (\Delta \pi_L)^2\right]$$

$$\pi^i = \pi^i_L + \pi^i_T, \quad \pi^i_L = \partial_i \pi_L \,, \quad \partial_i \pi^i_T = 0 \qquad c_s^2 = -\frac{U_{bb}}{\rho + p}$$

No ghost $\rho + p > 0$ No gradient instabilities $c_s^2 > 0$

Media and symmetries

rotational invariant SGM $\quad \Phi^a \to R^{ab}\,\Phi^b \quad R^{ab} \in SO(3)$

operators: rotational inv. in medium space and spacetime scalars to be used in the medium action

Operator	Definition
C^{AB}	$g^{\mu\nu}\,\partial_\mu\Phi^A\,\partial_\nu\Phi^B\,, \quad A,B=0,1,2,3$
B^{ab}	$g^{\mu\nu}\partial_\mu\Phi^a\,\partial_\mu\Phi^b\,, \quad a,b=1,2,3$
Z^{ab}	$C^{a0}\,C^{b0}$
X	C^{00}
W^{ab}	$B^{ab}-Z^{ab}/X$
b	$\sqrt{\det \boldsymbol{B}}$
Y	$u^\mu\partial_\mu\Phi^0$
y_n	$\mathrm{Tr}\,(\boldsymbol{B}^n\cdot\boldsymbol{Z})\,, \quad n=0,1,2,3$
τ_n	$\mathrm{Tr}\,(\boldsymbol{B}^n)\,, \quad n=1,2,3$
w_n	$\mathrm{Tr}\,(\boldsymbol{W}^n)\,, \quad n=0,1,2,3$
$\mathcal{O}_{\alpha\beta n}$	$(X/Y^2)^\alpha\,(y_n/Y^2)^\beta\,, \quad \alpha,\beta\in\mathbb{R}$
$\mathcal{O}_\alpha$	$(X/Y^2)^\alpha\,, \quad \alpha\in\mathbb{R}$