Energetic causal sets

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work with Marina Cortes

arXiv:1307.6167, arXiv:1308.2206, arXiv:1311.0186,
arXiv:1407.0032, and by Wolfgang Wieland

Thanks to, Laurent Freidel, Markus Muller, Stuart Kauffman,
Roberto Mangabeira Unger, Wolfgang Weiland

The unique events ontology

Time is fundamental and irreversible. Its activity is the process that creates new events from present events.

Spacetime is emergent and relational. Spacetime intervals reflect past causal relations among events created by the process that creates events.

Relations must relate something! The events must have intrinsic properties as well as relational properties.

Energy and momentum are these fundamental, intrinsic properties.

Energy and momentum are fundamental, intrinsic properties.

Relative locality teaches us that the primary geometry is the geometry of momentum space.

Causality is also primary.

Spacetime and its geometry are not primary: they are secondary and emergent.

Einstein taught us that concepts like simultaneity and locality are constructed from primary observations of energy and momentum.

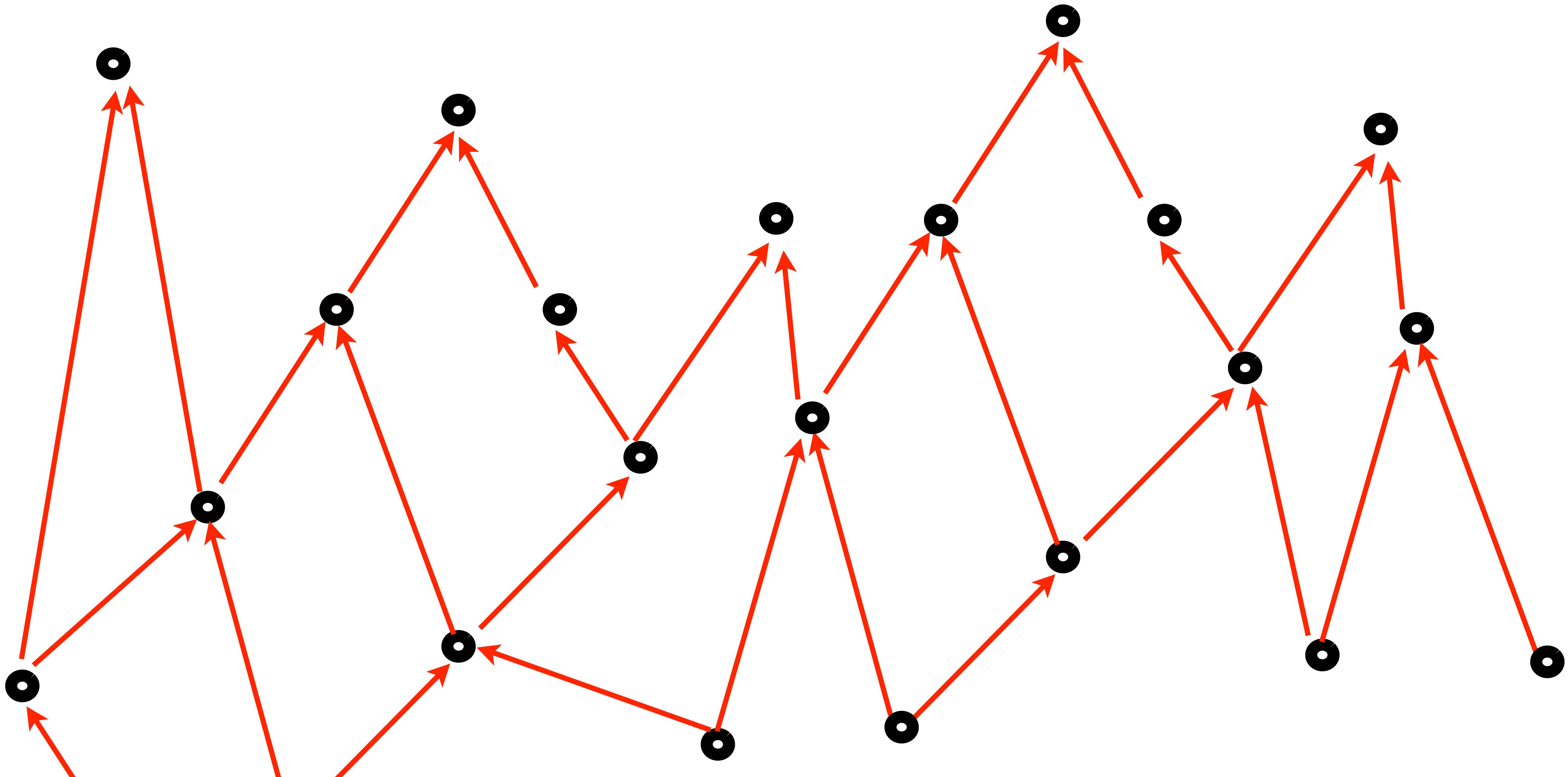
Spacetime is emergent and reconstructed only at the classical level.

If spacetime is emergent, then so is locality.
Fundamentally there is neither locality nor non-locality.
Just causality.

Fundamentally there is no non-commutivity, no uncertainty relations, no hbar. These all emerge with spacetime.

All we need of quantum is the amplitude law.

Causal sets:

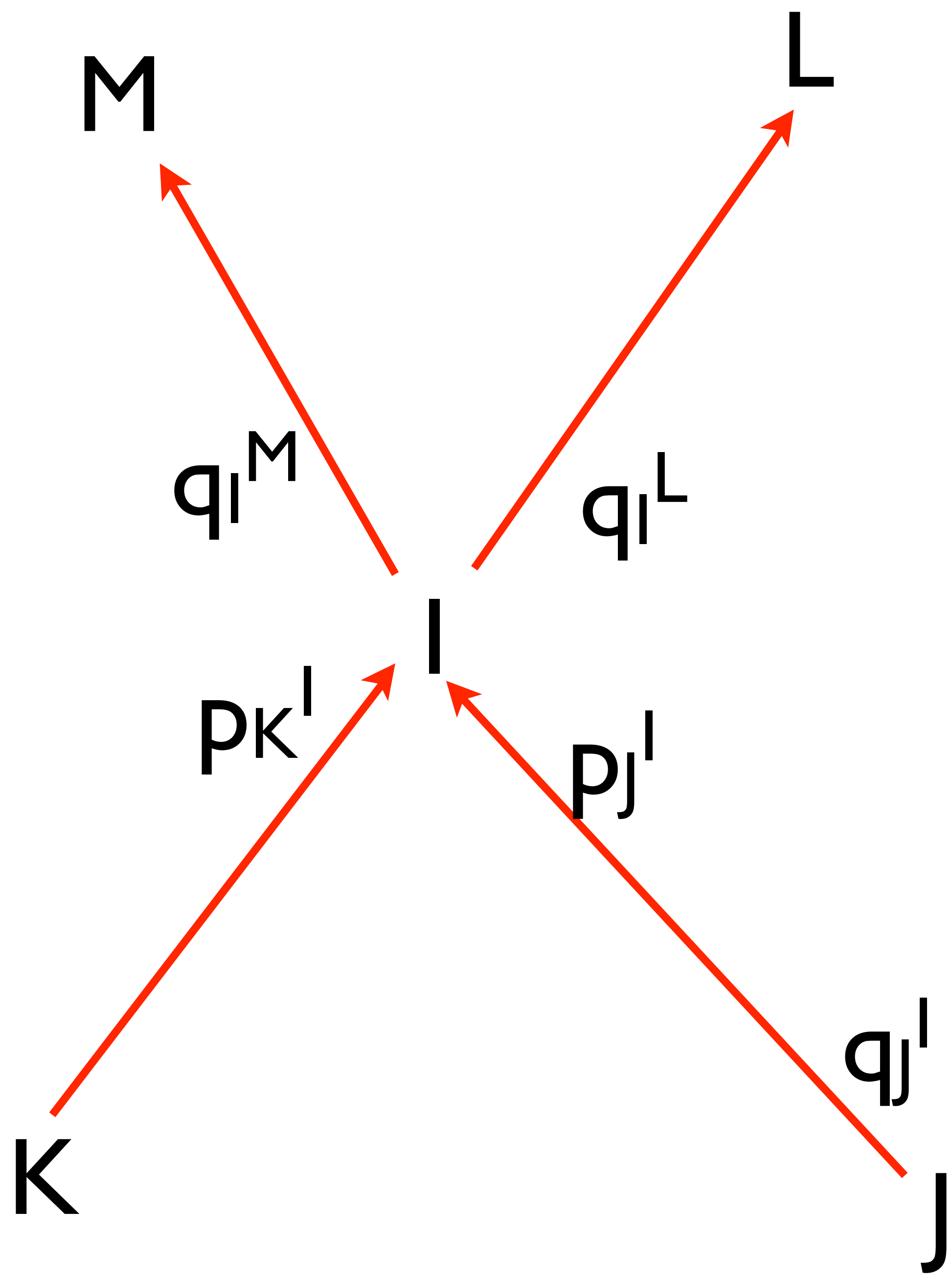


Energetic causal sets:

Each link, connecting E_I to one of its parents, E_j, has two momenta, an incoming momenta $p_J{}^I$ and an outgoing momentum $q_I{}^J$.

The total momenta of an event

$$P_a^I = \sum_J p^J_{aI}$$



Constraints:

The momenta are propagated to the new event and links by three constraints:

Conservation at each event:

$$\mathcal{P}_a^I = \sum_K p_{aK}^I - \sum_L q_{aI}^L = 0$$

Parallel transport on each edge:

$$\mathcal{R}_{aI}^K = p_{aI}^K - \mathcal{U}_{Ia}^{Kb} q_{bI}^K = 0$$

Energy-momentum relations:

$$\mathcal{C}_K^I = \frac{1}{2}\eta^{ab} p_{aK}^I p_{bK}^I + m^2 = 0$$

$$\tilde{\mathcal{C}}_K^I = \frac{1}{2}\eta^{ab} q_{aK}^I q_{bK}^I + m^2 = 0$$

Constraints: In the following we choose U=I and m=0

The momenta are propagated to the new event and links by three constraints:

Conservation at each event:

$$\mathcal{P}^I_a = \sum_K p^I_{aK} - \sum_L q^L_{aI} = 0$$

Parallel transport on each edge:

$$\mathcal{R}^K_{aI} = p^K_{aI} - \mathcal{U}^{Kb}_{Ia} q^K_{bI} = 0$$

Energy-momentum relations:

$$\mathcal{C}^I_K = \frac{1}{2}\eta^{ab} p^I_{aK} p^I_{bK} + m^2 = 0$$

$$\tilde{\mathcal{C}}^I_K = \frac{1}{2}\eta^{ab} q^I_{aK} q^I_{bK} + m^2 = 0$$

No spacetime.
The only geometry that comes in is the metric of momentum space.

Quantum dynamics:

All we need to define quantum dynamics is the amplitude law. Events, I, have amplitudes A_I.

Process P: $$\mathcal{A}[P] = \prod_I \mathcal{A}_I$$

Fix incoming and outgoing and sum:

$$\mathcal{A}[p_a^{in,\ I}; q_a^{out,\ I}] = \sum_P \mathcal{A}[P]$$

Probability:

$$\mathcal{P}[p_a^{in,\ I}; q_a^{out,\ I}] = \left|\mathcal{A}[p_a^{in,\ I}; q_a^{out,\ I}]\right|^2$$

The total amplitude is defined by integrating over internal momenta, imposing the constraints

$$\mathcal{A}[P] = \int \Pi_{(IJ)} dp_a^{IJ} dq_a^{IJ} \delta(\mathcal{C}_a^{IJ}) \delta(\mathcal{R}_I^J) \Pi_I \delta(\mathcal{P}_a^I) \Pi_I \mathcal{A}_I$$

This is the complete definition of the theory.

No hbar
No space or spacetime
No commutation relations
No uncertainty principle

The total amplitude is defined by integrating over momenta, imposing the constraints

$$\mathcal{A}[P] = \int \Pi_{(IJ)} dp_a^{IJ} dq_a^{IJ} \delta(\mathcal{C}_a^{IJ}) \delta(\mathcal{R}_I^J) \Pi_I \delta(\mathcal{P}_a^I) \Pi_I \mathcal{A}_I$$

We introduce lagrange multipliers to exponentiate the constraints:

$$\mathcal{A}[P] = N[\mathcal{C}] \int \Pi_{(IJ)} dp_a^{IJ} dq_a^{IJ} d\mathcal{N}_I^J d\tilde{\mathcal{M}}_I^J \Pi_I dZ_I^a \; e^{\imath S^0}$$

With an action that is pure constraints:

$$S = \sum_I z_I^a \mathcal{P}_a^I + \sum_{(I,K)} \left(x_K^{aI} \mathcal{R}_{aI}^K + \mathcal{N}_I^K \mathcal{C}_K^I - \tilde{\mathcal{N}}_I^K \tilde{\mathcal{C}}_K^I \right)$$

lagrange multipliers

$A_I=1$ for simplicity

Z^a has dimensions of inverse momenta. It is just a lagrange multiplier. Inessential for the theory. But it will emerge as spacetime coordinates.

We introduce lagrange multipliers to exponentiate the constraints:

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lagrange multipliers

Classical physics from the stationary phase approximation:

$$S = \sum_I z_I^a \mathcal{P}_a^I + \sum_{(I,K)} \left(x_K^{aI} \mathcal{R}_{aI}^K + \mathcal{N}_I^K \mathcal{C}_K^I - \tilde{\mathcal{N}}_I^K \tilde{\mathcal{C}}_K^I \right)$$

lagrange multipliers

Constraints:

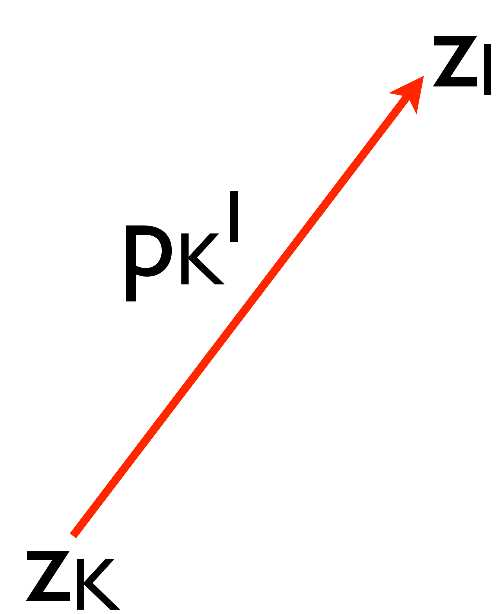
$$\mathcal{P}_a^I = \sum_K p_{aK}^I - \sum_L q_{aI}^L = 0 \qquad \mathcal{R}_{aI}^K = p_{aI}^K - \mathcal{U}_{Ia}^{Kb} q_{bI}^K = 0$$

$$\mathcal{C}_K^I = \frac{1}{2}\eta^{ab} p_{aK}^I p_{bK}^I = 0 \qquad \tilde{\mathcal{C}}_K^I = \frac{1}{2}\eta^{ab} q_{aK}^I q_{bK}^I = 0$$

Equations of motion:

$$z_I^a - z_K^a = p_K^{aI} \mathcal{M}_I^K$$

$$\mathcal{M}_I^K = \tilde{\mathcal{N}}_I^K - \mathcal{N}_I^K$$



The emergence of spacetime

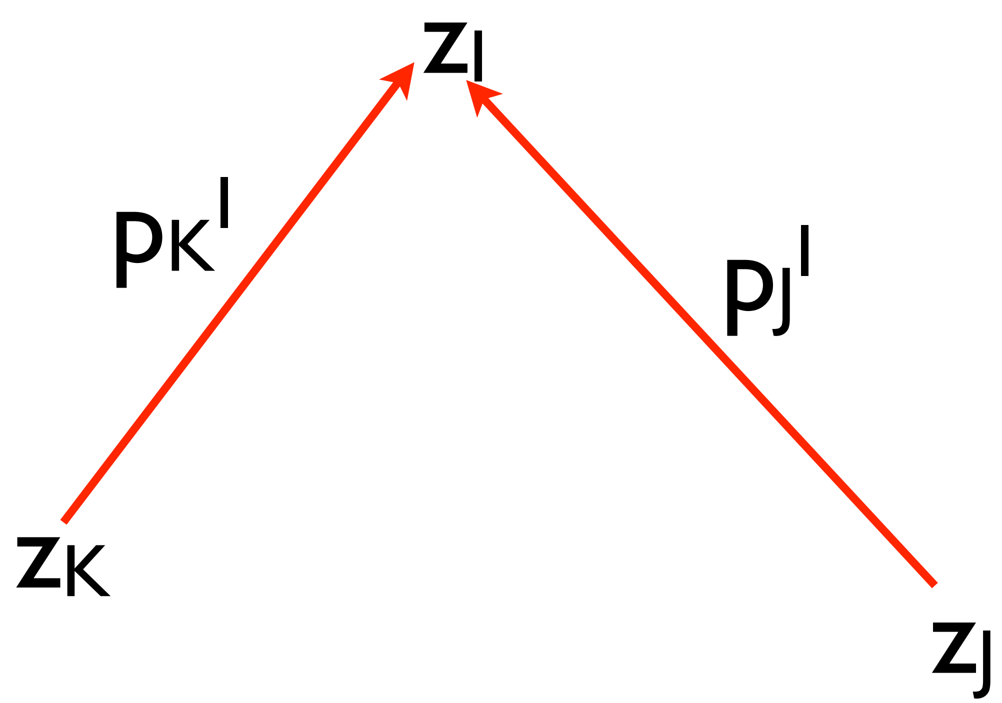
from the stationary phase approximation

Spacetime emerges when there are consistent solutions to all the equations:

$$z_I^a - z_K^a = p_K^{aI} \mathcal{M}_I^K$$

rescale z --> z/h to give spacetime coordinates units of length.

h is purely conventional.



Spacetime inherits its metric from momentum space:

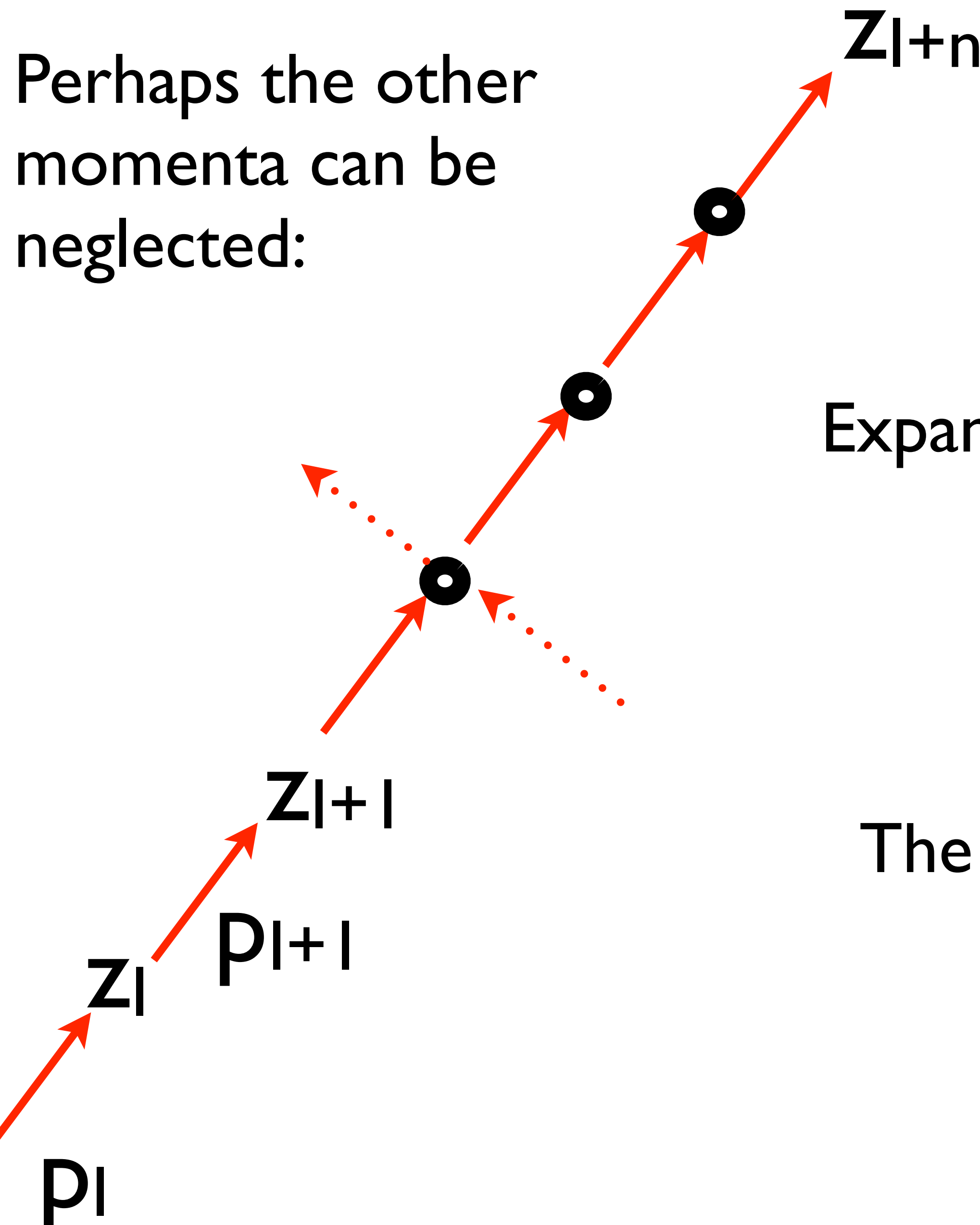
$$|z_I^a - z_K^a|^2 = (z_I^a - z_K^a)(z_I^b - z_K^b)\eta_{ab}$$
$$= (\mathcal{M}_I^K)^2 |p_K^{aI}|^2 = 0$$

U=1 gives flat spacetime

The emergence of massless particles and relativistic dynamics

Consider a long chain of simple events (one in and one out):

Perhaps the other momenta can be neglected:



Equations of motion:

$$p^I_a = p^{I+1}_a = p_a$$
$$z^a_{I+1} - z^a_I = p^{aI}\mathcal{M}_I$$

Expand in a small time interval: $z^a_{I+1} = z^a_I + \dot{z}^a(t)\Delta t$

The EoM is now: $\dot{z}^a(t) = \dfrac{\mathcal{M}_I}{\Delta t} p^a_I = n p^a_I$

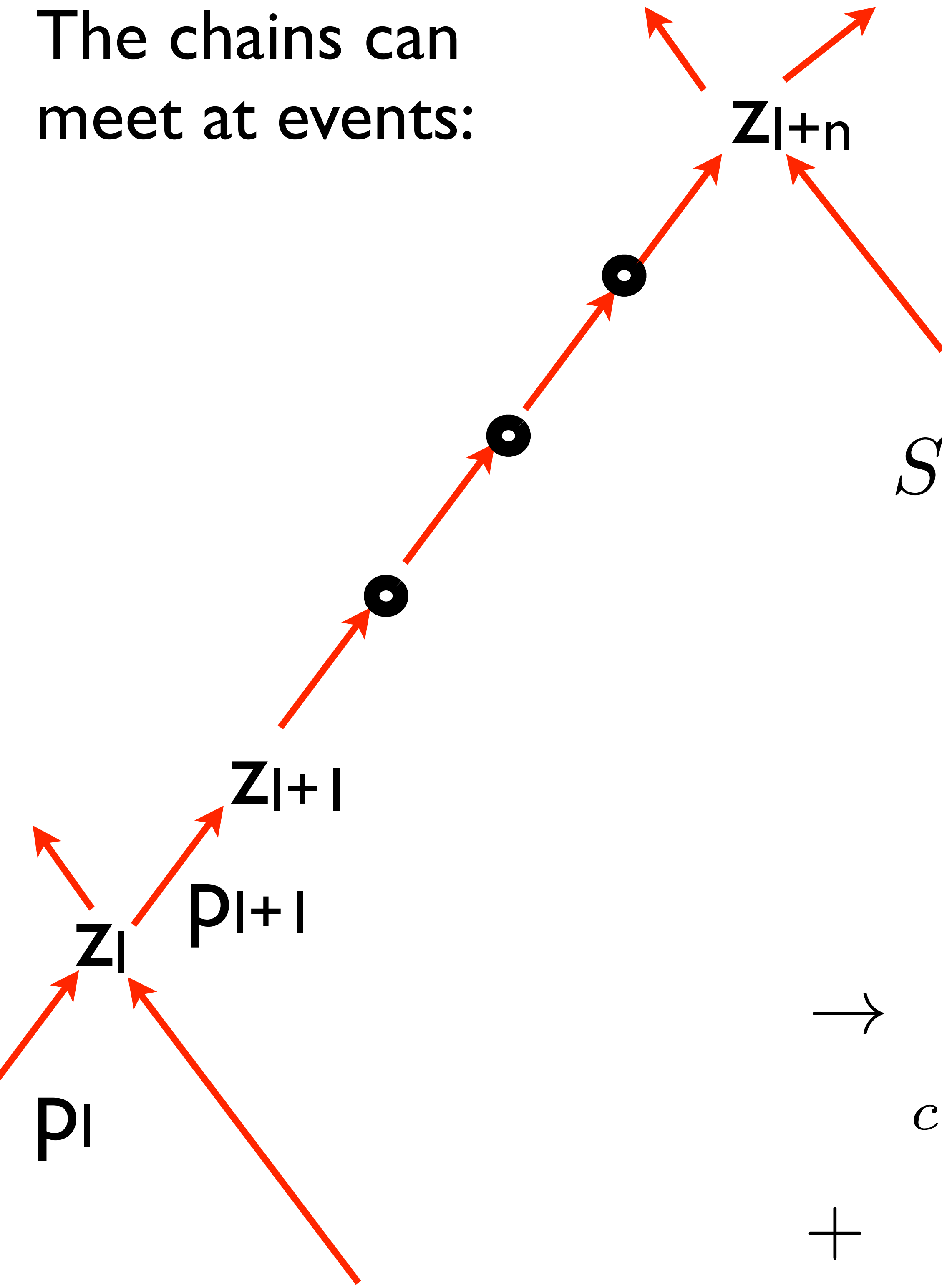
The action is now:

$$S = \sum_I p^I_a (z^a_{I+1} - z^a_I) - \frac{1}{2}\mathcal{M}_I p^2_I$$

A continuum action that gives the same classical physics: $\rightarrow \displaystyle\int dt \left(p_a(t)\dot{z}^a(t) - \frac{1}{2} n(t) p(t)^2 \right)$

which is the action for a free relativistic massless particle:

The chains can meet at events:



"pre-relative locality"

$$S = \sum_{chains} \sum_{I} p_a^I (z_{I+1}^a - z_I^a) - \frac{1}{2} \mathcal{M}_I p_I^2$$

$$+ \sum_{interactions} z_I^a \mathcal{P}_a^I$$

$$\rightarrow \sum_{chains} \int dt \left(p_a(t) \dot{z}^a(t) - \frac{1}{2} n(t) p(t)^2 \right)$$

$$+ \sum_{interactions} z_I^a \mathcal{P}_a^I$$

which is the action for relativistic particles with local interactions.

Input:
- Energetic causal set
- momentum space
- conservation laws
- amplitude law.

No:

- spacetime,
- non-commutivity,
- hbar
- uncertainty relations.

No locality or non-locality, only causality.

Emerges as output:

- embedding of causal set in emergent spacetime
- lagrange multipliers become emergent coordinates.
- spacetime geometry inherited from momentum space
- dynamics of interacting relativistic particles
- relative locality framework
- Discrete dynamics is totally constrained. Poisson structure emerges in classical, continuum limit.

Thank you

Dynamics: *What causes or chooses the causal set?*

No previous answer: CS models are stochastic or quantum.

- In our first paper, we propose a globally deterministic dynamics that answers why events take place, based on the insight that in a relational structure each event must have a unique causal past.
- In our second paper, we give quantum dynamics-sum over all possible causal structure with fixed initial and final conditions.
- In our third paper we describe an energetic causal set that is also a spin foam.

Stochastic dynamics are based on an heuristic Principle:

Leibnitz's identity of the indiscernible.

Principle of the identity of the indiscernible (PII): *any two events or objects with isomorphic relational properties are to be identified.*

- *Global symmetries cannot be fundamental. Indeed GR has none and all the global symmetries in the standard model are accidental or broken.*

- *Relative locality: Localization is a consequence of identity, ie something is uniquely localized if it is distinguished by having a unique causal neighborhood.*

- *Hypothesis: the fundamental geometry is built from distinctiveness based on causal neighborhoods. Distance is a consequence of having disimilar causal neighborhoods.*

- *There are defects in this causal geometry. Two systems with very similar causal neighborhoods are nearby causally, even if distant in the coarse grained macroscopic metric. Hence they interact.*

- *The interactions induced between two similar systems are repulsive in that they act to increase their distinctiveness. Thus the PII is protected dynamically.*

The PII forces local physics to be non-deterministic:

- *By the PII each event has a unique causal neighborhood* (arXiv:1307.6167)*:*
    - *Suppose two events A and B have isomorphic causal pasts:*

$$P(A) = P(B)$$

Then to prevent a violation of the PII their causal futures must be different

$$\Rightarrow F(A) \neq F(B)$$

Thus the same causal past implies a different causal future. Hence local physics cannot be deterministic. It must be anti-deterministic.

The basic hypothesis: there is a non-local interaction between similar systems which acts to increase their differences. This is the origin of quantum physics. This interaction is driven by a potential energy which measures the distinctiveness of all the pairs of similar subsystems in nature.

Put in interactions through A_I

Twistorial formulation

Twistorial formulation

- Assume momenta are null: replace momenta by spinors:

$$p_a^{IJ} \leftrightarrow \pi_{A'}^{IJ} \bar{\pi}_A^{IJ} \qquad q_a \leftrightarrow \chi_{A'}^{IJ} \bar{\chi}_A^{IJ}$$

- The redshift constraints are now:

$$\mathcal{R}_{AI}^K = \pi_{AI}^K - U_A^B \chi_{BI}^K = 0$$

- The conservation law at each event is:

$$\mathcal{P}_{AA'}^I = \sum_K \bar{\pi}^I{}_{AK} \pi^I{}_{A'K} - \sum_L \bar{\chi}_{AI}^L \chi_{A'I}^L = 0$$

- There is a new constraint, fixing the helicity:

$$\mathcal{D} = \omega^A \bar{\pi}_A + \bar{\omega}^{A'} \pi_{A'} - 2S = 0$$

- Take U=1 and solve R

Twistorial formulation

$$p_a^{IJ} \leftrightarrow \bar{\pi}^{IJ}_{A'}\pi^{IJ}_A \quad q_a \leftrightarrow \bar{\chi}^{IJ}_{A'}\chi^{IJ}_A$$

- The action is now:

$$S^{twistor} = \sum_I z_I^{AA'}\mathcal{P}^I_{AA'} + \sum_{(I,K)} +\Omega_I^K \mathcal{D}^I_K$$

(The mass shell constraint and R=0 have been solved for)

$$\mathcal{P}^I_{AA'} = \sum_K \bar{\pi}^I{}_{AK}\pi^I{}_{A'K} - \sum_L \bar{\chi}^L_{AI}\chi^L_{A'I} = 0$$

$$\mathcal{D} = \omega^A\bar{\pi}_A + \bar{\omega}^{A'}\pi_{A'} - 2S = 0$$

- The eom include the twistor incidence relation:

$$\Omega^I_K\omega^{AI}_K = z_I^{AA'}\bar{\pi}^I{}_{A'K}$$

$$z^{AA'}(\lambda) = z_0^{AA'} + \lambda\pi^A\bar{\pi}^{A'}$$

Relation to Wolfgang Wieland's twistorial spin foams 1301.5859

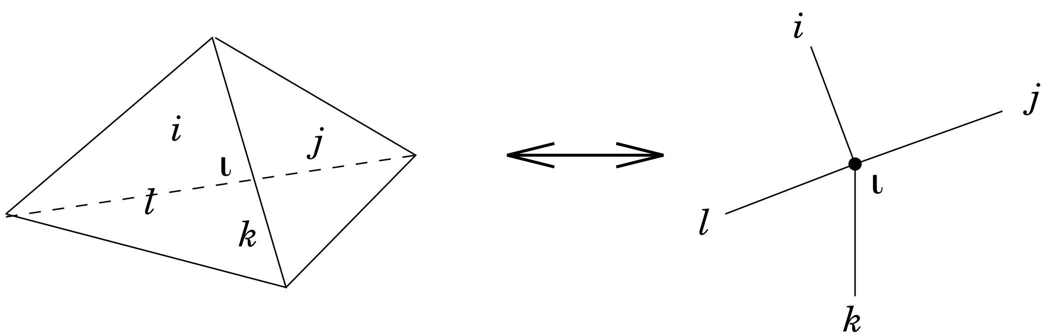
(work in progress w Wolfgang and Marina)

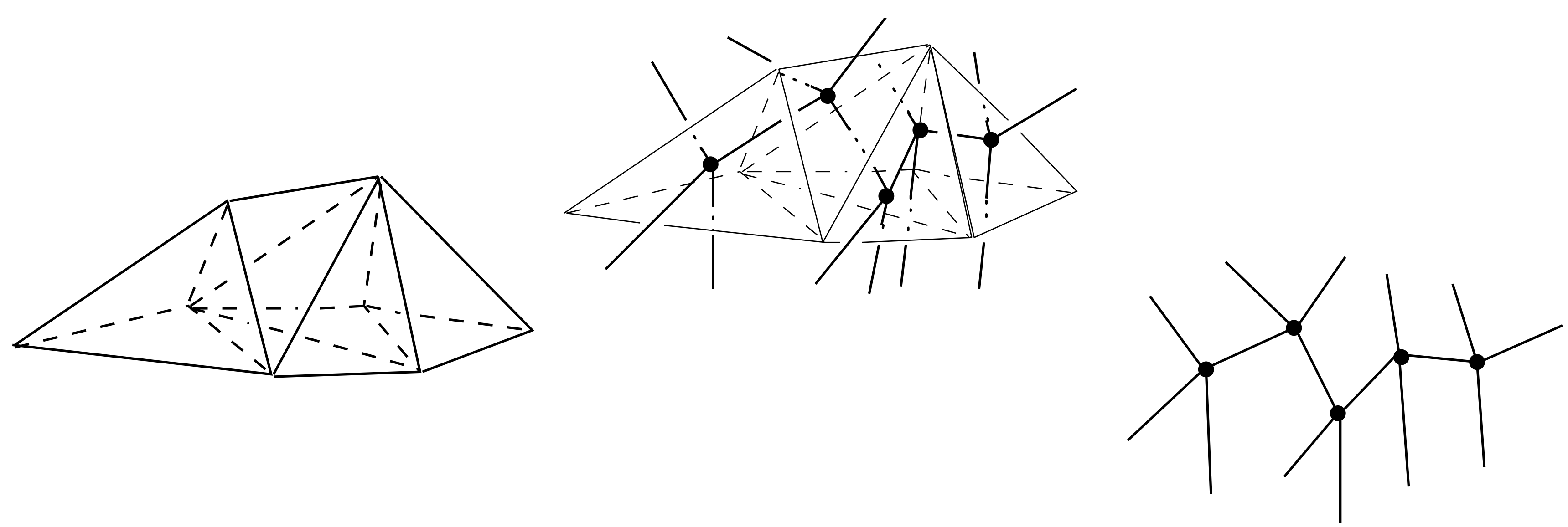
Fotini Markopoulou's dual spinnetwork evolution

arXiv:gr-qc/9704013

Causal spin network evolution

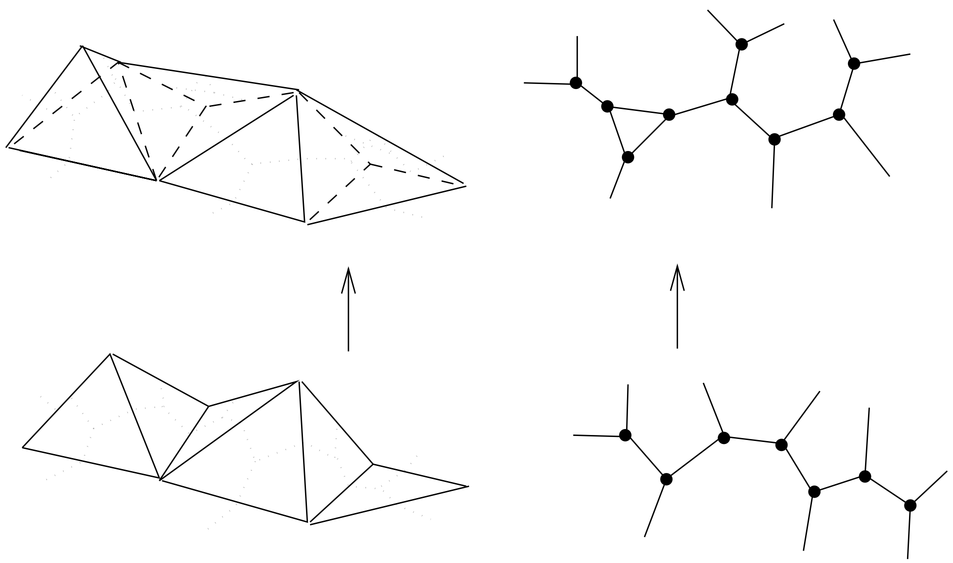
•The initial state is a four-valent spin network G embedded in a topological three manifold M. From G we define a dual triangulation, T, whose faces are labeled by SU(2) spins and whose tetrahedra are labeled by intertwiners.

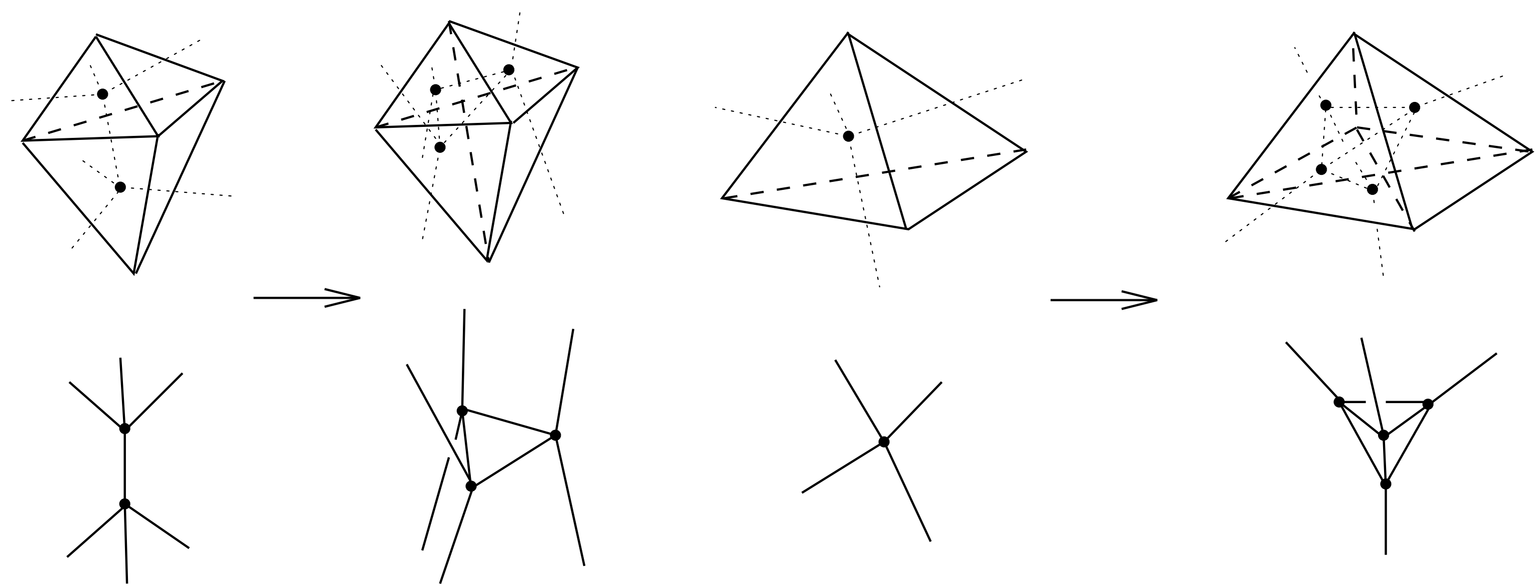




LQG states represent spatial diffeo equivalence classes.

•An evolution move is a Pachner move, which are generated by a four simplex V_I placed on top of a set of n=1 - 4 adjacent tetrahedra. One erases those tetrahedra and replaces the with the complement tetrahedra inside of V. Additional labels are introduced as needed. These define an n --> 5-n move. The resulting triangulation is the new state. The initial n tetrahedra are labeled the past set of V_I, called P_I, the new 5-n are called the future set F_I.





- This is done many times to generate a causal spin foam SF.

- The resulting four dimensional simplicial complex has the structure of a causal set.
    - Each four simplex, V_I is an event.
    - Two events V_I and V_J have an immediate causal link, L_{IJ} if a tetrahedron in the future set of I is also in the past set of J. The causal link L_{IJ} can then contain several tetrahedra. We say event K is to the future of event I, K>I if there is a chain of immediate future pointing causal links beginning on I and ending on K.

- Except for the initial triangulation, every tetrahedron is uniquely in the future set of one four simplex. Except for the final triangulation, every tetrahedron is uniquely in the past set of one four simplex.

- Every tetrahedron and each triangle in a causal spin foam is space-like.

- Dual to each four simplex V_I is an event, E_I in a dual complex.

  - Dual to each tetrahedron, T is a timeline link, l_T connecting two events which contain T as part of the future or past set.

- Two events can be connected by more than one timeline link, each dual to a tetrahedron in the future set of one and the past set of the other.

- Dual to each triangle is a spacelike face, or an edge in the three space orthogonal to l_T.

Volume as momentum

Wolfgang: 1301.5859

- Endow the triangles with flux and connection variables.
- Normal to every tetrahedra is a timelike 4-vector, call it a momenta, p_a
- Construct the volume of the tetrahedra, V =V(fluxes)
- The volume plays the role of mass, in the mass-shell constraint:

$$C = p_a p^a + V^2 = 0$$

- The four-momenta are conserved in Pachner moves:

$$\mathcal{P}^I_a = \sum_{T \in \text{past set of I}} p^T_a - \sum_{T \in \text{future set of I}} p^T_a = 0$$

Details:

•Each triangle τ in T (or its dual spacelike link) has an initial flux Π in sl(2,C), a final flux, Π_f and dual holonomy g_τ in SL(2,C). These are related by the constraint

$$\mathcal{R}^{\tau}_{ab} = \tilde{\Pi}^{\tau}_{ab} - (g_{\tau}^{-1} \cdot \Pi_{\tau} \cdot g_{\tau})_{ab} = 0$$

•Each tetrahedron T has a timeline four momentum, p_a, which are vectors in an internal momentum space,

- The four momenta and fluxes are related by two constraints:
  - The simplicity constraint:

$$\mathcal{S}^b_\tau = p^T_a \Pi^{ab}_\tau = 0$$

and the volume shell constraint

$$\mathcal{C}^T = p_a p_b \eta^{ab} + V_T^2(\Pi) = 0$$

η^{ab} is a metric on the internal momentum space

The three volume of a tetrahedron, T is a function of the fluxes across three of its four faces.

$$V_T = \frac{\sqrt{2}}{3}\sqrt{|\epsilon_{ijk}\Pi^i(\tau_1)\Pi^j(\tau_2)\Pi^k(\tau_3)|}$$

$\Pi^i(\tau)$ is the dual, in the three-space orthogonal to p^a, of the flux $\Pi^{ab}(\tau)$

Twistors and the simplicity constraints:

One solves:

$$\mathcal{R}^{\tau}_{ab} = \tilde{\Pi}^{\tau}_{ab} - (g_{\tau}^{-1} \cdot \Pi_{\tau} \cdot g_{\tau})_{ab} = 0$$

In terms of two twistors:

$$T_{\alpha} = (\omega^{A}, \bar{\pi}_{A'}), \qquad \tilde{T}_{\alpha} = (\tilde{\omega}^{A}, \tilde{\bar{\pi}}_{A'})$$

Laurent, Etera, Simone, Wolfgang

$$\Pi_{AB} = \omega_{(A}\pi_{B)}$$

$$\tilde{\Pi}_{AB} = \tilde{\omega}_{(A}\tilde{\pi}_{B)}$$

$$g_{A}^{B} = \frac{\omega_{A}\tilde{\pi}^{B} - \tilde{\omega}_{A}\pi^{B}}{\sqrt{\omega_{A}\pi^{A}}\sqrt{\tilde{\omega}_{B}\tilde{\pi}^{B}}}$$

Poisson brackets: $\{\omega^{\tau}_{A}, \pi^{B}_{\tau'}\} = \delta^{B}_{A}\delta_{\tau\tau'}$

The simplicity constraints become a twisted helicity constraint:

$$\mathcal{D} = \frac{1}{\beta + \imath}(\omega_A \pi^A) + cc = 0$$

and a linear constraint:

$$\mathcal{F} = p^{AA'} \omega_A \bar{\pi}_{A'} = 0$$

The spin foam/energetic causal set action:

$$S^{ctf} = \sum_I Z_I^a \mathcal{P}_a^I + \sum_T \mathcal{N}^T \mathcal{C}^T + \sum_\tau (\Omega \mathcal{D} + \rho \mathcal{F})$$

$$\mathcal{P}_a^I = \sum_{T \in \text{past set of I}} p_a^T - \sum_{T \in \text{future set of I}} p_a^T = 0$$

$$\mathcal{C}^T = p_a p_b \eta^{ab} + V_T^2(\Pi) = 0$$

$$\mathcal{D} = \frac{1}{\beta + \imath}(\omega_A \pi^A) + cc = 0$$

$$\mathcal{F} = p^{AA'} \omega_A \bar{\pi}_{A'} = 0$$

The partition function:

$$Z = \int \prod_{V_I} dZ_I \prod_T dN_T dp_a^T \prod_\tau d\Omega_\tau d\rho_\tau d\omega_\tau^A d\pi_{A'}^\tau e^{\imath S^{ctf}}$$

The spin foam/energetic causal set action:

$$S^{ctf} = \sum_I Z_I^a \mathcal{P}_a^I + \sum_T \mathcal{N}^T \mathcal{C}^T + \sum_\tau (\Omega \mathcal{D} + \rho \mathcal{F})$$

$$\mathcal{P}_a^I = \sum_{\tau \in \text{past set of I}} p_a^T - \sum_{\tau \in \text{future set of I}} p_a^T = 0$$

$$\mathcal{C}^T = p_a p_b \eta^{ab} + V_T^2(\Pi) = 0$$

$$\mathcal{D} = \frac{1}{\beta + \imath}(\omega_A \pi^A) + cc = 0$$

$$\mathcal{F} = p^{AA'} \omega_A \bar{\pi}_{A'} = 0$$

The EOM from varying the p_a are:

$$Z_{T+}^a - Z_{T-}^a = N_T p_T^a + \sigma^a_{AA'} \sum_{\tau \in T} \rho_\tau \omega_\tau^A \bar{\pi}_\tau^{A'}$$

T^+ and T^- are the four simplices to the future and past of the tetrahedron T.

Thus, a spin foam model ***is*** *an energetic causal set.*

•Enhanced by holonomy and flux variables, which can be represented as twistors

•It's EOM include embeddings of events (dual to 4-simplices) in a spacetime:

$$Z^a_{T^+} - Z^a_{T^-} = N_T p^a_T + \sigma^a_{AA'} \sum_{\tau \in T} \omega^A_\tau \bar{\pi}^{A'}_\tau$$

•The existence of solutions remains to be investigated. These are necessary for the semiclassical limit to exist.

• Causal histories that have consistent solutions define a semiclassical approximation in which a spacetime emerges with an embedded causal process.

•Double the momenta and free up U(T):

- Each tetrahedra now has two timelike normals
  pa: normal wrt the past 4-simplex of T
  qa: normal wrt the future 4-simplex of T

They are related by

$$\mathcal{R}^K_{aI} = p^K_{aI} - \mathcal{U}^{Kb}_{Ia} q^K_{bI} = 0$$

The action is now

$$S^{ctf} = \sum_I Z^a_I \mathcal{P}^I_a + \sum_T (X^a_T \mathcal{R}^T_a + \mathcal{N}^T \mathcal{C}^T + \tilde{\mathcal{N}}^T \tilde{\mathcal{C}}^T) + \sum_\tau (\Omega \mathcal{D} +$$

The equations of motion become:

$$Z^a_{T+} - U(T)^a_b Z^b_{T-} = N_T p^a_T + \sigma^a_{AA'} \sum_{\tau \in T} \rho_\tau \omega^A_\tau \bar{\pi}^{A'}_\tau$$

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The partition function becomes:

$$Z = \int \prod_{V_I} dZ_I \prod_{T} dX_T^a dN_T d\tilde{N}_T dp_a^T d\tilde{p}_a^T dU_T \prod_{\tau} d\Omega_\tau d\rho_\tau d\omega_\tau^A d\pi_{A'}^\tau e^{i}$$

Conclusions:

•Energetic causal sets define a quantum theory of spacetime where momenta, energy and causal structure are intrinsic.
•There is a new mechanism for spacetime to be emergent in the stationary phase approximation.
•hbar, locality, uncertainty etc are all emegent with spacetime.

•Using Markopoulou and Wieland's results, a spin foam model can be recast as an energetic causal set.
• Volume is mass.
•This gives a new mechanism for the emergence of spacetime in spin foam models to be explored.

The action is now

$$S^{ctf} = \sum_I Z_I^a \mathcal{P}_a^I + \sum_T (X_T^a \mathcal{R}_a^T + \mathcal{N}^T \mathcal{C}^T + \tilde{\mathcal{N}}^T \tilde{\mathcal{C}}^T) + \sum_\tau (\Omega \mathcal{D} + \rho$$

The equations of motion become:

$$Z_{T+}^a - U(T)_b^a Z_{T-}^b = N_T p_T^a + \sigma_{AA'}^a \sum_{\tau \in T} \omega_\tau^A \bar{\pi}_\tau^{A'}$$

Twistorial formulation

- Assume momenta are null: replace momenta by spinors:

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$$\mathcal{R}_{AI}^{K} = \pi_{AI}^{K} - U_A^B \chi_{BI}^{K} = 0$$

- The conservation law at each event is:

$$\mathcal{P}_{AA'}^{I} = \sum_K \pi_{AK}^{I} \bar{\pi}_{A'K}^{I} - \sum_L \chi_{AI}^{L} \bar{\chi}_{A'I}^{L} = 0$$

- There is a new constraint, fixing the helicity:

$$\mathcal{D} = \omega^A \bar{\pi}_A + \bar{\omega}^{A'} \pi_{A'} - 2S = 0$$