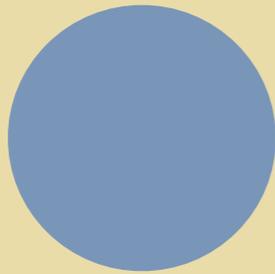
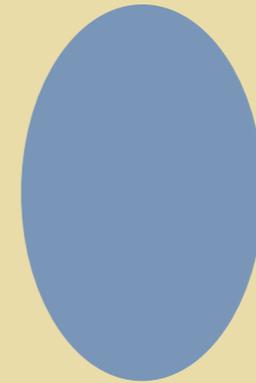


# Polarized Black Holes in AdS



Lauren Greenspan



[hep-th/1511.08505 Costa, LG, Oliveira, Penedones, Santos]

# Motivation

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- Find interesting new geometries in AdS.
- Study Black Hole Polarization
- What can we learn about the dual field theories?

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# Idea

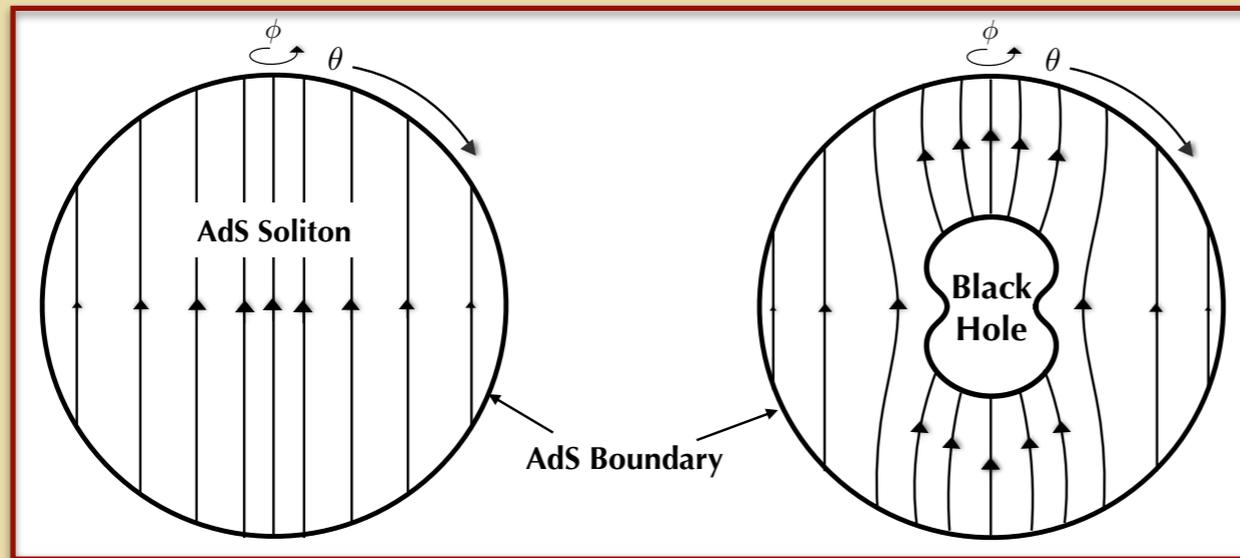
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- Study the thermodynamics of deformed 4-dimensional black holes subject to a dipolar potential  $A_\tau = i\mathcal{E} \cos \theta$  that are dual to a 3-dimensional field theory.

# Gravity Set-up

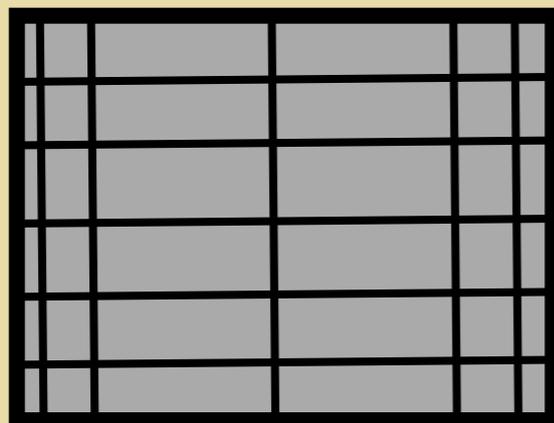
$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left( R + \frac{6}{l^2} - F_{\alpha\beta} F^{\alpha\beta} \right) + \frac{1}{8\pi G_N} \int d^3x \sqrt{h} K$$

- gauge field  $\tilde{A}_\mu$
- metric  $g_{\mu\nu}$



Study numerical solutions  
Einstein-Maxwell gravity  
with a dipolar potential  
source.

$$R_{\mu\nu} + \frac{3}{l^2} g_{\mu\nu} = 2F_{\mu\alpha} F_{\nu}^{\alpha} - \frac{1}{2} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}, \quad d \star F = 0.$$



- ❖ Descretize PDEs chebyshev x fourier
- ❖ Solve with Spectral Methods  
(exponential convergence)

# Ansätze

$$ds_{sol}^2 = \frac{1}{(1-r^2)^2} \left\{ A(r, x) d\tau^2 + \frac{4G(r, x)dr^2}{2-r^2} + r^2(2-r^2) \left[ \frac{4C(r, x)}{2-x^2} \left( dx + \frac{H(r, x)dr}{r} \right)^2 + B(r, x)(1-x^2)^2 d\phi^2 \right] \right\}$$

$$\tilde{A}_\tau^{sol} = -ir D(r, x) d\tau$$

$$ds_{BH}^2 = \frac{1}{(1-r^2)^2} \left( d\tau^2 r^2 A(r, \theta) f_{BH}(r) + R^2 \left( \frac{4dr^2 G(r, \theta)}{f_{BH}(r)} + C(r, \theta)(d\theta + 2rdrH(r, \theta))^2 + d\phi^2 B(r, \theta) \sin^2 \theta \right) \right)$$

$$f_{BH}(r) = (1-r^2)^2 - Q^2(1-r^2)^3 + R^2(3-3r^2+r^4)$$

$$\tilde{A}_\tau^{BH} = -ir^2 D(r, \theta) d\tau$$

Boundary Conditions:

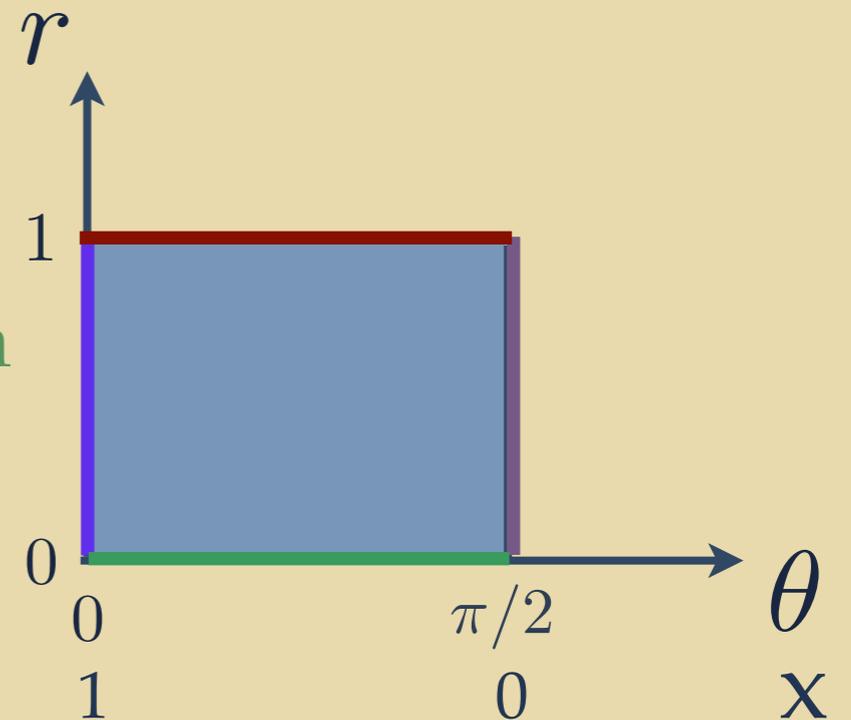
✦ Regularity at Axes of Symmetry

hor / origin  
equator  
pole

✦ At **infinity**  $A = G = C = B = 1$

$$H = 0$$

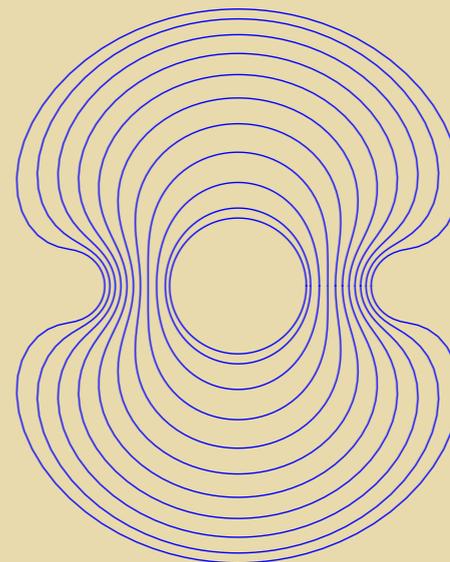
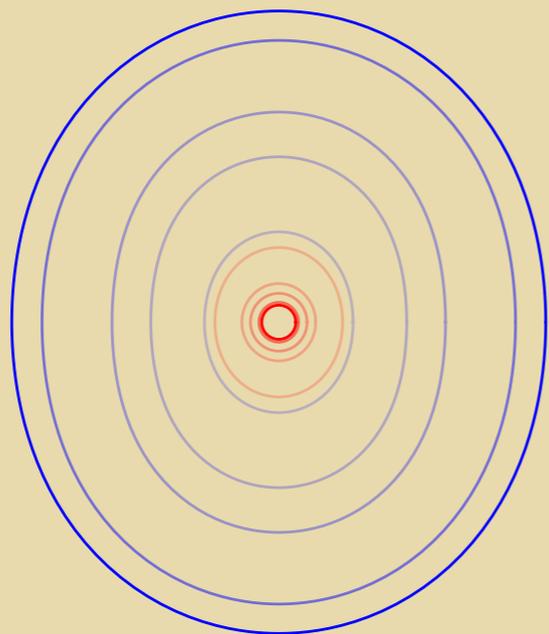
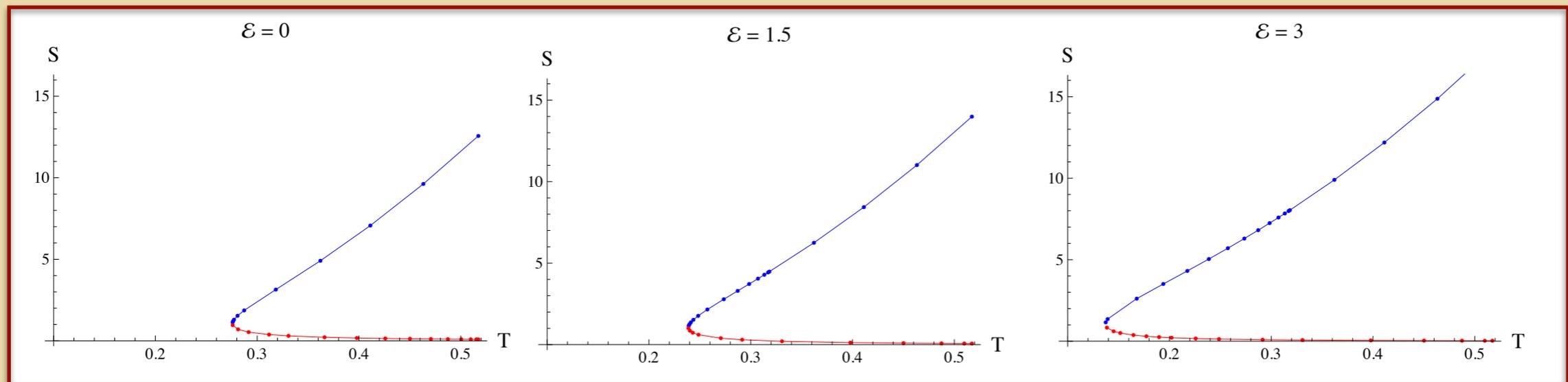
$$D = \mathcal{E} \cos \theta$$



# Black Hole Polarization

What happens to the shape of the horizon when we change  $\varepsilon$  and T?

$$S = \frac{\mathcal{A}}{4G_N} = \frac{\pi R^2}{G_N} \int_0^{\frac{\pi}{2}} d\theta \sin \theta \sqrt{C(0, \theta)B(0, \theta)}$$



# Free Energy

---

$$T_{\mu\nu} = \frac{2}{\sqrt{h}} \frac{\delta S}{\delta h^{\mu\nu}} = \frac{1}{8\pi G_N} (K_{\mu\nu} - K h_{\mu\nu} + G_{\mu\nu} - 2h_{\mu\nu})$$

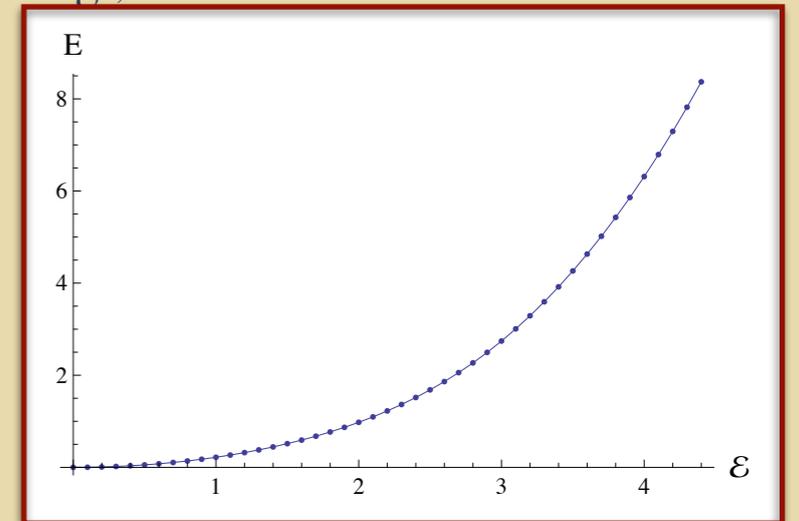
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$$E = \int d\Omega_2 T_t^t$$

e.g., for the soliton...



# Free Energy

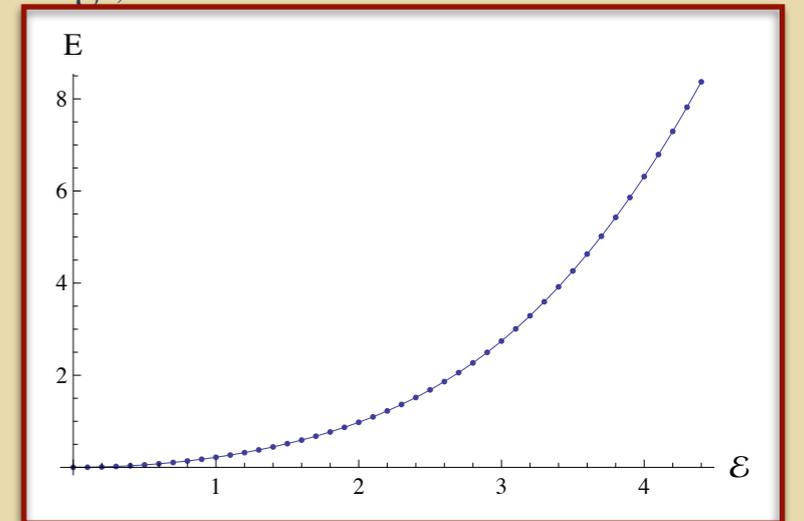
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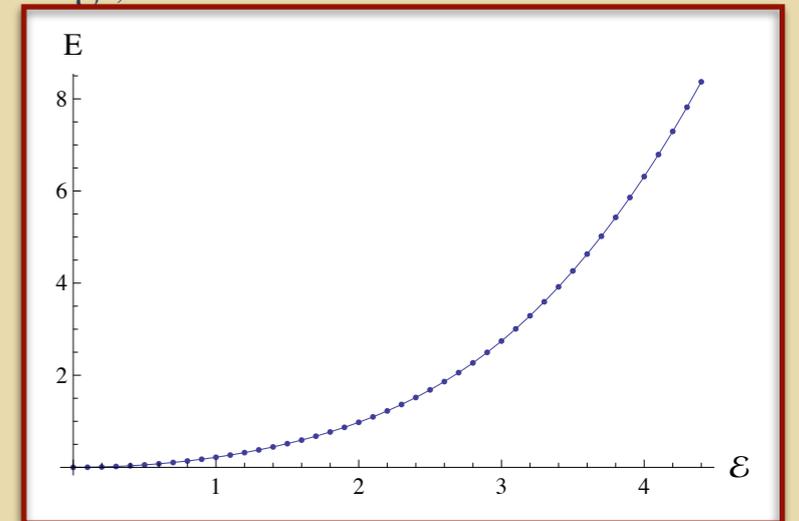
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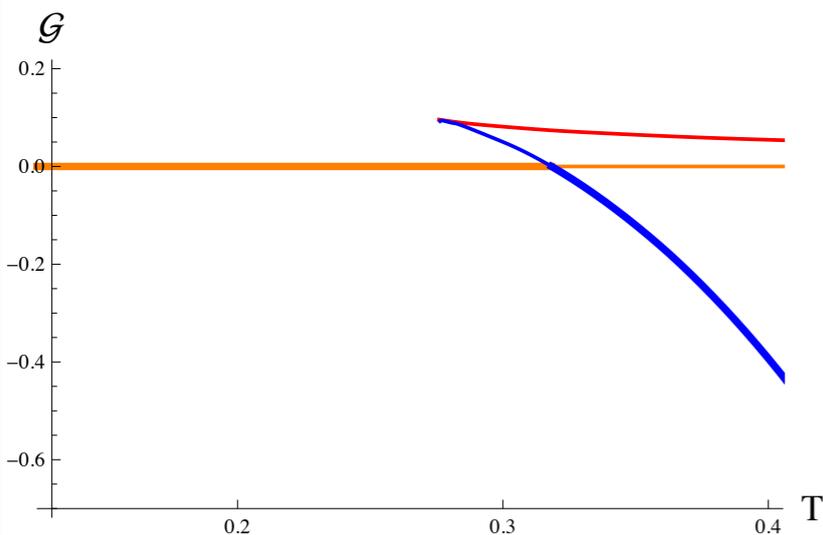
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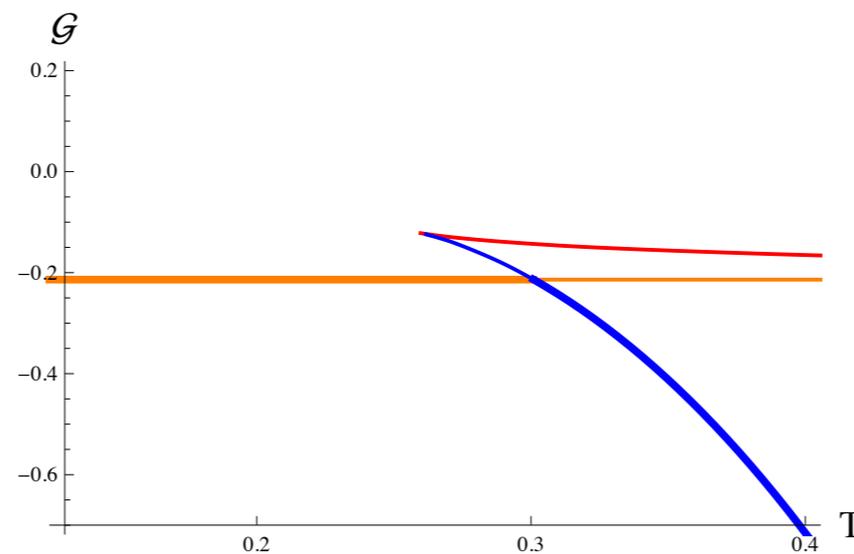


$$\mathcal{G} = E - TS - 4\pi \int_0^{\pi/2} d\theta \sin \theta \rho(\theta) \mathcal{E} \cos \theta$$

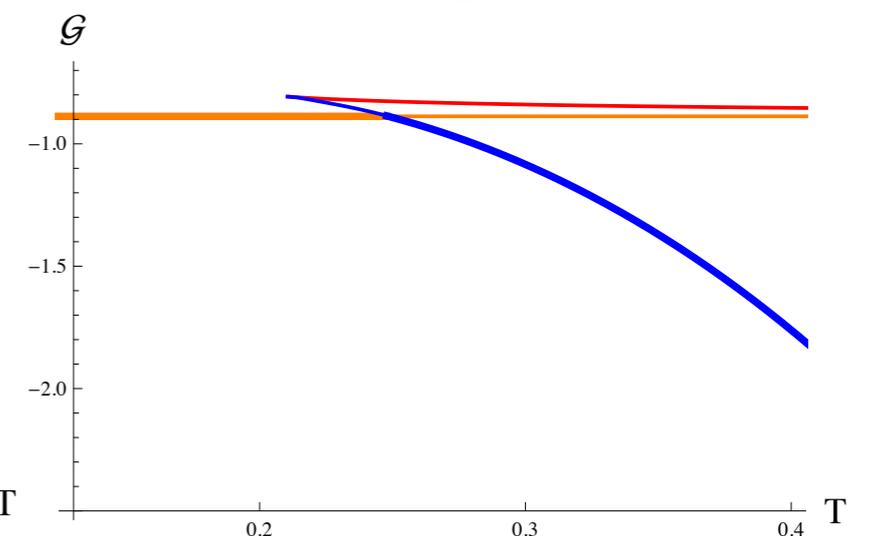
$\epsilon = 0$



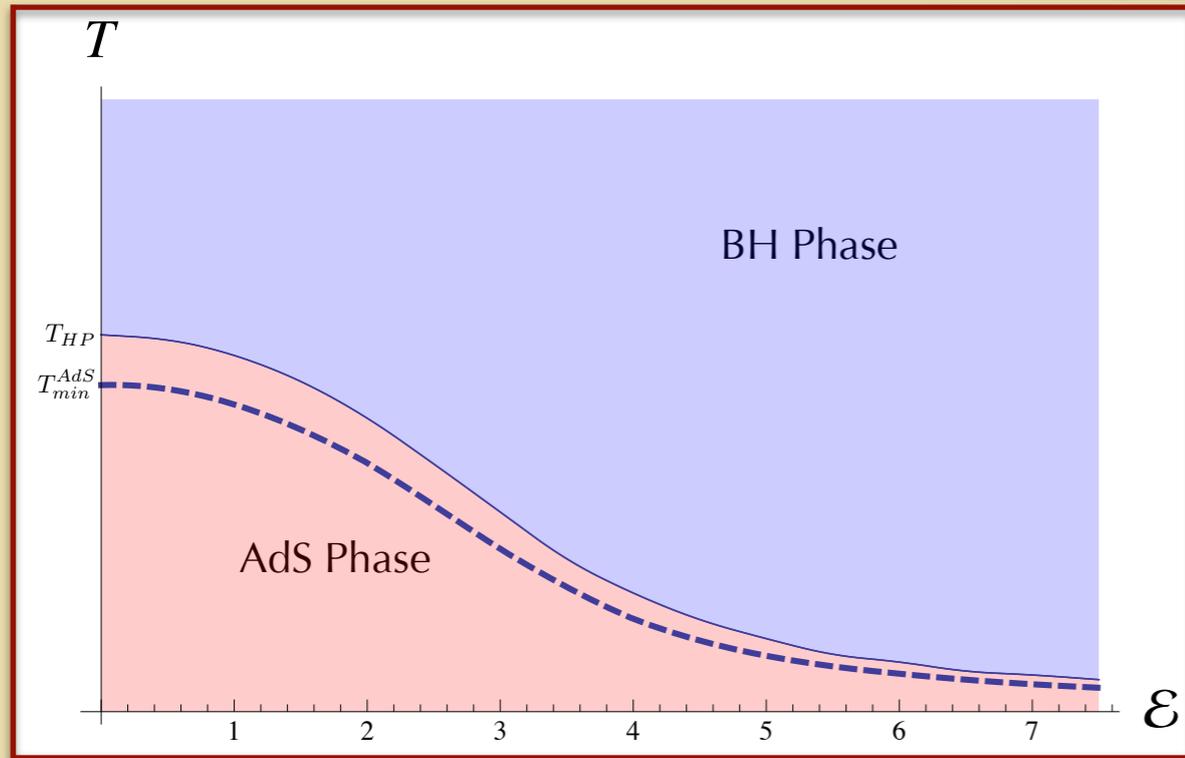
$\epsilon = 1$



$\epsilon = 2$

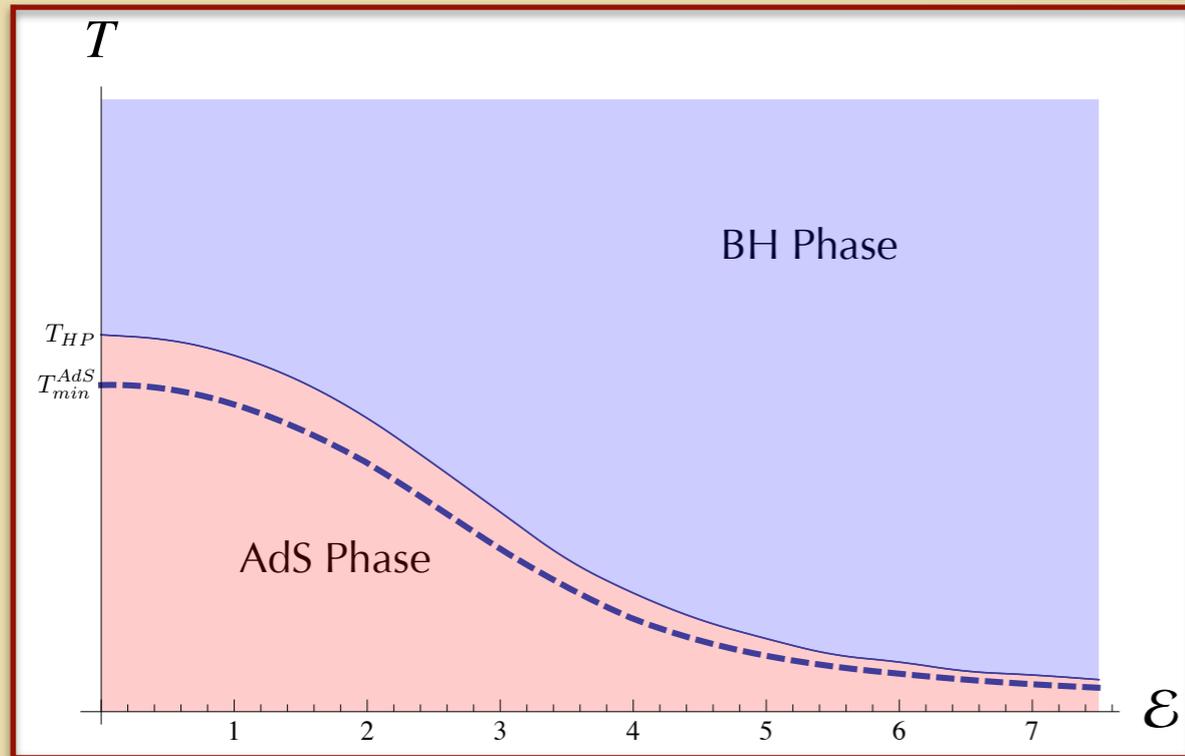


# Phase diagram



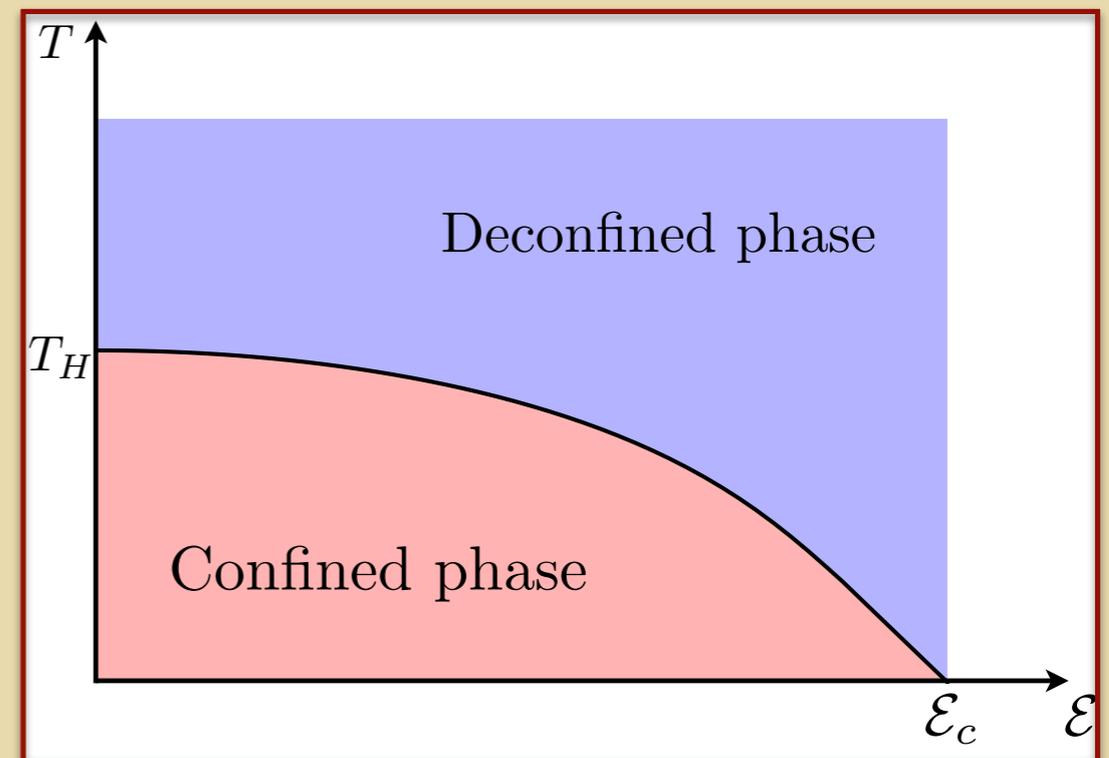
- Strong coupling picture of some dual theory

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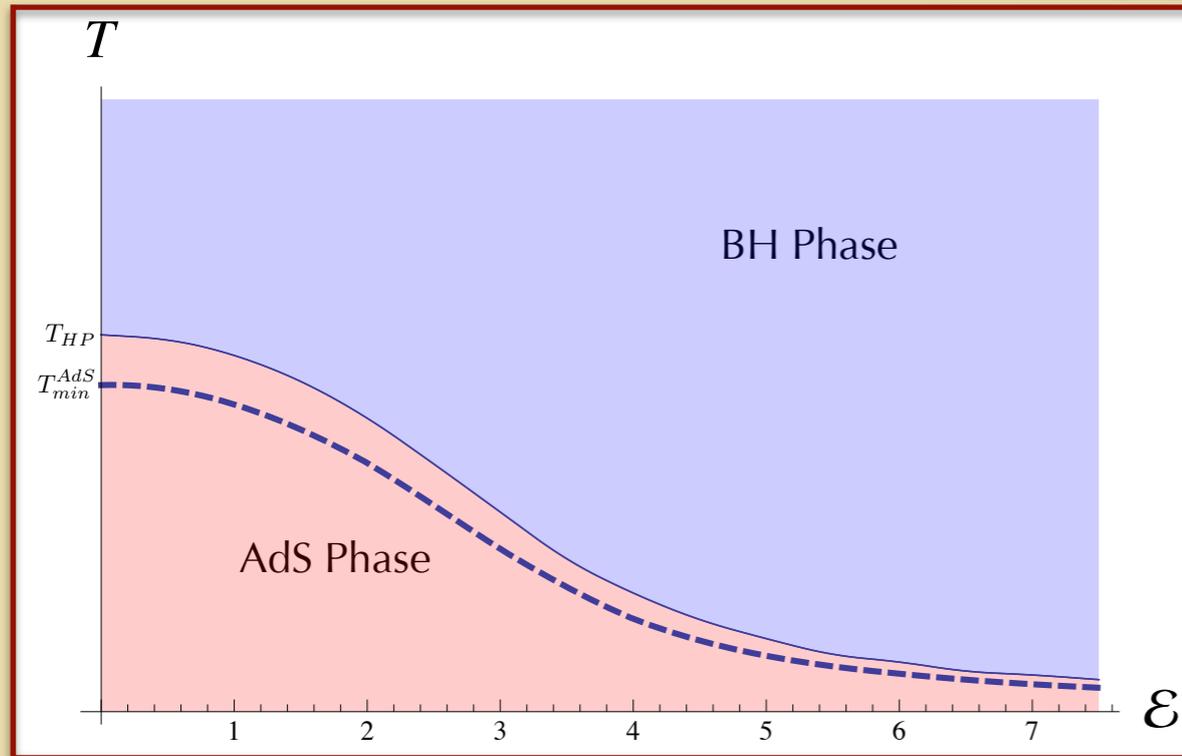


- ? Try looking at the partition function of free Bosons on a sphere to get weak coupling picture

- Strong coupling picture of some dual theory

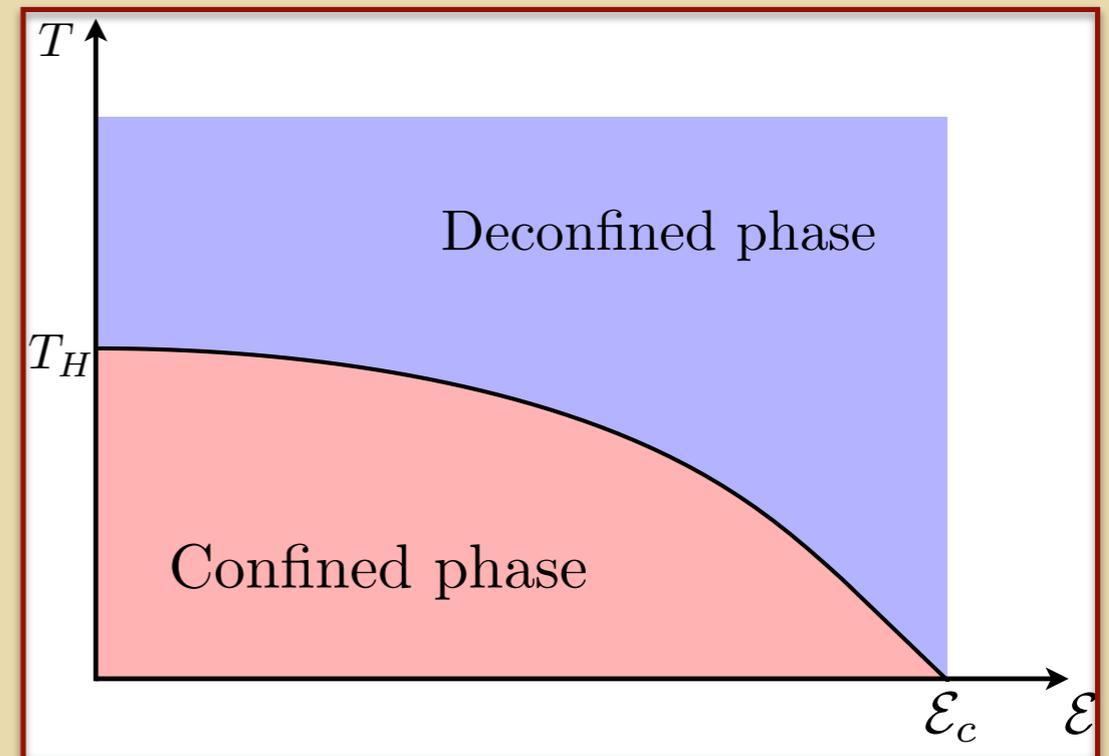


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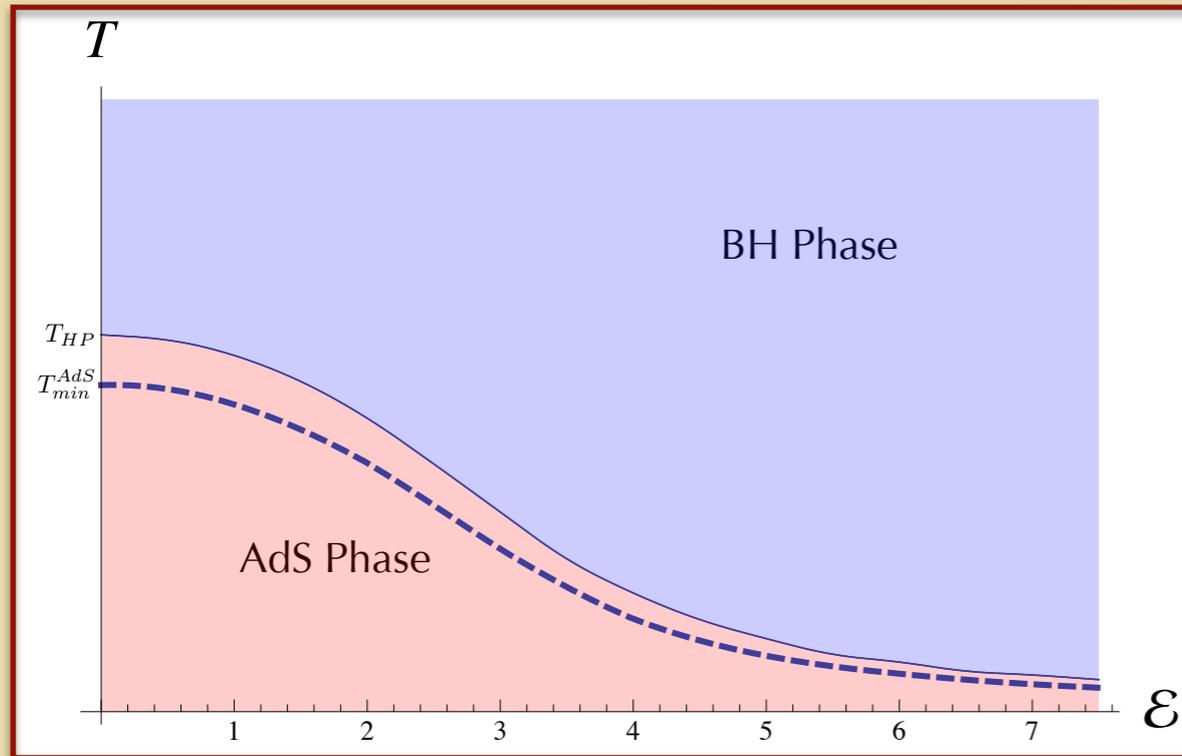


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maximum electric field

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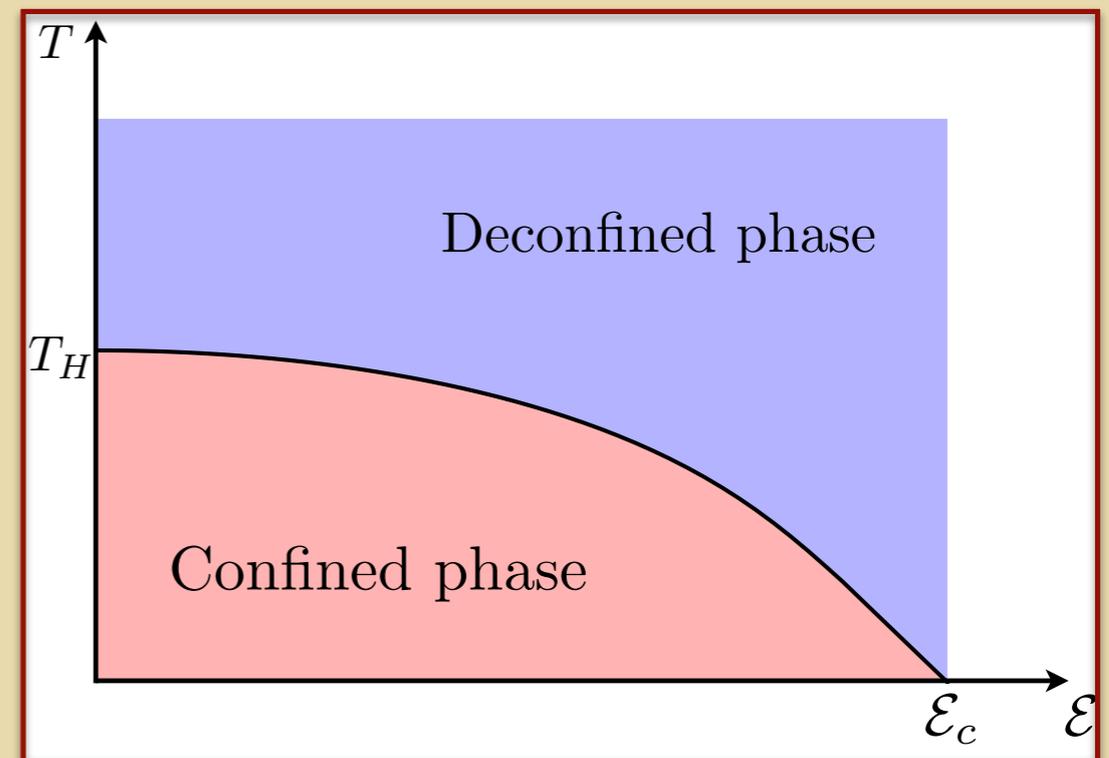


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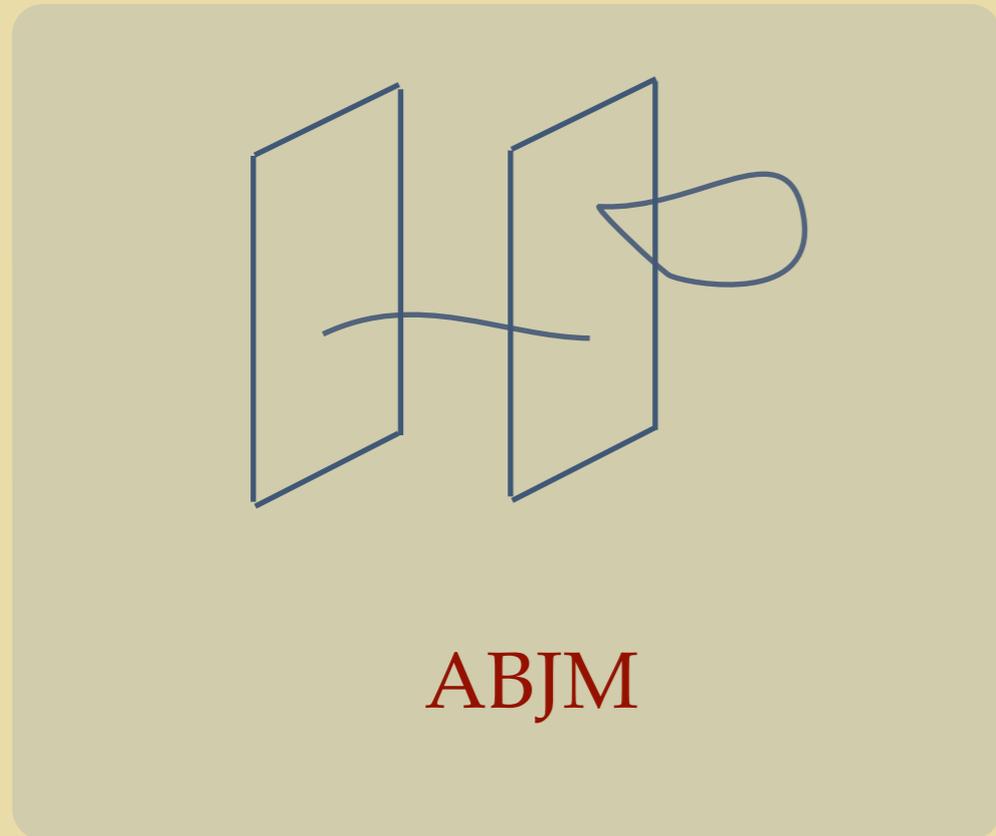
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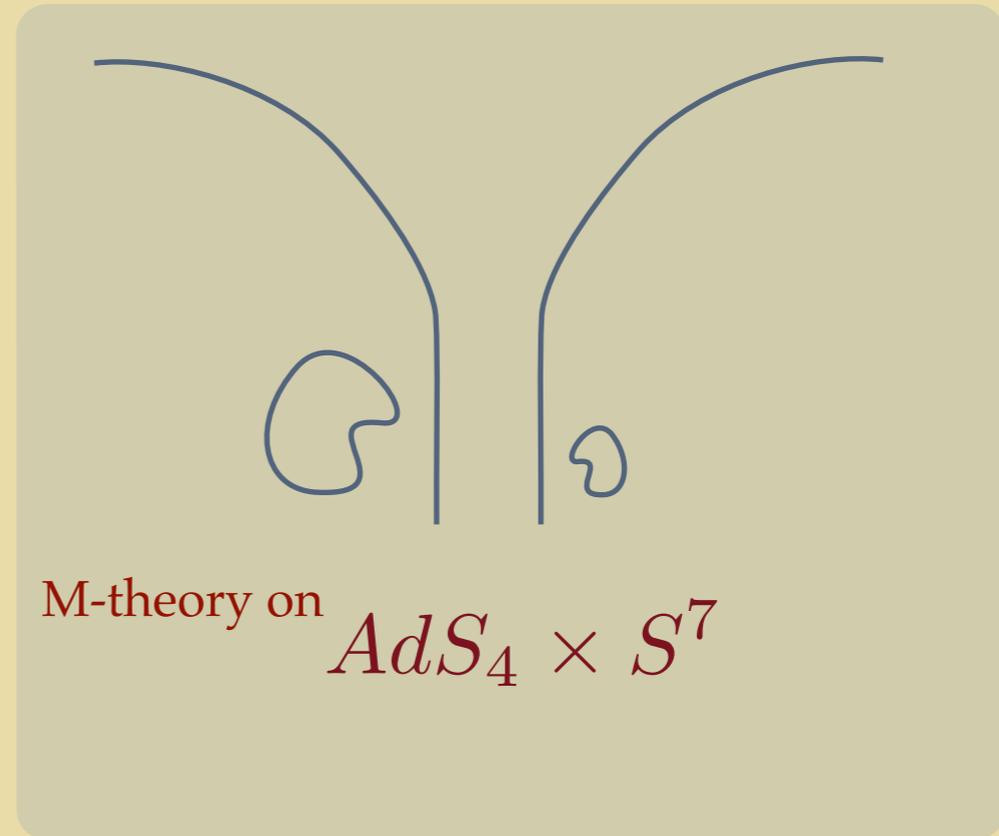
What is the correct gauge / gravity description for polarized black holes?

# Current Work: ABJM [Aharony, Berenstein, Jafferis, Maldacena 2008]

A precise dual of BH polarization is deformed 3 d ABJM theory

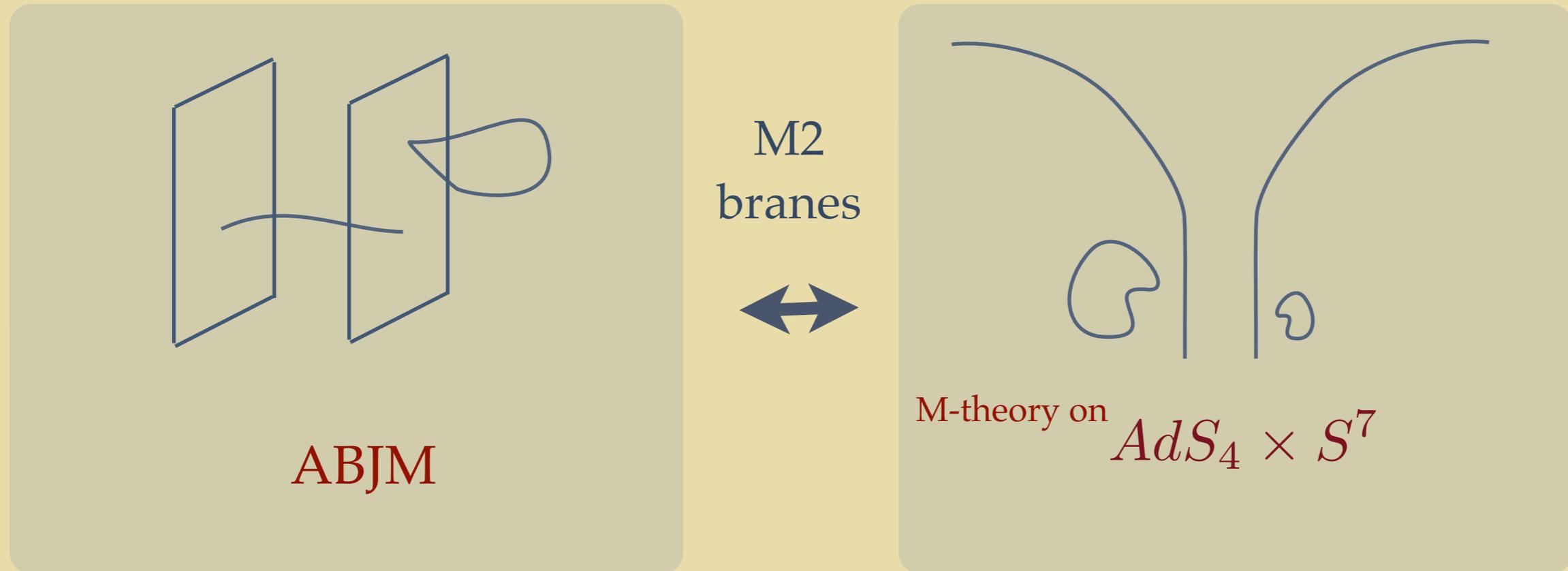


M2  
branes



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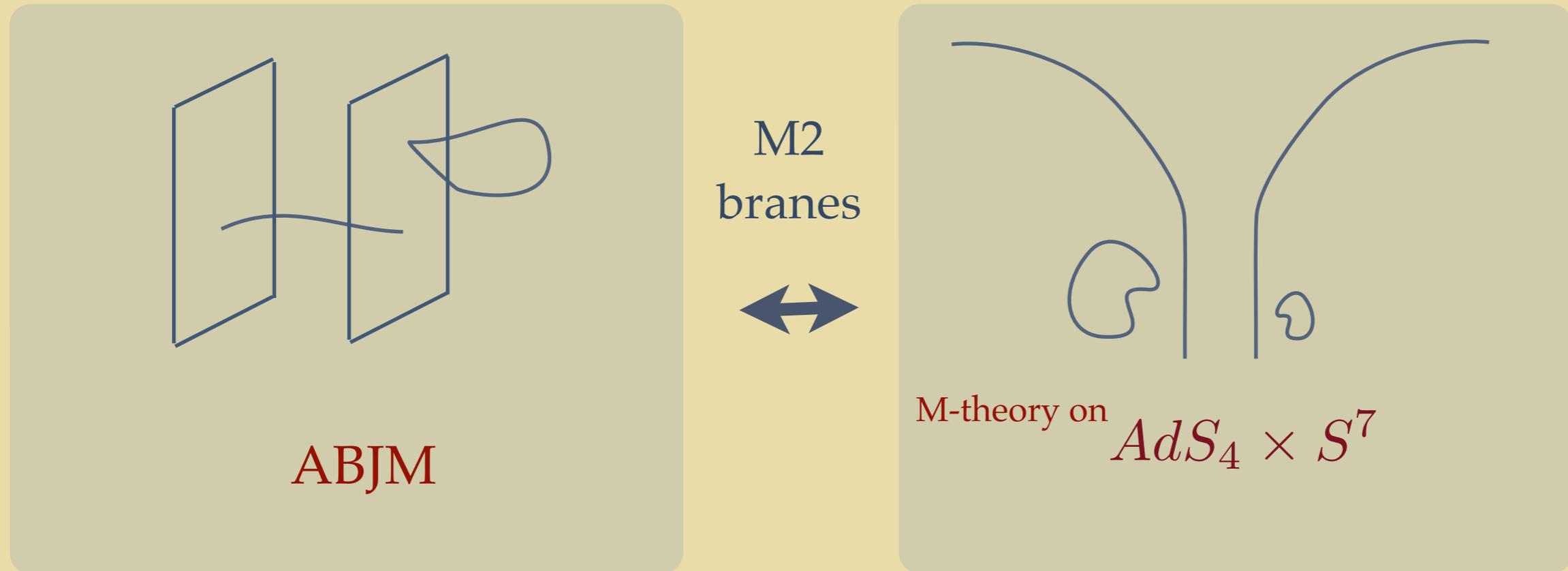
$AdS_4 \times S^7$   $\rightarrow$  4 d gauged SUGRA that can be truncated to include
 

- gauge field  $\tilde{A}_\mu$   $\leftarrow$  Non-normalizable mode
- scalar field  $\Phi$
- metric  $g_{\mu\nu}$

[Cvetic, Duff, Hoxha et. al. 1999]

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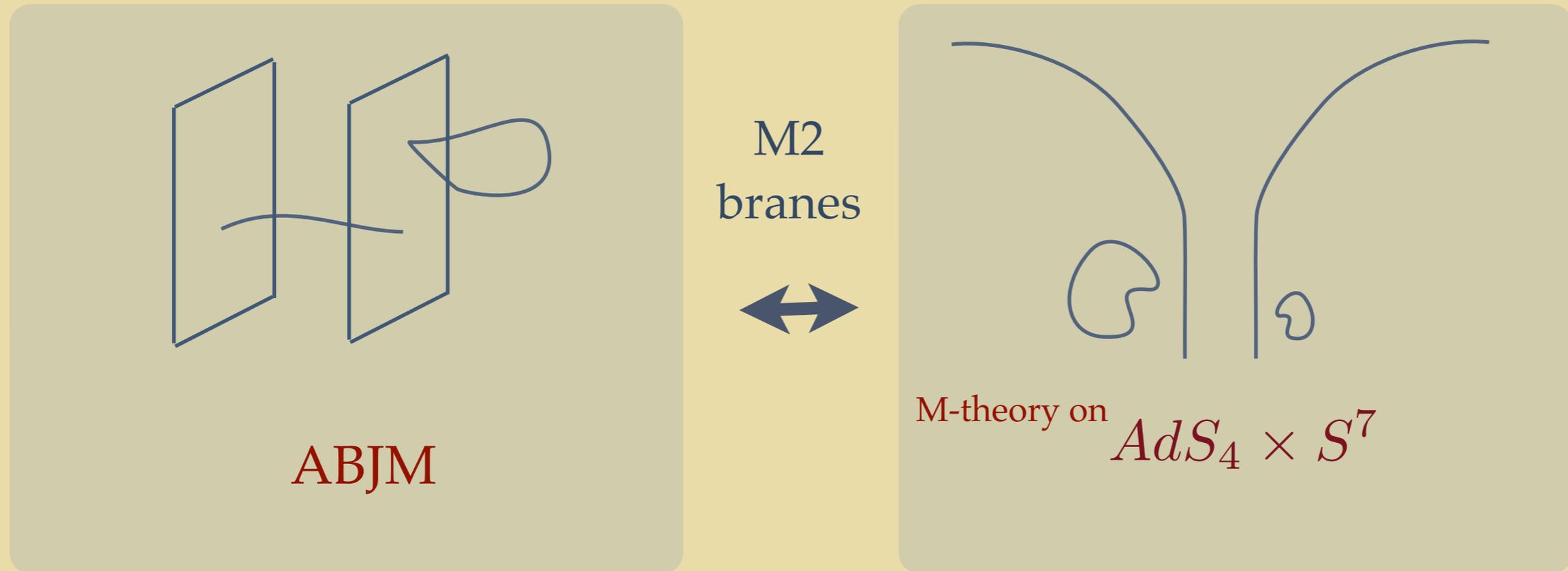
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[Cvetič, Duff, Hoxha et. al. 1999]    **Maximum electric field!**

**Concrete Realization of the gauge/gravity duality!**

[Itzhaki, Maldacena, Sonnenschein, Yankielowicz '98]

# Current / Future Work

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- ❖ study dynamical stability of BH
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Thank You