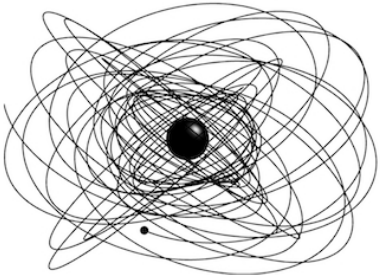
Gravitational self-force
in extreme-mass-ratio binaries



Leor Barack
University of Southampton

In memory of

Steven Detweiler

1947-2016

20 years ago...

Gravitational radiation reaction to a particle motion

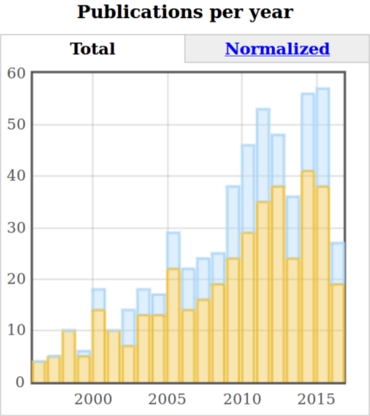
Yasushi Mino*
Department of Earth and Space Science, Graduate School of Science, Osaka University, Toyonaka 560, Japan and Department of Physics, Faculty of Science, Kyoto University, Kyoto 606-01, Japan

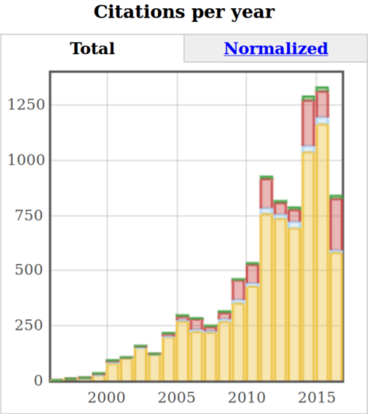
Misao Sasaki[†] and Takahiro Tanaka[‡]
Department of Earth and Space Science, Graduate School of Science, Osaka University, Toyonaka 560, Japan
(Received 10 June 1996)

Axiomatic approach to electromagnetic and gravitational radiation reaction of particles in curved spacetime

Theodore C. Quinn and Robert M. Wald
Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 S. Ellis Avenue, Chicago, Illinois 60637-1433
(Received 24 October 1996)

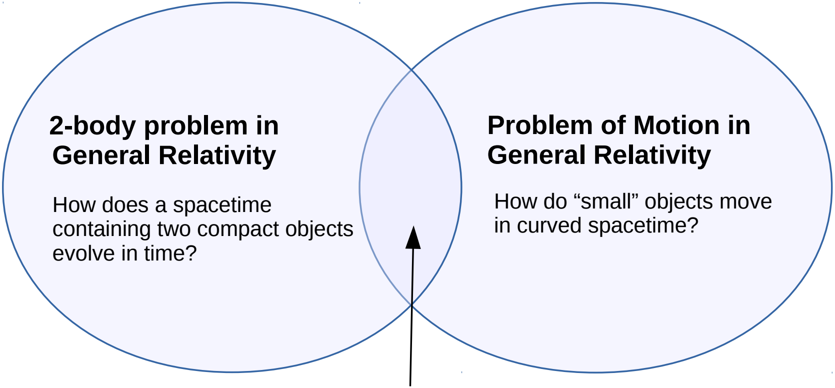
Self-force literature 1996-2016*





*ADS summary for papers containing "self-force" in title or abstract (1 July 2016)

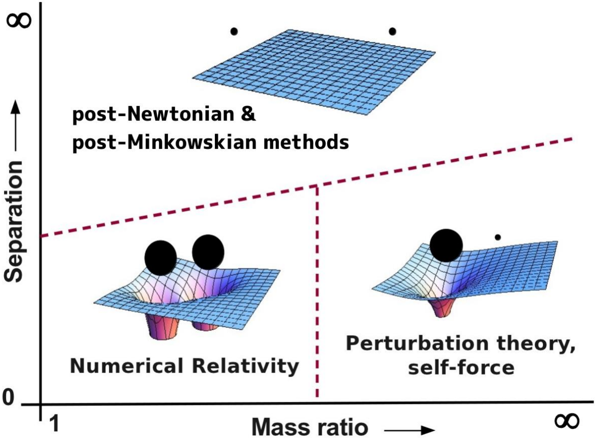
Physical context(s)



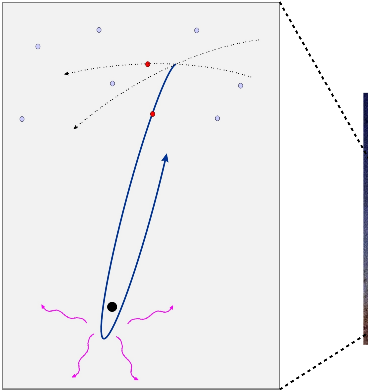
This talk:

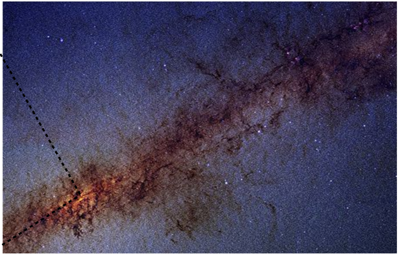
Gravitational self-force in extreme-mass-ratio binaries

Domains of the 2-body problem in GR



Extreme-Mass-Ratio Inspirals (EMRIs) in Nature





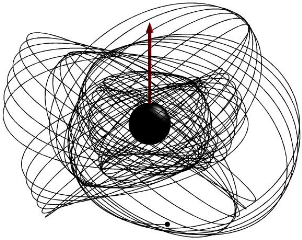
- eLISA sensitive to $M_{\rm MBH} \sim 10^{5.5}$-$10^{7.5} M_\odot \Rightarrow$ mass ratio $\eta \sim 1 : 10^4$-10^7.
- eLISA sees 10s-1000s(?) EMRIs out to $z \sim$ a few.
- $(T_{\rm orb} \sim \textit{hour}) \ll (T_{\rm RR} \sim T_{\rm orb}/\eta \sim \textit{yrs})$

EMRIs as probes of strong-field geometry

Assuming central object is a Kerr BH:

- Orbit tri-periodic (1 rotation + 2 librations)
- Orbit ergodic (space-filling) in general
- Principal elements drift in time $\rightarrow$ radiation
- Positional elements drift in time $\rightarrow$ precession

[movie]



credit: S. Drasco

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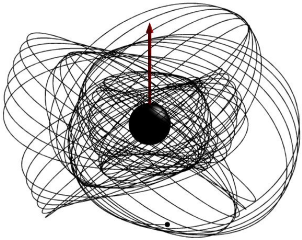
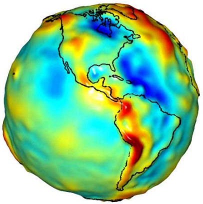


credit: S. Drasco



credit: NASA

- Excellent probe of strong-field geometry:
  - Precision "black-hole geodesy"
  - Tests of GR
- Need accurate templates for matched filtering!

"Capra Programme"

- Calculate EMRI orbits and waveforms, phase-accurate over $T_{\rm RR}$
- Strong field (no resort to PN)
- Generic eccentricity, inclination, spins
- Accuracy requirement for local self-force:
  $\Phi = \Phi_0 + \Omega\Delta t + \dot{\Omega}\Delta t^2 + \ldots$
  To keep $\delta(\dot{\Omega}\Delta t^2) \lesssim 1$ over $\Delta t = T_{\rm RR}$
  need $\delta(\dot{\Omega}) \lesssim T_{\rm RR}^{-2} = O(\eta^2)$
  $\Rightarrow$ Second-order self-force

More recent pursuits:

- Feed into PN theory
- Calibrate EOB potentials
- Other applications (cosmic censorship,...)

“Capra Programme”

participants of the
17th Capra meeting
(Caltech 2014)

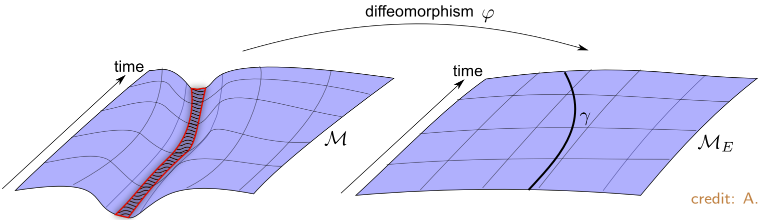
Plan for rest of the talk

- **Self-force: From foundations to computation**
  - Derivation of the equation of motion
  - Calculation methods
- **A sample of results:**
  - radiative evolution
  - "post-geodesic" physical effects: ISCO shift, periastron & spin precession, self-tides, redshift,. . .)
  - contact with other approaches
  - self-force as a "cosmic censor"
- **Survey of what the talk did not cover**
- **Summary of progress & outlook**

From Foundations to Computation

Problem of motion

FIELD degrees of freedom → PARTICLE degrees of freedom



credit: A. Pound

Body in exact spacetime

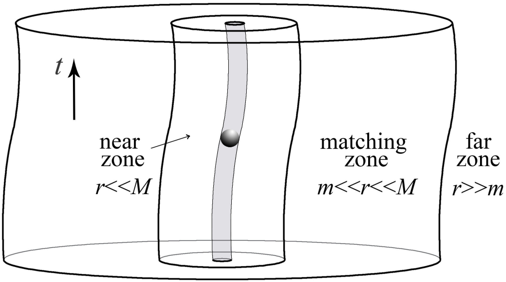
Effective representation of motion in background spacetime

Guiding principle:
"point particles" don't make sense as fundamental objects in GR,
but "point particle equation of motion" does — in a certain effective way.

Matched Asymptotic Expansions

Mino, Sasaki & Tanaka (1997), Poisson (2003)

building on early works by Burke, d'Eath, Kates, Thorne & Hartle,. . .

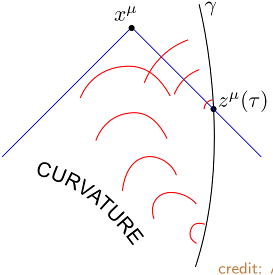


- Trajectory defined on background spacetime using a suitable far-zone limit; constrained by matching near & far expansions of the metric in the matching zone.
- No resort to "point particles": notion *derived* rather than assumed
- More rigorous derivation by Gralla & Wald (2008) using a 1-parameter metric family (extending work by Geroch & Ehlers on geodesic motion).

Equation of Motion at 1st post-geodesic order

Metric perturbation at x^μ is a sum of "**direct**" and "**tail**" contributions:

$$g^{\rm full}_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h^{\rm direct}_{\alpha\beta} + h^{\rm tail}_{\alpha\beta}$$



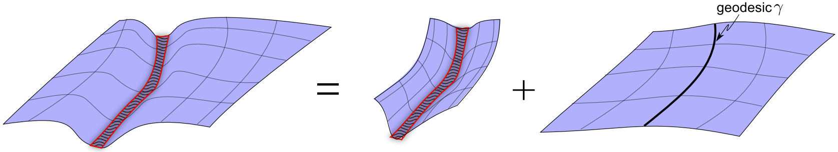
credit: A. Pound

$$\begin{aligned} \ddot{z}^\alpha &= -\frac{1}{2}(g^{\alpha\beta}_{(0)} + u^\alpha u^\beta)\, u^\gamma u^\delta \left(2\nabla^{(0)}_\delta h^{\rm tail}_{\beta\gamma} - \nabla^{(0)}_\beta h^{\rm tail}_{\gamma\delta}\right)\Big|_{z(\tau)} \\ &=: \; F^\alpha_{\rm self}/m \end{aligned}$$

“R field” reformulation (Detweiler & Whiting 2003)

- $h^{\rm tail}_{\alpha\beta}$ is **not** a vacuum solution of the linearized Einstein equations
- But one can construct a vacuum solution $h^{R}_{\alpha\beta}$ [associated with a certain (a-causal) Green function in the Hadamard representation] such that

$$\begin{aligned} F^{\alpha}_{\rm self} &= m\nabla^{\alpha\beta\gamma} h^{\rm R}_{\beta\gamma} \\ &= m\nabla^{\alpha\beta\gamma}\left(h_{\beta\gamma} - h^{\rm S}_{\beta\gamma}\right) \end{aligned}$$

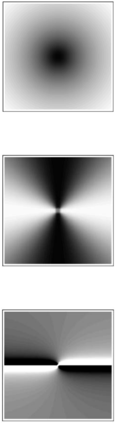


full metric $g^{\rm full}_{\alpha\beta}$ = “self-field” $h^{\rm S}_{\alpha\beta}$ + effective metric $g^{(0)}_{\alpha\beta} + h^{\rm R}_{\alpha\beta}$

- **Interpretation:** orbit is a geodesic in the effective metric.
- Similar result for extended material objects (Harte 2010), 2nd-order self-force (Pound 2012), non-perturbative (Harte 2012)

Self-force and gauge

- Self-force is gauge-dependent,
  but $\{F^{\alpha}_{\rm self}, h_{\alpha\beta}\}$ contain invariant information
- EoM originally formulated in Lorenz gauge, $\nabla^{\beta}\bar{h}_{\alpha\beta} = 0$.



- **Generalizations:**
  - Continuous deformations of Lorenz (LB & Ori 2001)
  - Direction-dependent (bounded) deformations of Lorenz (Gralla & Wald 2008)
  - Parity-regular gauges (Gralla 2011)
  - Radiation gauges (Pound, Merlin & LB 2014)

Last generalization allows convenient calculation via Teukolsky's formalism.

Practical schemes in black-hole spacetimes:
I. Mode-sum method (LB & Ori 2000)

- Subtraction of $h^S_{\alpha\beta}$ done mode-by-mode in a multipole expansion about large BH:

$$
\begin{aligned}
F^{\alpha}_{\rm self}(z(\tau)) &= m\sum_{\ell=0}^{\infty}\left[(\nabla^{\alpha\beta\gamma}h_{\beta\gamma})^{\ell}-(\nabla^{\alpha\beta\gamma}h^{S}_{\beta\gamma})^{\ell}\right]_{x\to z(\tau)} \\
&= \sum_{\ell=0}^{\infty}\left[m(\nabla^{\alpha\beta\gamma}h_{\beta\gamma})^{\ell}_{x\to z(\tau)}-A^{\alpha}(z)\ell-B^{\alpha}(z)-C^{\alpha}(z)/\ell\right]-D^{\alpha}(z)
\end{aligned}
$$

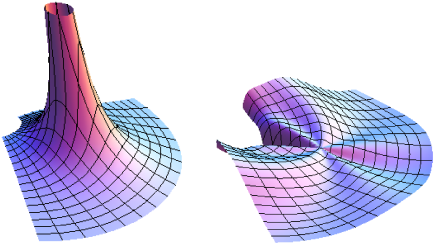
- Regularization parameters derived analytically from local form of $h^S_{\alpha\beta}$; known for generic orbits in Kerr (LB & Ori 2000-03)
- Higher-order parameters improve convergence (Heffernan, Ottewill, Wardell 2012-14)
- Numerical input: Modes of $h_{\beta\gamma}$ obtained by solving metric perturbation equations with a particle (delta function) source and retarded boundary conditions.

Practical schemes in black-hole spacetimes:
II. Puncture (or "effective source") method

- Analytically construct Puncture field $h^P_{\alpha\beta} \approx h^S_{\alpha\beta}$ so that $\nabla h^P = \nabla h^{\mathrm{S}}$ at particle.
- Write linearized field equation $\delta G_{\mu\nu}(h) = T_{\mu\nu}$ in "punctured" form

$$\delta G_{\mu\nu}(h - h^P) = T_{\mu\nu} - \delta G_{\mu\nu}(h^P) =: S^{\mathrm{eff}}_{\mu\nu}$$

- Numerically solve for Residual field $h^{\mathrm{Res}} := h - h^P$. Then $F_{\mathrm{self}} = m\nabla h^{\mathrm{Res}}$



credit: J. Thornburg & B. Wardell

- Implementations (2007–) by
  - LB, Golbourn, Dolan, Thornburg,...
  - Detweiler, Vega, Diener, Wardell,...

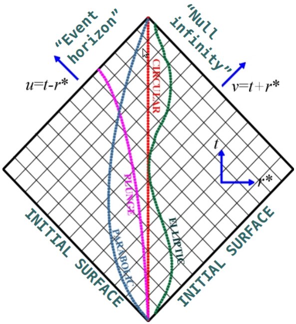
Numerical implementation strategies

Time-domain approach

E.g.: Discretize linearized Einstein Field Equation in Lorenz-gauge on a characteristic grid and evolve in 1+1d. (LB & Lousto; LB and Sago)

Variants:

- 2+1d in Kerr (Dolan, Wardell & LB)
- finite elements (Canizares & Sopuerta)
- Mesh refinement & compactification (Thornburg)



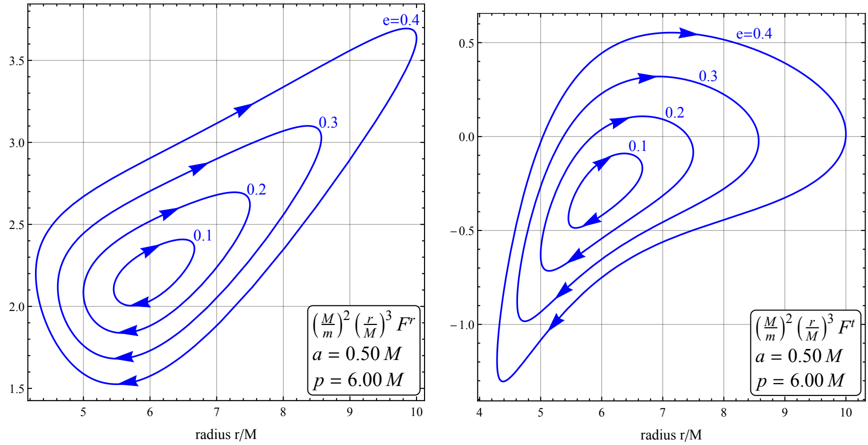
Frequency-domain approach

- In Schwarzschild: solve ODEs for Fourier modes of metric perturbation (Burko, Detweiler, LB, Warburton, Akcay, Kavanagh, Ottewill, Evans, Hopper,...)
- In Kerr: Reconstruct metric perturbation from Fourier modes of curvature scalars (Friedman, Keidl, Shah, van de Meent,...)

A SAMPLE OF RESULTS

Self-force along fixed geodesic orbits

sample results for equatorial orbits in Kerr ($a = 0.5M$)

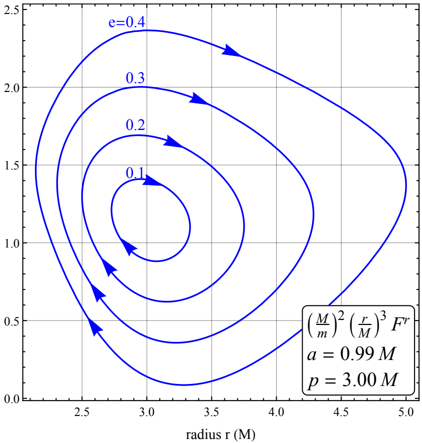


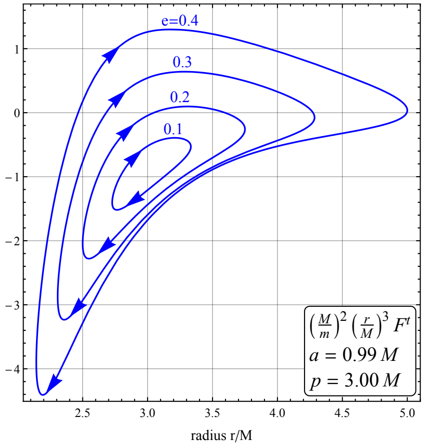
Maarten van de Meent (2016)

using numerical implementation of Mano-Suzuki-Takasugi method
+ metric reconstruction + mode-sum regularization.

Self-force along fixed geodesic orbits

sample results for equatorial orbits in Kerr ($a = 0.99M$)





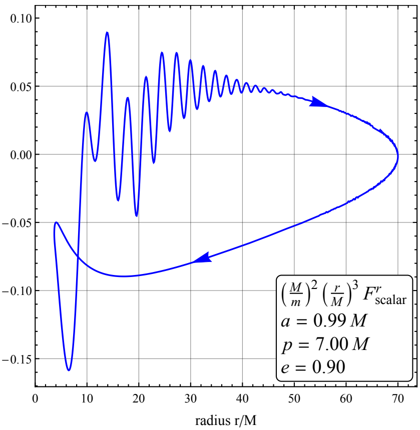
Maarten van de Meent (2016)

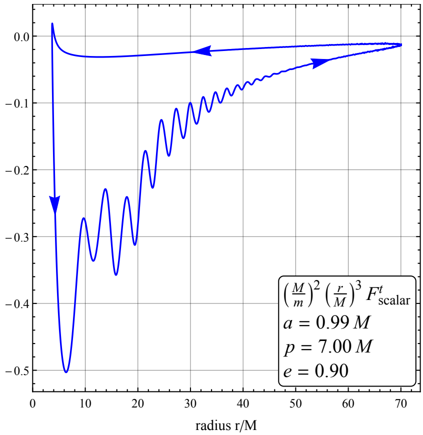
using numerical implementation of Mano-Suzuki-Takasugi method

$+$ metric reconstruction $+$ mode-sum regularization.

Self-force along fixed geodesic orbits

sample results for equatorial orbits in Kerr ($a = 0.99M, e = 0.9$)





Thornburg and Wardell (2016)

Scalar-field self-force, using a 2+1d implementation of the puncture method

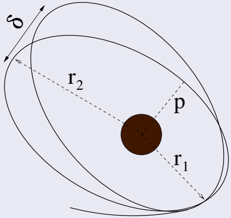
Self-forced orbital evolution (in Schwarzschild)

Warburton, Akcay, LB, Gair & Sago (2012); Osburn, Warburton & Evans (2016)

"Instanteneous" geodesic parametrized by $\{p, e, \chi_0\}$:

$$p = \frac{2r_1 r_2}{r_2 + r_1}, \qquad e = \frac{r_2 - r_1}{r_2 + r_1}$$

$$r(t; p, e, \chi_0) = \frac{p}{1 + e\cos(\chi(t) - \chi_0)}$$



Method of osculating geodesics (Pound & Poisson 2008)

Inspiral orbit reconstructed as a smooth sequence of tangent geodesics:

$$p \to p(t): \qquad \frac{dp}{dt} = \text{terms involving } F_{\text{self}}(\chi(t); p, e, \chi_0)$$

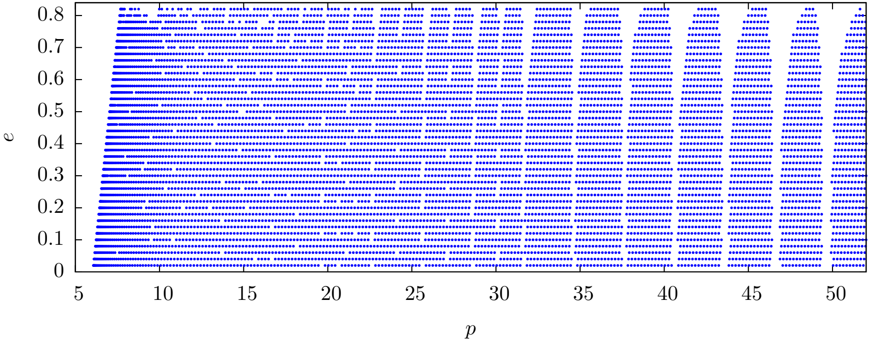
$$e \to e(t): \qquad \frac{de}{dt} = \cdots$$

$$\chi_0 \to \chi_0(t): \qquad \frac{d\chi_0}{dt} = \cdots$$

Self-forced orbital evolution (in Schwarzschild)

Warburton, Akcay, LB, Gair & Sago (2012); Osburn, Warburton & Evans (2016)

Preparing the self-force data...

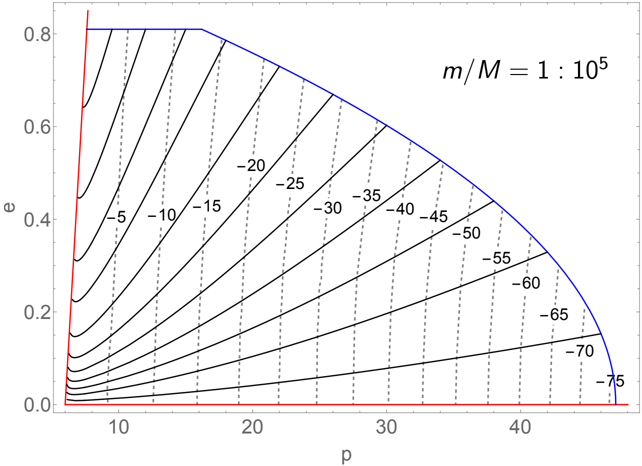


For each p, e write $F_{\rm self}(\chi - \chi_0; p, e)$ as a Fourier sum of $\chi - \chi_0$ harmonics.
Then interpolate coefficients over p, e plane.

This approximated self-force, calculated on fixed geodesics, differs by an amount of $O(m^3)$ from the true self-force acting on the evolving orbit .

Self-forced orbital evolution (in Schwarzschild)

Warburton, Akcay, LB, Gair & Sago (2012); Osburn, Warburton & Evans (2016)



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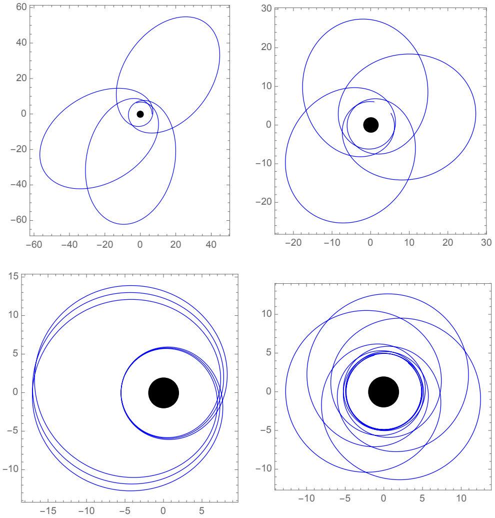


For $m/M = 10 : 10^6 M_\odot$, snapshots show orbit 2115.5, 500, 100 and 1 days to plunge.

Conservative effects of the self-force

Now "turn off" dissipation:

$$m u^\beta \nabla_\beta u^\alpha = F^\alpha_{\rm cons} := \frac{1}{2}\left[F^\alpha_{\rm self}(h^{\rm ret}) + F^\alpha_{\rm self}(h^{\rm adv})\right]$$

Motivation

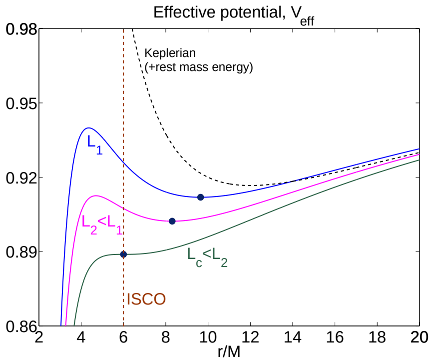
- Study secular effect of conservative piece on phase evolution
- Clean quantitative description of post-geodesic (finite-mass) effects
- Allows comparison with post-Newtonian predictions
- Strong-field calibration data for EOB potentials

O(m) shift in the ISCO frequency: Schwarzschild

Restoring force $\sim -mV''_{\rm eff} + F_{\rm self}$

vanishes at $r_{\rm isco} = 6M + O(m/M)$.

$$\Rightarrow \Omega_{\rm isco} = \sqrt{\frac{M}{r_{\rm isco}^3}} + \Delta\Omega_{\rm isco}$$



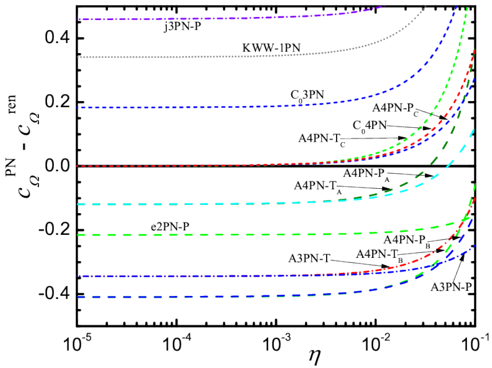
$$\left(\frac{\boldsymbol{\Delta\Omega_{\rm isco}}}{\boldsymbol{\Omega_{\rm isco}}}\right)_{\bf SF} = \mathbf{0.2513(6)\, m/M} \qquad \text{(LB \& Sago 2009)}$$

$$= 0.25101546(5)\, m/M \qquad \text{(Akcay, LB, Damour \& Sago 2012)}$$

$$\left(\frac{\Delta\Omega_{isco}}{\Omega_{isco}}\right)_{3PN} = 0.434913\ldots\; m/M$$

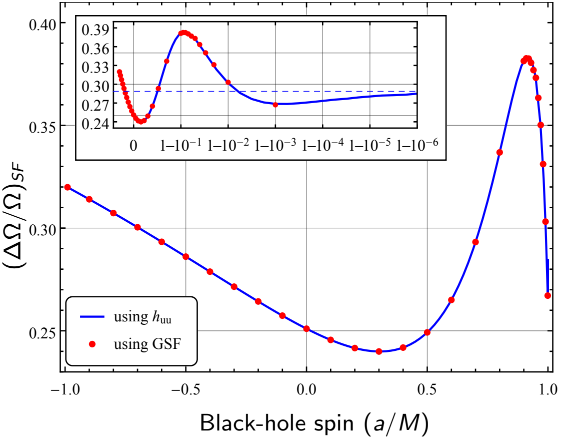
ISCO shift as an accurate strong-field benchmark

Method	$c_\Omega^{\rm PN}$	Δ_{c_Ω}
A4PN-P$_A$	1.132	-0.0955
A4PN-T$_A$	1.132	-0.0955
$C_0$3PN	1.435	0.1467
e2PN-P	1.036	-0.1717
KWW-1PN	1.592	0.2726
A3PN-P	0.9067	-0.2754
A3PN-T	0.9067	-0.2754
A4PN-P$_B$	0.8419	-0.3272
A4PN-T$_B$	0.8419	-0.3272
j3PN-P	1.711	0.3671
j2PN-P	0.6146	-0.5088
KWW-S	0.5610	-0.5515
$C_0$2PN	0.5833	-0.5338
E_h3PN	0.4705	-0.6240
e3PN-P	2.178	0.7409
A2PN-P	0.2794	-0.7767
A2PN-T	0.2794	-0.7767
E_h2PN	0.0902	-0.9279
E_h1PN	-0.01473	-1.011
E_h-S	-0.05471	-1.044
HH-S	-0.1486	-1.119
j1PN-P	-0.1667	-1.133
KWW-2PN	-1.542	-2.232
j-P-S	-2.104	-2.682
KWW-3PN	4.851	2.877
HH-1PN	6.062	3.844
HH-2PN	-12.75	-11.19
HH-3PN	25.42	19.32



Results from Favata 2010

O(m) shift in the ISCO frequency: Kerr



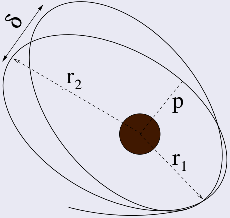
Isoyama, LB, Dolan, Le Tiec, Nakano, Shah, Tanaka, Warburton (2014) by minimizing $E_{\rm binding}$ derived from interaction Hamiltonian $H = -\frac{1}{2m} h^{\alpha\beta}_{R,sym} p_\alpha p_\beta$.

Van de Meent (2016) using a direct self-force calculation

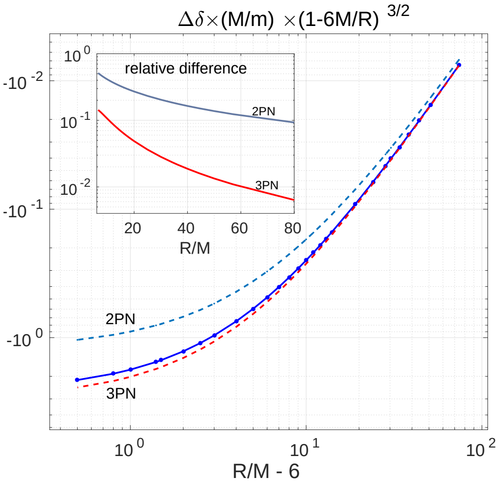
Warburton, Casals, Kavanagh, Ottewill & Wardell (2016) obtained $a \to M$ limit analytically: $(\Delta\Omega/\Omega)_{SF} \to \frac{1}{2\sqrt{3}}$

O(m) correction to the periastron advance in slightly eccentric orbits (Damour 2010; LB, Damour & Sago 2010)

$$\delta = 2\pi\left[\left(1-\frac{6M}{R}\right)^{-1/2}-1\right]+\Delta\delta(R)$$

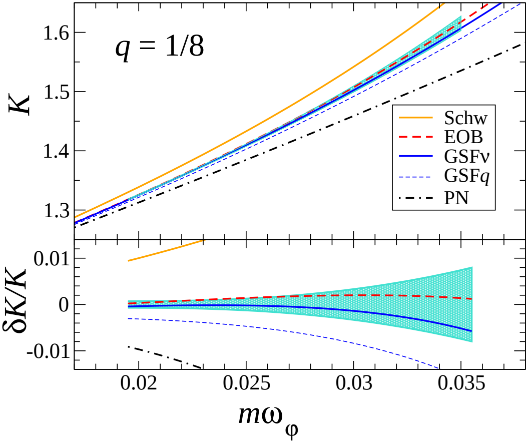


Use $R := \left(\frac{M+m}{\Omega^2}\right)^{1/3}$ as an "invariant" parametrization of the limiting circular orbit.



Periastron advance: comparison with full NumRel

(Le Tiec, Mruoe, LB, Buonanno, Pfeiffer & Sago 2011)



$K = \delta + 1$

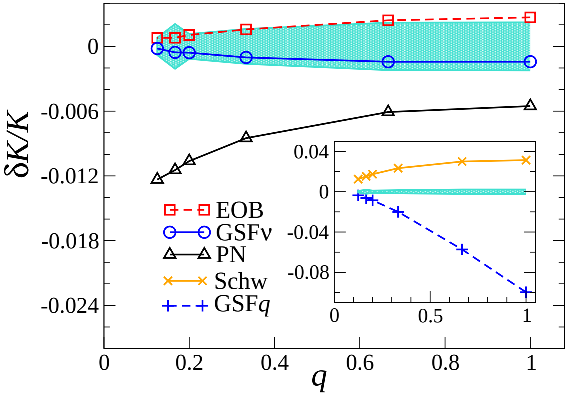
$q = m/M$

GSFq: $\Delta\delta \propto \frac{m}{M}$

GSFν: $\Delta\delta \propto \frac{mM}{(M+m)^2}$

Periastron advance: comparison with full NumRel

(Le Tiec, Mruoe, LB, Buonanno, Pfeiffer & Sago 2011)



Numerical Relativity simulations can be used to **"predict"** the O(m) precession effect where direct self-force results are not yet available. For Kerr: Le Tiec, Buonanno, Mroue, Pfeiffer, Hemberger, Lovelace, Kidder, Scheel, Szilagyi, Taylor & Teukolsky (2013).

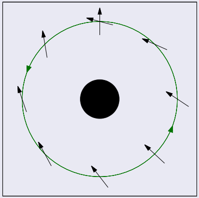
"Self-torque" and spin precession

(Dolan, Warburton, Harte, Le Tiec, Wardell & LB 2014)

In limit $s \ll m^2$, spin is parallel-transported along geodesic of $\underline{g + h^R}$:

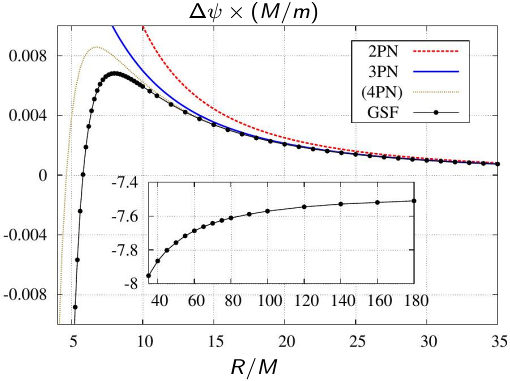
$$u^\beta \nabla_\beta^{(R)} u_\alpha = 0, \quad u^\beta \nabla_\beta^{(R)} s_\alpha = 0 \qquad \text{(Harte 2012)}$$

Circular orbit in Schwarzschild:
Spin undergoes simple precession:



$$\psi(R) = 1 - \sqrt{1 - 3M/R} + \boldsymbol{\Delta\psi}$$

Precession angle per radian angular motion



"Self-tides": $O(m)$ contribution to tidal field

Quadrupolar invariants (Dolan et al 2014); Octopolar invariants (Nolan et al 2015)

Circular orbit in Schwarzschild:

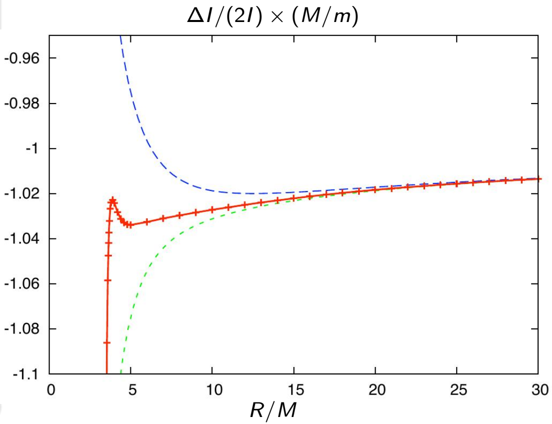
$\mathcal{E}_{\alpha\gamma} = R_{\alpha\beta\gamma\delta} u^\beta u^\delta$ tidal field

$\mathcal{B}_{\alpha\gamma} = R^*_{\alpha\beta\gamma\delta} u^\beta u^\delta$ frame-drag field

($R_{\alpha\beta\gamma\delta}$ corresponding to $g + h^R$) give 4 ind. invariants (3 eigenvalues + 1 angle between eigenbases), from which other curvature invariants can be constructed.

E.g., the Kretschmann Scalar

$$I := C^{\alpha\beta\gamma\delta} C_{\alpha\beta\gamma\delta} = \frac{3M^2}{R^6} + \Delta I(R)$$



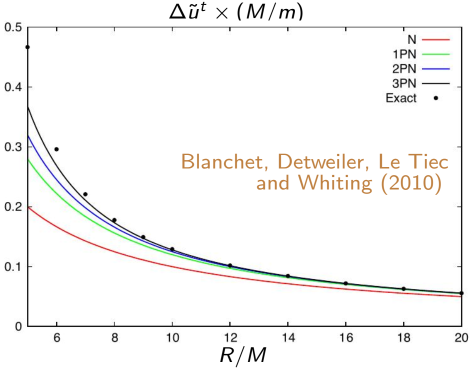
Detweiler's redshift

Circular orbits Detweiler (2008); eccentric orbits LB & Sago (2011)

$$\tilde{u}^t := \frac{dt}{d\tilde{\tau}} = \left(1 - \frac{3M}{R}\right)^{-1/2} + \Delta\tilde{u}^t(R)$$

where $\tilde{u}^\alpha$ is 4-velocity in smooth effective metric: $(g^{(0)}_{\alpha\beta} + h^R_{\alpha\beta})\tilde{u}^\alpha\tilde{u}^\beta = -1$

- First contact with PN theory (Detweiler 2008)
- Comparison between self-force calculations in different gauges (Sago, LB & Detweiler 2008)
- $\Delta\tilde{u}^t$ related to **binding energy** in PN theory (Le Tiec *et al* 2012)
- $\Delta\tilde{u}^t$ related to **interaction Hamiltonian** in perturbation theory (Isoyama *et al*, in prep.)

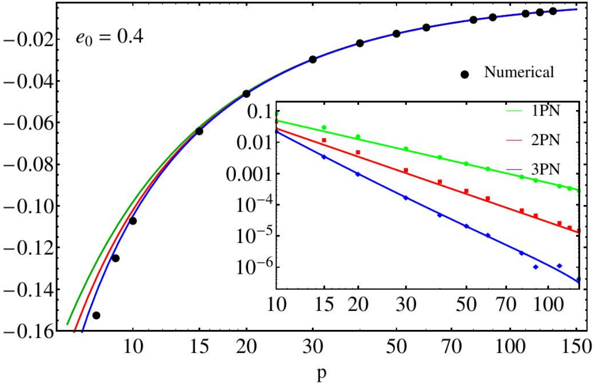


Orbit-averaged redshift

$$\langle \tilde{u}^t \rangle = \frac{t \text{ period}}{\tilde{\tau} \text{ period}} = \langle \tilde{u}^t \rangle_0(\Omega, \omega_r) + \Delta\langle \tilde{u}^t \rangle(\Omega, \omega_r)$$

using rotational and epicyclic frequencies (or $\{p, e\}$ defined from these frequencies) as "invariant" orbital parameters

$\Delta\langle \tilde{u}^t \rangle$



Results (for Schwarzschild) from Akcay, Le Tiec, LB, Sago & Warburton (2015)

Similar results for Kerr: van de Meent & Shah (2015).

Post-Newtonian expansion of the self-force

- Numerical extraction of high-order PN parameters using arbitrary-precision computer algebra (Shah et al; Johnson-McDaniel et al 2014-16)
- Analytical calculation of high-order PN parameters using Mano-Suzuki-Takasugi method (Bini & Damour; Kavanagh et al; Hopper et al 2013-2016)
- calculation in Kerr through 8.5PN, $O(e^2)$ and $O(a^2)$: Bini, Damour and Geralico (2016)

n	δU^{e^n}	Refs.
0	22.5PN	Kavanagh et al.[25]
2	9.5PN	This paper
4	9.5PN	This paper
6	4PN	Hopper et al.[28]
8	4PN	Hopper et al.[28]
10	4PN	Hopper et al.[28]
12	4PN	This paper
14	4PN	This paper
16	4PN	This paper
18	4PN	This paper
20	4PN	This paper

Table from Bini, Damour and Geralico (2016)

Self-force and weak cosmic censorship

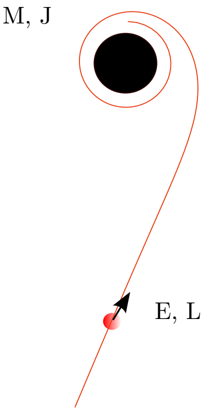
Can we overspin a Kerr black hole by throwing a small particle into it?

Is it possible to achieve $J + mL > (M + mE)^2$?

Wald (1974):
No, if $J = M^2$ and self-force is ignored.

Jacobson & Sotiriou (2009):
Yes, if $J = M^2 - \epsilon^2$ and self-force is ignored.

Barausse, Cardoso & Khanna (2010):
Yes, in $J = M^2 - \epsilon^2$ case, even if radiation loses are taken into account!



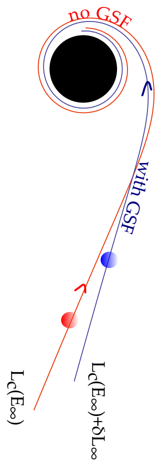
Self-force and weak cosmic censorship

Can we overspin a Kerr black hole by throwing a small particle into it?

Colleoni & LB (2015);
Colleoni, LB, Shah & van de Meent (2015):

No, if full effect of the self-force is taken into account!

However, by fine-tuning the initial parameters it is possible to reach extremality
$\Rightarrow$ need higher-order self-force information.



- Similar conclusions for problem of overcharging a Reissner-Nordström black hole
  Isoyama, Sago & Tanaka (2011)
  Zimmerman, Vega, Poisson & Haas (2013)

What I have not covered in this talk

- Calibration of EOB potentials using self-force data (Damour 2010, and work by many others since)
- The 2nd-order self-force:
  - "UV" regularization (Detweiler 2012; Gralla 2012; Pound 2012-14)
  - "IR" regularization (Pound 2015-16)
  - effect of internal structure (Flanagan and Moxon, preliminary)
  - first numerical calculation (Miller, Pound, Warburton, Wardell & LB — coming soon!)
- EMRI as a dynamical system; Hamiltonian formulation (Vines & Flanagan 2015; Isoyama, Pound, Tanaka et al 2016)
- Dynamical effects of resonant crossing (Flanagan, Hughes & Ruangsri 2014; van de Meent 2014)
- Miscellanea: self-force in higher dimensions; dependence on internal structure of central object; inspiral into extremal black holes; self-force on unbound orbits, . . .

Summary of progress and an outlook

2004
GR17 @ Dublin

- 1st-order self-force formulation
- Mode-sum method
- Numerical calculations with scalar-field toy model in spherical symmetry

2016
GR21 @ NYC

- Rigorous 1st-order self-force formulation
- 2st-order self-force formulation
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- Extraction of post-geodesic dynamical effects
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- Countdown for eLISA launch! (2029?)