Bulk amplitude and degree of divergence in 4d Spin Foams

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arXiv: 1602.01825 [gr-qc]

The background is based on the collaboration with A.Banburski, J. Hnybida and L. Freidel
Phys.Rev.D 92, 124014(2015),   arXiv: 1512.05331 [gr-qc]
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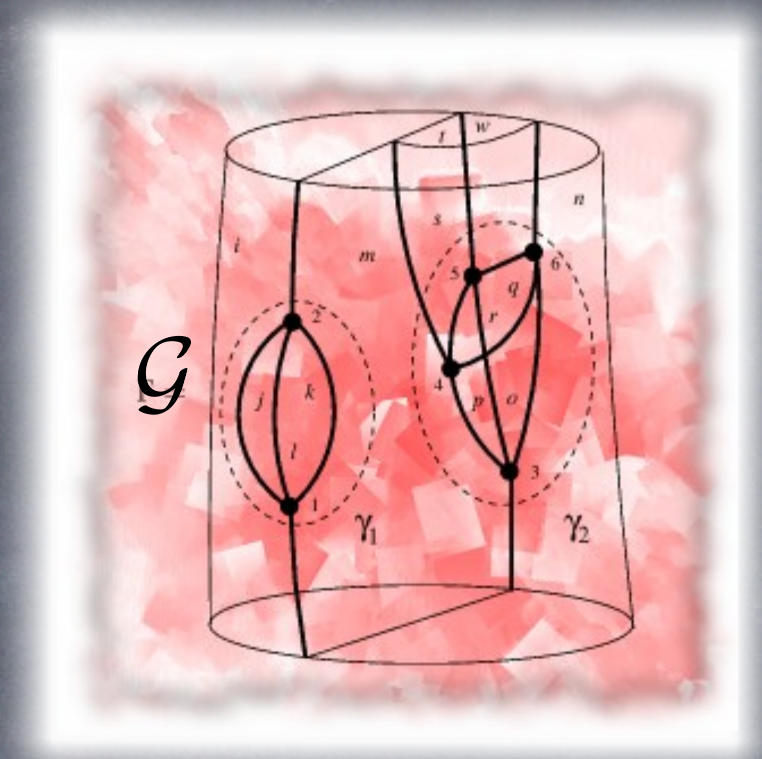
A spin foam partition function $Z(\mathcal{G})$ is defined by a path integral over all geometrical degrees of freedom with a given boundary $\partial\mathcal{G}$.



Graph credit: www.fuw.edu.pl/~kostecki/school.html

Questions ?

- How to compute arbitrary amplitudes efficiently?
- Can we learn the degree of divergence of an amplitude simply by its graphic properties?
- What type of geometry in the bulk has the dominant contribution to the partition function?

This talk will address those questions within a 4-d Riemannian Spin Foam model in terms of holomorphic representation with vanishing cosmological constant.

Strategy:

- Holomorphic representation and coherent state
- Imposing the simplicity constraints on the Spin(4) projector
- Some calculation tricks: the Homogeneity map

We will finally arrive at an expression which gives efficient approximation of arbitrary amplitude, and also a formula for degree of divergence simply based on number of vertices, faces, edges etc.

4-d Holomorphic Spin Foam Model and Diagrammatic notation

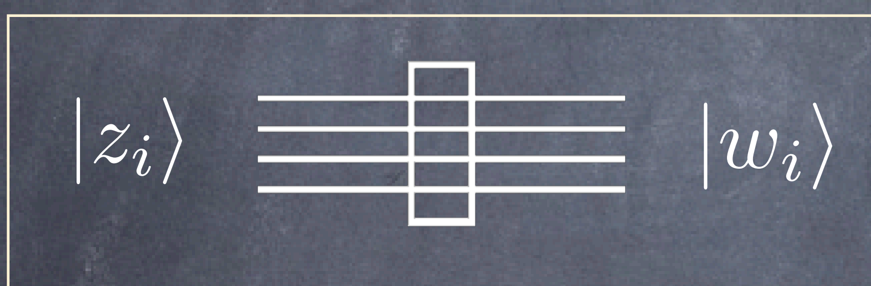
- In the holomorphic representation of SU(2) [V.Bargmann 1962, J.Schwinger 1952]

  If we use the notation of spinors $|z\rangle \equiv (\alpha, \beta)^T, \quad |\check{z}\rangle = |z] \equiv (-\overline{\beta}, \overline{\alpha})^T$

  the kernel of SU(2) coherent projector:

$$P(z_i; w_i) = \int_{\mathrm{SU}(2)} dg\, e^{\sum_i [z_i|g|w_i\rangle}$$

[M.Dupuis&E.Livine, 2011]



$$= \sum_J \frac{\left(\sum_{i<j} [z_i|z_j\rangle [w_i|w_j\rangle\right)^J}{J!(J+1)!}$$

[J.Hynbida &L.Freidel, 2012]

- We impose the holomorphic simplicity constraint [M.Dupuis&E.Livine, 2011]

  on the Spin(4) BF projector: (slight difference with EPRL/FK)

$$\tilde{P}_\rho(z_i; w_i) := P(z_i; w_i) \cdot P(\rho z_i; \rho w_i)$$

$$= \sum_J F_\rho(J) \frac{\left(\sum_{i<j} [z_i|z_j\rangle [w_i|w_j\rangle\right)^J}{J!(J+1)!}$$

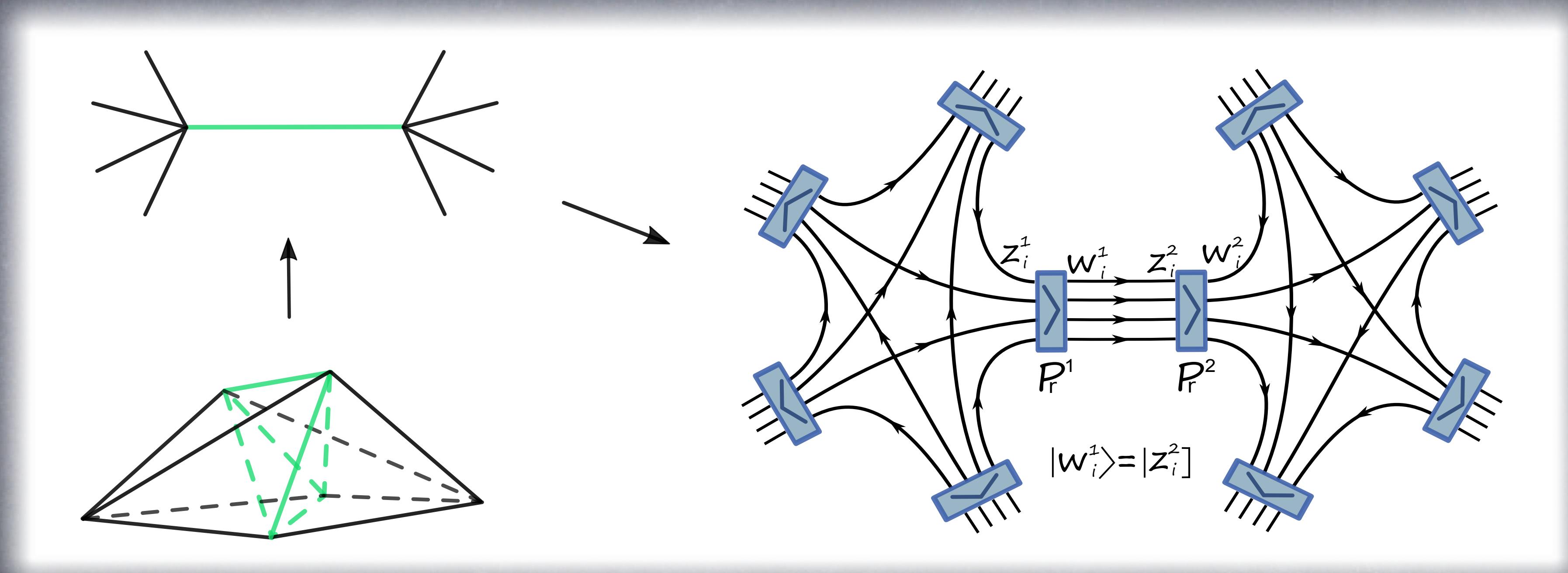
where $\rho^2 = |1-\gamma|/(1+\gamma)$ when $\rho \to 0, F_0(J) \to 1$ BF limit

- The partition function:

$$\mathcal{Z}(\Delta^*) = \prod_{f\in\Delta^*} \sum_{j_f} A_f(j_f) \int \prod_i d\mu_\rho(z_i, w_i) \prod_{e\in\Delta^*} \tilde{P}_\rho(z_i^e; w_i^e)$$

where the face weight $A_f(j_f)$ is a function of spin: $(2j_f+1)^\eta$

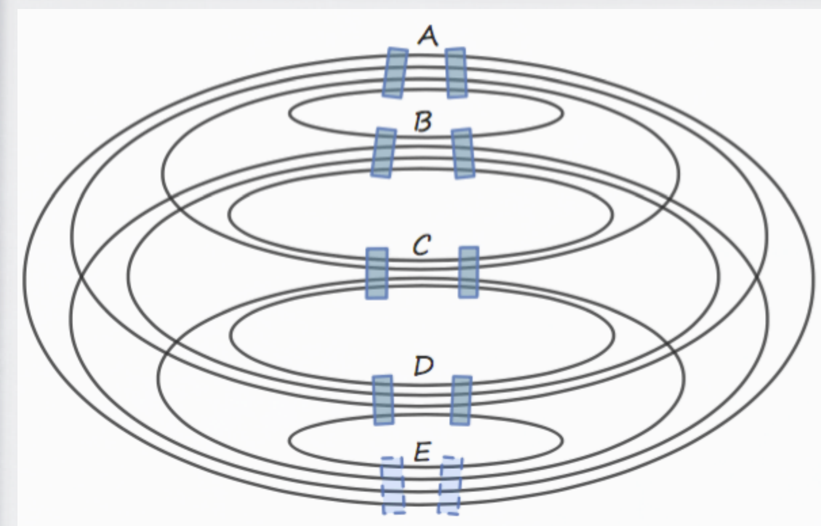
- parameters: $\rho^2 = |1-\gamma|/(1+\gamma)$; face weight η.

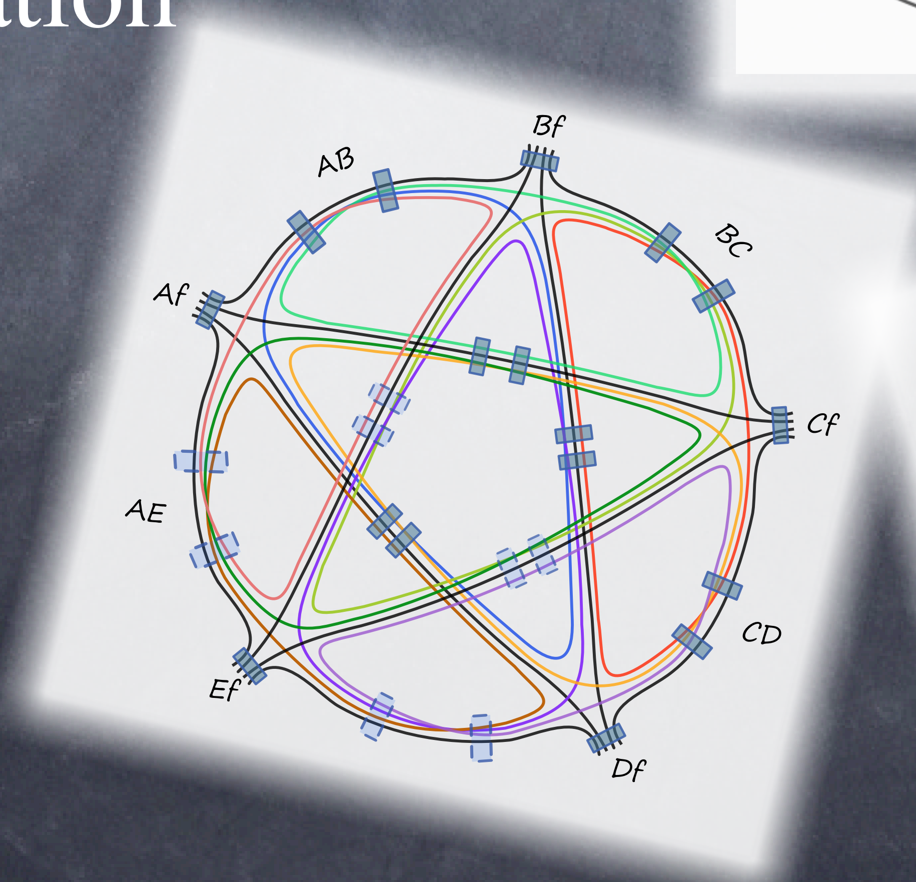


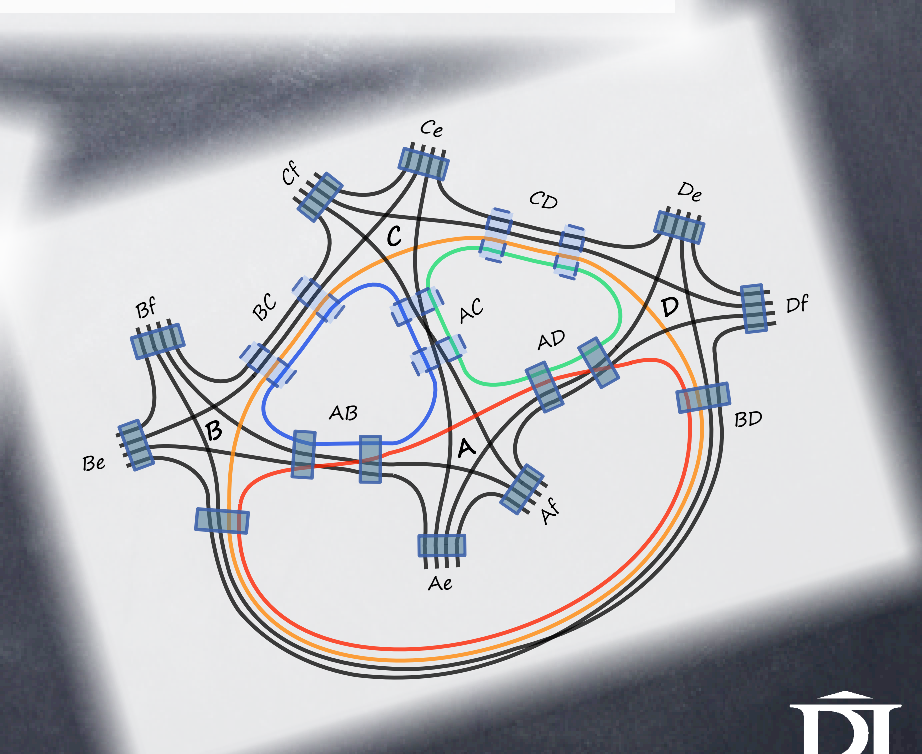
- At the leading order, for a single 4-simplex, it has the same semiclassical limit as EPRL/FK model. arXiv: 1512.05331 [gr-qc]
- It can be immediately generalized to arbitrary type of lattice, not just restricted to the simplicial one.

Sketch: Method of evaluating the amplitude

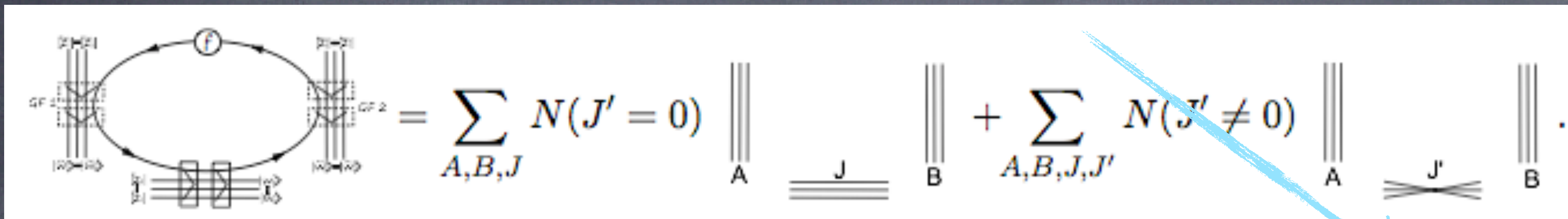
- Gauge fixing
- Loop identity and truncation
- Spinors integration





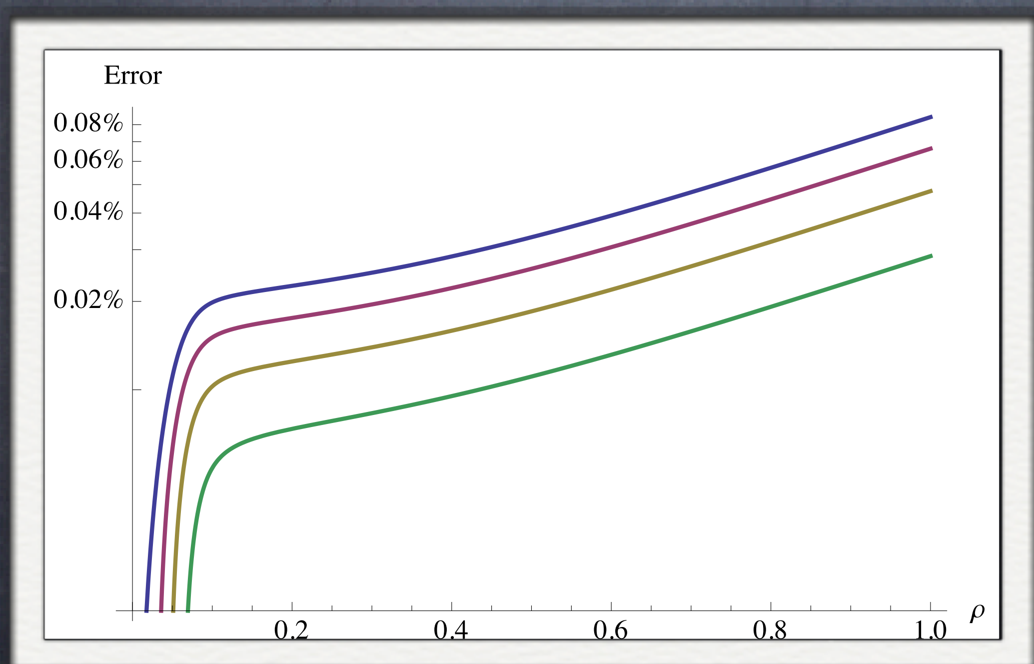


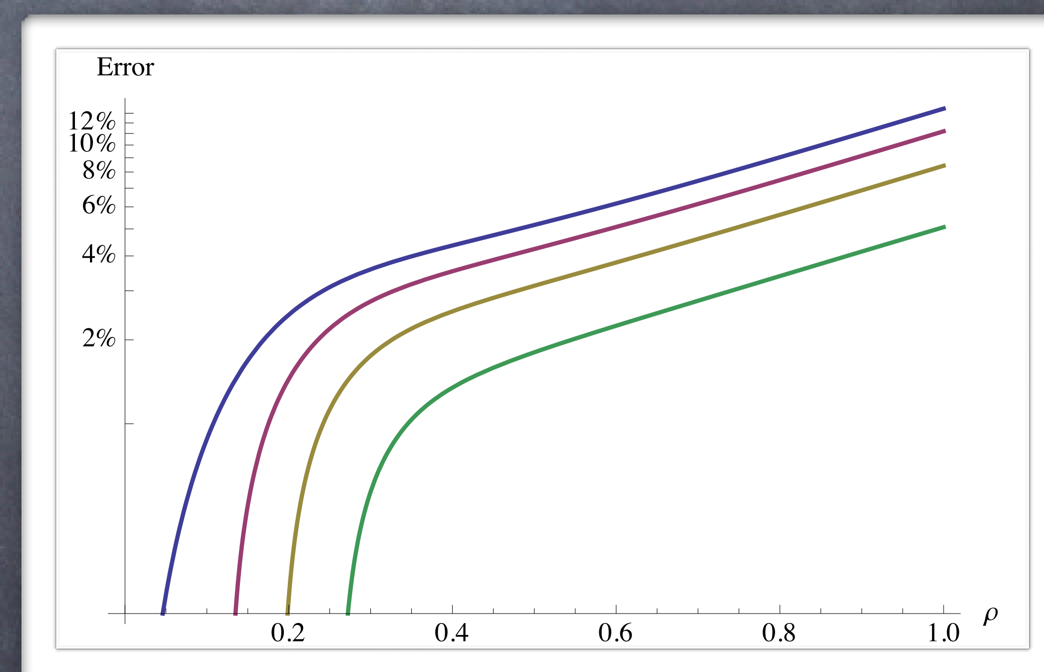
- The most basic step of evaluating amplitude is integrating out the common faces in the 2-complex, which correspond to loops in cable diagram.
- The loop identity and its truncation: [A.Banburski, L.-Q.C, L. Freidel and J.Hnybida, Phys.Rev.D 92, 124014(2015)]



where A, B, J are the total spins in the shared tetrahedra.

- The error from truncation:





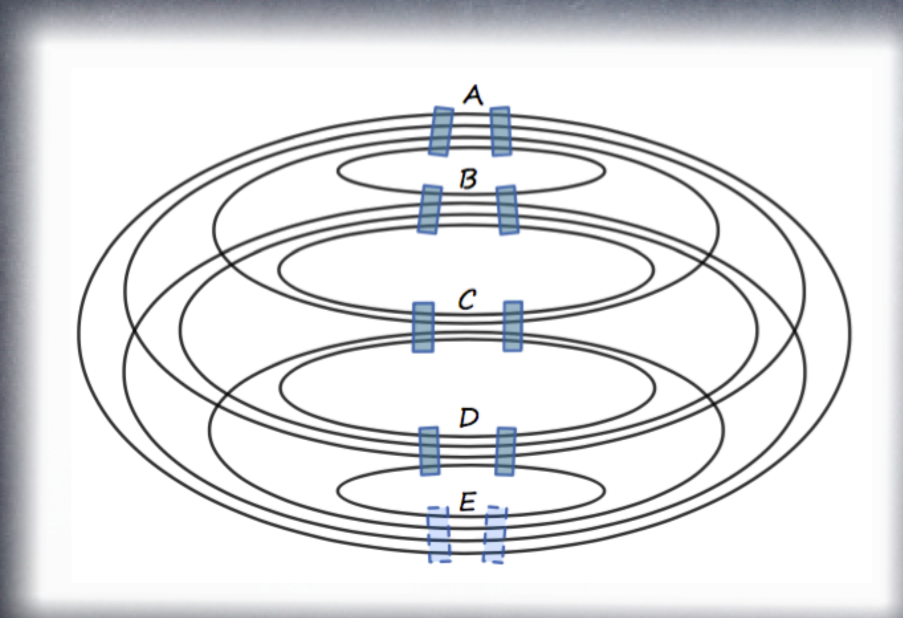
Large spin: Total Spin on each tetrahedra is 100

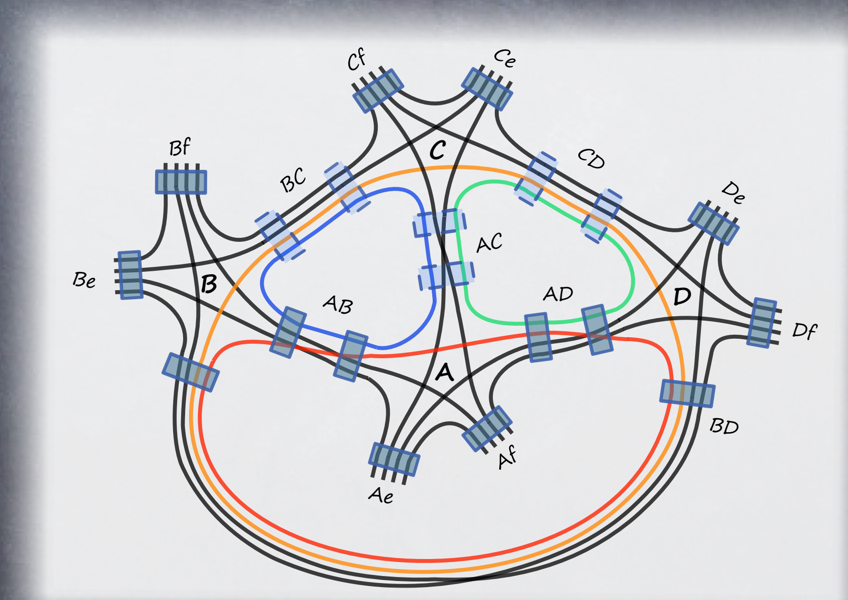
Small spin: Total Spin on each tetrahedra is 5

Blue, red, yellow and green lines correspond to face weight equal to 1,2,3,4 respectively.

Bulk amplitude A_{bulk}

The evaluation of the partition function on a fully contracted 2-complex, or on a connected 2-complex with zero boundary spins.





Truncated bulk amplitude $\tilde{A}_{bulk}$

We have completely integrated out the intertwiner degrees of freedom !

$$\tilde{A}_{bulk} = \sum_{\{j_l\}\in Z/2} \prod_f \underbrace{\frac{(2j_f+1)^{\eta+1}}{(1+\rho^2)^{2j_f N_f}}}_{\#:|F|-|C_T|} \cdot \prod_\alpha \underbrace{(\sum\nolimits_{l\in\tilde{\Phi}_\alpha} 2j_l+1)^{\eta-1}}_{\#:|C_T|} \cdot \prod_e \underbrace{F_\rho^2\,(\sum\nolimits_{l\in\tilde{\Theta}_e} 2j_l)}_{\#:|E|}$$

where $\quad F_\rho(J) = {}_2F_1(-J-1,-J;2;\rho^4)$; face weight η .

$|E|$ $|F|$ and $|C_T|$ label the number of edges, faces and fundamental cycles;

Only need to keep track of combinatorics!

There are $|F|-|C_T|$ free summations of spins.

It allows us to write down the value of bulk amplitudes simply based on graph properties!

Let us extract some physics!

- Given a fixed boundary, what type of geometry in the bulk has the dominant contribution to the partition function?

- Can we learn the degree of divergence of an amplitude simply by its graphic properties?

Degree of divergence $D(\mathcal{G}) = \Lambda^{(\eta+2)|F|-6|E|+3|V|-3}$

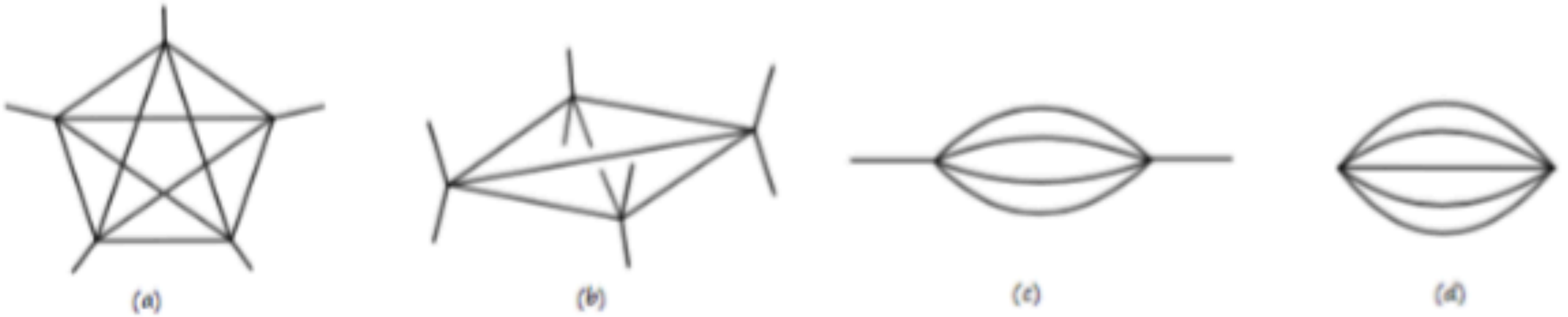


FIG. 9: A few simple examples we are considering in the following table.

	$\|V\|$	$\|E\|$	$\|F\|$	Degree of divergence
5–1 move (a)	5	10	10	$\Lambda^{10\eta-28}$
4–2 move (b)	4	6	4	$\Lambda^{4\eta-19}$
Elementary melon (c)	2	4	6	$\Lambda^{6\eta-9}$
Fully contracted melon (d)	2	5	10	$\Lambda^{10\eta-7}$

- When $\eta = 3$, (a) has exactly Λ^2 divergence, while (b) is finite, which is the expected behavior if the model has diffeomorphism invariance.

[Related with earlier work of A. Riello, C. Perini, C. Rovelli, S. Speziale etc.]

- Degree of a graph (the sum of the genera of its jackets), non-negative.

$$\omega_{4d}(\mathcal{G})/3 = 3|V| - |F| + 4.$$

[R.Gurau, V.Rivasseau, J.P. Ryan, V.Bonzom etc. in the series work of 1/N expansion of colored tensor models]

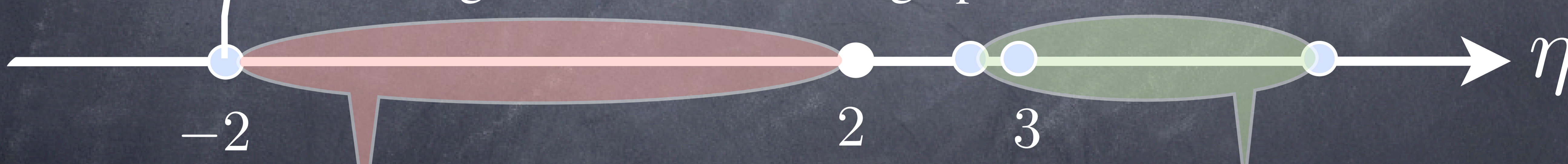
- Then we can write degree of divergence in terms of $\omega_{4d}(\mathcal{G}), |F|$:

$$D(\mathcal{G}) = \Lambda^{(\eta-2)|F| - 4\omega_{4d}(\mathcal{G})/3 + 13},$$

Or in terms of $\omega_{4d}(\mathcal{G}), |V|$: $D(\mathcal{G}) = \Lambda^{3(\eta-2)|V| - (2+\eta)\omega_{4d}(\mathcal{G})/3 + 4\eta + 5}$.

- **Indication of different phases:**

Dominated by $\omega_{4d}(\mathcal{G}) = 0$: spherical topology and degree zero -- melonic graphs



The amplitude is increasingly **suppressed** by higher number of vertices.

$\eta = 2$ same as the Colored Tensor Model: **Branched polymers phase**

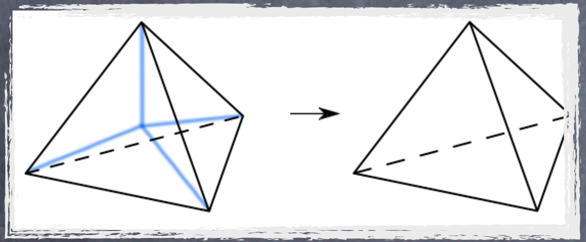
The region of physical interest: 5-1 is divergent and 4-2 is convergent.

Discussion

- The divergence is expected to encode information of diffeomorphisms in Spin Foam models (with zero cosmological constant).

- Recall: Discretized Riemannian 3-d gravity: [L.Freidel & D. Louapre,Nucl.Phys.B 662,279 (2003)]

Diffeomorphism = Local Lorentz + Translation symmetry of the vertex



Λ^3 or $\delta_{SU(2)}(\mathbb{1})$ divergence

- The degree of divergence of Ponzano-Regge model:

$$D_{SU(2)BF}(\mathcal{G}) = \Lambda^{3|F|-3|E|+3|V|-3} = \Lambda^{3|V|/2-3\omega_{3d}(\mathcal{G})+6}$$

	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert F\rvert$	Degree of divergence
4–1 Pachner move	4	6	4	Λ^3
A fully contracted melon	2	4	6	Λ^9

- Without properly gauge fixing the vertex translation symmetry, the most divergent graphs also have degree zero, hence are melonic.

Discussion

The implications of the result are different for two approaches towards continuum limit: summing over all the possible diagrams, or refining the partition function on a fixed lattice.

- For summation approach:

1) The melonic dominance might be resolved by proper gauge fixing, as in 3d.

2) Sum of all the vacuum bubbles as a normalization factor for the physical correlation functions;

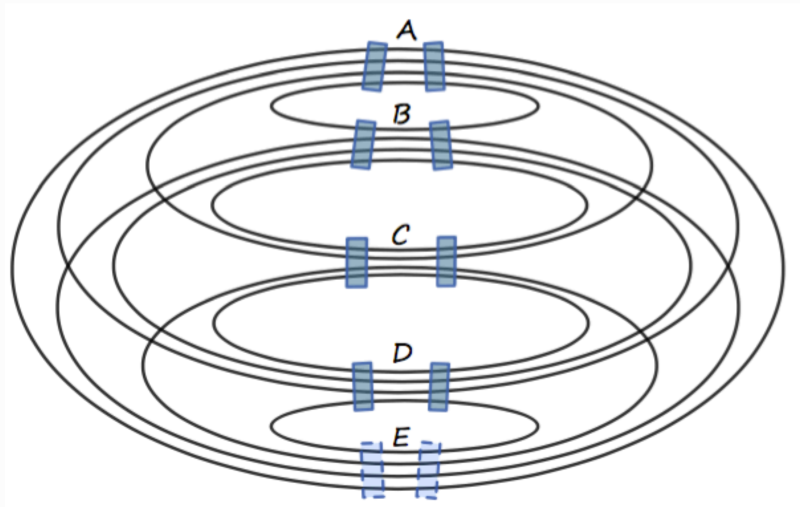
3) Restricting on the space of diagrams which are to be summed over. A complete classification of 2-complexes which are dual to non-degenerate geometries?

- For refining approach:(à la Dittrich)

There is no concern of the melonic dominance. Possible phase transition is indicated by the distinct behaviors of the model in different range of η.

Thank you!

A simple example:



Cable diagram of a super melon.

We can simply read out its amplitude through the graph structure!

$$\tilde{A}_{melon} = \sum_{\{j_{AB},j_{AC}\dots,j_{CD}\}} \prod_{l} \frac{(2j_l+1)^{\eta+1}}{(1+\rho^2)^{24 j_l}} \cdot F_\rho^2\,(\sum_{l\in\Theta_E} 2j_l) \cdot \prod_{\alpha} \left[F_\rho^2\,(\sum_{l\in\Phi_\alpha} 2j_l) \cdot (\sum_{l\in\Phi_\alpha} 2j_l+1)^{\eta-1} \right]$$

where $\alpha \in \{A,B,C,D\}$ the set $\Theta_E = \{AB, BC, AC, AD, BD, CD\}$,

$\Phi_A = \{AB, AC, AD\}, \Phi_B = \{AB, BC, BD\}, \Phi_C = \{BC, CD, AC\}, \Phi_D = \{CD, BD, AD\}.$

(Extra slides)