Suppressing the primordial tensor amplitude without changing the scalar sector in quadratic curvature gravity

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in collaboration with

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Based on Phys. Rev. D **92**, 103503

[arXiv : 1508.07412]

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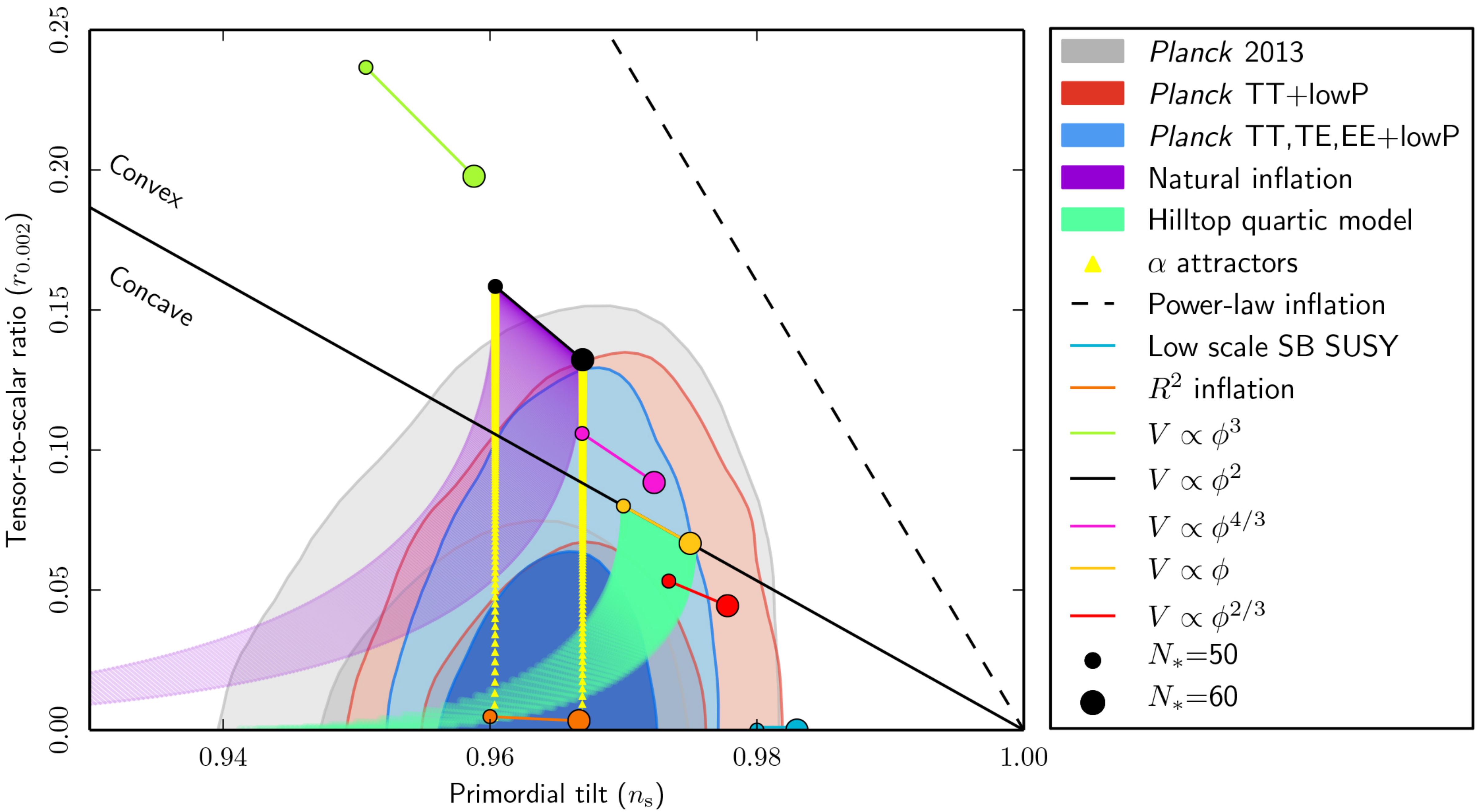
July 13, 2016

Constraints on Inflation model



Planck 2015 results. XX
arXiv:1502.02114

Question

Can we modify only tensor modes without changing the scalar sector?

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without changing the scalar sector?

Yes, we can
with higher curvature corrections

Construction of theories

Action: $S = S_{\mathrm{EH}} + S_\phi + S_{\mathrm{higher}}$

$$S_{\mathrm{EH}} = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\mathcal{R}, \qquad \kappa = 8\pi G$$

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}P(\phi, \partial^\mu\phi\partial_\mu\phi),$$

$$S_{\mathrm{higher}} = \frac{1}{\kappa}\int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{M^2}\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} + \cdots\right).$$

Construction of theories

Theories we want have the properties as follows:

- No ghost degrees of freedom

- Changing the dynamics of tensor perturbations while the scalar perturbations is left unchanged

Construction of theories

Construction with

$$\text{the unit normal to constant } \phi \text{ hypersurfaces}$$

$$u_\mu := -\frac{\partial_\mu \phi}{\sqrt{-\partial^\nu \phi \partial_\nu \phi}},$$

the induced metric

$$\gamma_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu,$$

for example: $\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}_{\mu'\nu'\rho'\sigma'}\gamma^{\mu\mu'}\gamma^{\nu\nu'}\gamma^{\rho\rho'}u^\sigma u^{\sigma'}$

Construction of theories

ADM decomposition

taking constant ϕ hypersurfaces
as constant time hypersurfaces,

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \gamma_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right).$$

quadratic curvature terms

$$\sqrt{\gamma}N \ \times \ \{K^4,\ K_{ij}K^{ij}K^2,\ \cdots,\ R^2,\ R_{ij}R^{ij},\\ K^2R,\ KK^{ij}R_{ij},\ \cdots,\ D_iK_{jk}D^iK^{jk},\ \cdots\}$$

Cosmological perturbations

$$N = 1 + \delta N, \quad N_i = \partial_i \chi + \chi_i, \quad \gamma_{ij} = a^2 e^{2\zeta} \left(e^h\right)_{ij},$$

About scalar perturbations

$$K_i^{\ j} = H\delta_i^{\ j} + \frac{1}{3}\delta K \delta_i^{\ j} + \delta \widetilde{K}_i^{\ j},$$

where

$$\delta K = -3H\delta N + 3\dot{\zeta} - \frac{1}{a^2}\partial^2\chi,$$

$$\delta \widetilde{K}_i^{\ j} = -\frac{1}{a^2}\left(\partial_i\partial^j - \frac{1}{3}\delta_i^{\ j}\partial^2\right)\chi,$$

and

$$\delta R_i^{\ j} = -\frac{1}{a^2}\left(\partial_i\partial^j + \delta_i^{\ j}\partial^2\right)\zeta.$$

Combinations for which the scalar variables are canceled out

$$2\partial_i\delta\widetilde{K}_{jk}\partial^i\delta\widetilde{K}^{jk}-3\partial_i\delta\widetilde{K}^{ik}\partial^j\delta\widetilde{K}_{jk},$$

and

$$\delta R_{ij}\delta R^{ij}-\frac{3}{8}\delta R^2,$$

Including vector and tensor perturbations

$$2\partial_i\delta\widetilde{K}_{jk}\partial^i\delta\widetilde{K}^{jk}-3\partial_i\delta\widetilde{K}^{ik}\partial^j\delta\widetilde{K}_{jk}$$
$$=\frac{1}{2a^2}\left(\partial_i\dot{h}_{jk}\right)^2+\frac{1}{4a^6}\left(\partial^2\chi_i\right)^2,$$

$$\delta R_{ij}\delta R^{ij}-\frac{3}{8}\delta R^2=\frac{1}{4a^4}\left(\partial^2 h_{ij}\right)^2.$$

Construction of Lagrangian

$$\mathcal{L}_1' = \frac{\sqrt{\gamma}N}{M^2}\left(2D_i\widetilde{K}_{jk}D^i\widetilde{K}^{jk} - 3D_i\widetilde{K}^{ik}D^j\widetilde{K}_{jk}\right),$$
$$\mathcal{L}_2 = \frac{\sqrt{\gamma}N}{M^2}\left(R_{ij}R^{ij} - \frac{3}{8}R^2\right),$$

As alternated for $\mathcal{L}_1'$

$$\mathcal{L}_1 = \frac{\sqrt{\gamma}N}{M^2}(2D_i\widetilde{K}_{jk}D^i\widetilde{K}^{jk} - D_i\widetilde{K}^{ik}D^j\widetilde{K}_{jk} - 2D_i\widetilde{K}_{jk}D^j\widetilde{K}^{ik})$$

this can be written as

$$\mathcal{L}_1 = \frac{\sqrt{\gamma}N}{M^2}W_{ijk}W^{ijk}, \qquad W_{ijk} = 2D_{[i}\widetilde{K}_{j]k} + D_l\widetilde{K}^l{}_{[i}\gamma_{j]k}.$$

Construction of Lagrangian

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$$= \frac{\sqrt{-g}}{M^2}C_{\mu\nu\rho\sigma}C_{\mu'\nu'\rho'\sigma'}\gamma^{\mu\mu'}\gamma^{\nu\nu'}\gamma^{\rho\rho'}u^\sigma u^{\sigma'}$$

N. Deruelle, M. Sasaki, Y. Sendouda and A. Youssef, JHEP **09**, 009 (2012)

Tensor amplitudes in $\mathcal{L}_1$ and $\mathcal{L}_2$ model

$\mathcal{L}_1$ model

$$S = S_{\mathrm{EH}} + S_\phi + S_{\mathrm{higher}}$$

$$S_{\mathrm{higher}} = \frac{1}{\kappa}\int \mathrm{d}^4x \mathcal{L}_1$$

$$\mathcal{L}_1 = \frac{\sqrt{\gamma}N}{M^2}\left(2D_i\widetilde{K}_{jk}D^i\widetilde{K}^{jk} - D_i\widetilde{K}^{ik}D^j\widetilde{K}_{jk} - 2D_i\widetilde{K}_{jk}D^j\widetilde{K}^{ik}\right)$$

for tensor perturbations

$$S = \frac{1}{8\kappa}\int \mathrm{d}t\mathrm{d}^3x\, a^3\left[\dot{h}_{ij}^2 - \frac{1}{a^2}(\partial_k h_{ij})^2 + \frac{4}{M^2a^2}(\partial_k\dot{h}_{ij})^2\right]$$

$\mathcal{L}_1$ model

$$f_k^\lambda(t) = \left(\frac{1}{4\kappa}\right)^{1/2} a^{3/2} \left(1 + \frac{4k^2}{M^2 a^2}\right)^{1/2} h_k^\lambda$$

$$\ddot{f}_k + \omega_k^2(t) f_k = 0$$

$$\omega_k^2 := -\frac{1}{4}\left(H^2 + 2\dot{H}\right) + \frac{k^2/a^2 - 2H^2 - \dot{H}}{1 + 4k^2/M^2a^2} - \frac{4H^2k^2/M^2a^2}{(1 + 4k^2/M^2a^2)^2}$$

WKB solution for $k^2/a^2 \gg H^2, M^2$

$$f_k \simeq \frac{1}{\sqrt{2\omega_k}} \exp\left[-\mathrm{i}\int^t \omega_k(t')\mathrm{d}t'\right]$$

$\mathcal{L}_1$ model

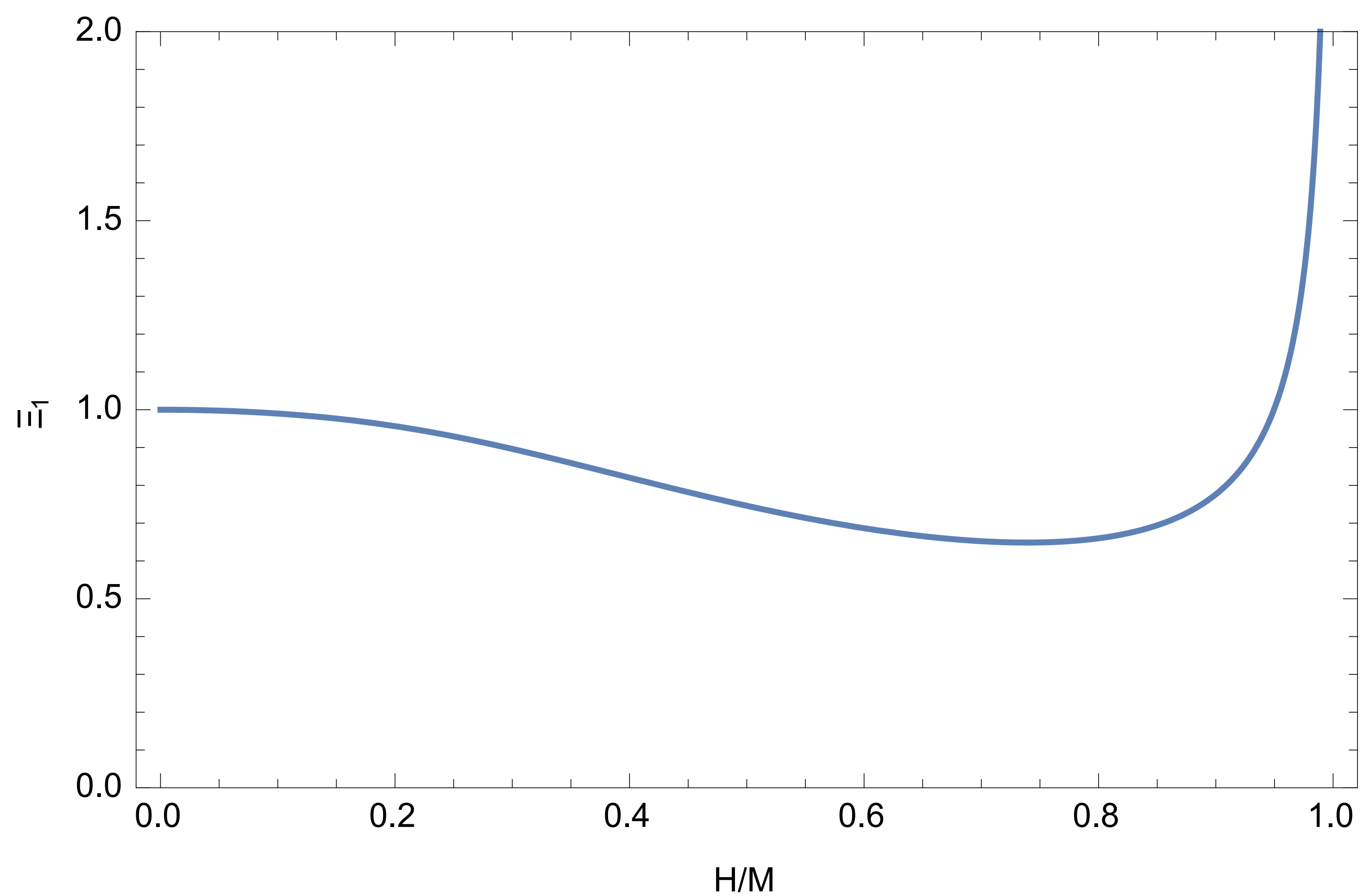
$$\mathcal{P}_T(k) = \frac{k^3}{\pi^2}\left|h_k\right|^2$$

N. Deruelle et al. JHEP **09**, 009 (2012)

$$\mathcal{P}_T = \frac{2\kappa H^2}{\pi^2}\Xi_1(H/M),$$

where

$$\Xi_1(x) := \frac{\cosh(\pi\nu/2)\coth(\pi\nu/2)|\Gamma(-1/4+\mathrm{i}\nu/4)|^4}{128\pi^2 x^3},$$



$$\nu := \sqrt{x^{-2}-1}$$

$\mathcal{L}_2$ model

$$S = S_{\rm EH} + S_\phi + S_{\rm higher}$$

$$S_{\rm higher} = -\frac{1}{2\kappa}\int \mathrm{d}^4x \mathcal{L}_2$$

$$\mathcal{L}_2 = \frac{\sqrt{\gamma}N}{M^2}\left(R_{ij}R^{ij} - \frac{3}{8}R^2\right),$$

for tensor perturbations

$$S = \frac{1}{8\kappa}\int \mathrm{d}t\mathrm{d}^3x\, a^3\left[\dot{h}_{ij}^2 - \frac{1}{a^2}(\partial_k h_{ij})^2 - \frac{1}{M^2a^4}(\partial^2 h_{ij})^2\right]$$

$\mathcal{L}_2$ model

$$v_k^\lambda := (4\kappa)^{-1/2} a h_k^\lambda$$

$$\frac{\mathrm{d}^2 v_k}{\mathrm{d}\eta^2} + \omega_k^2(\eta) v_k = 0$$

$$\omega_k^2 := k^2 + \frac{k^4}{M^2 a^2} - \frac{1}{a}\frac{\mathrm{d}^2 a}{\mathrm{d}\eta^2}$$

WKB solution

$$v_k \simeq \frac{1}{\sqrt{2\omega_k}} \exp\left[-\mathrm{i}\int^\eta \omega_k(\eta')\mathrm{d}\eta'\right]$$

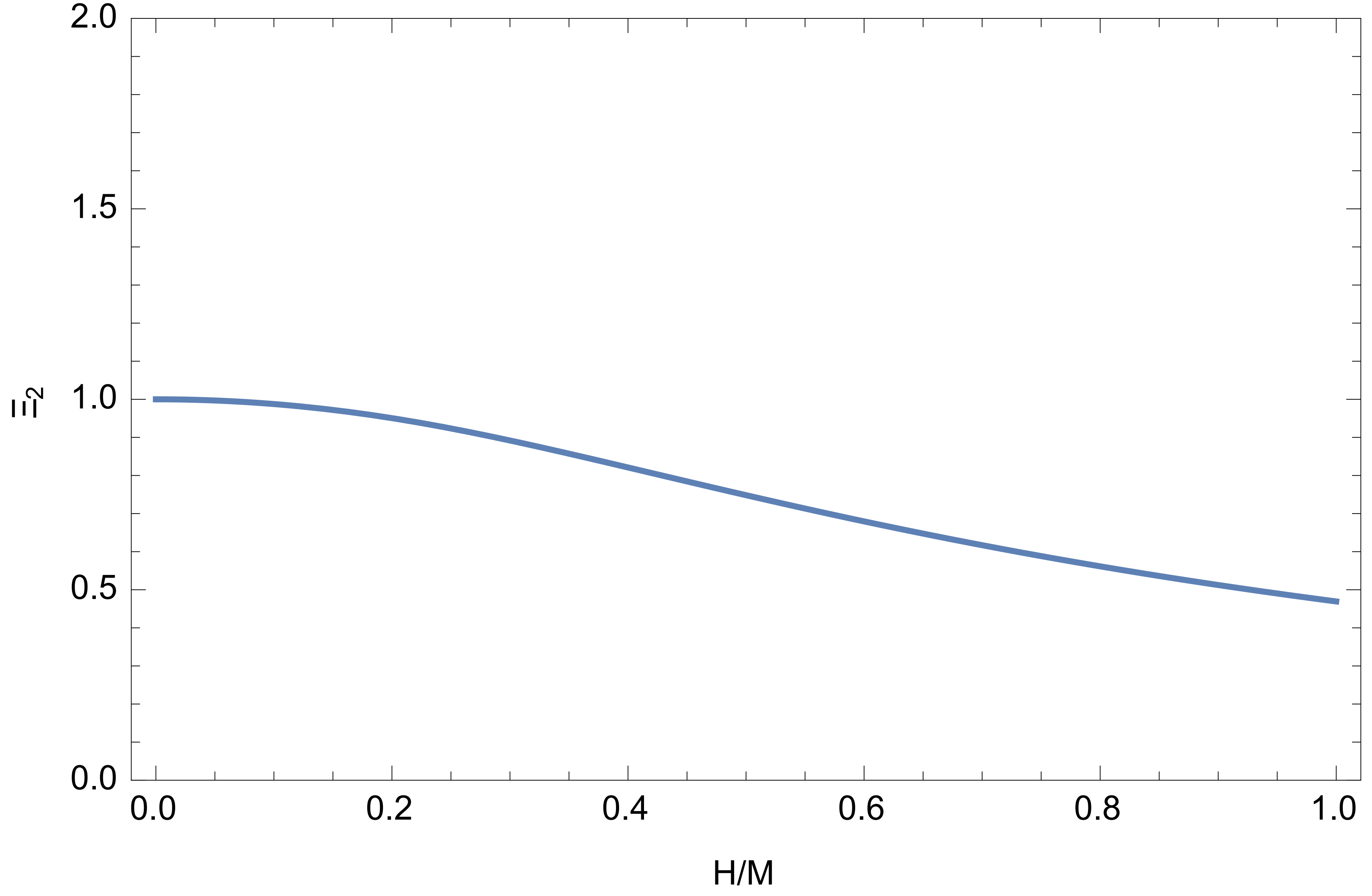
solution at large k in de Sitter background

$$v_k = \frac{e^{-\pi/8x} W_{\mathrm{i}/4x,3/4}(-\mathrm{i}xk^2\eta^2)}{(-2xk^2\eta)^{1/2}}$$

$\mathcal{L}_2$ model

$$\mathcal{P}_T = \frac{2\kappa H^2}{\pi^2}\Xi_2(H/M)$$

$$\Xi_2(x) := \frac{\pi}{4}\left[e^{\pi/(4x)}x^{3/2}\left|\Gamma(5/4+\mathrm{i}/(4x))\right|^2\right]^{-1}$$



$\mathcal{L}_2$ model

$\mathcal{L}_2$ contains

$$\mathcal{L}_2 \sim \frac{1}{M^2}\zeta(\partial^2\zeta)^2$$

non-Gaussianity generated by this term

$$f_{NL} \sim \frac{H^2}{\epsilon M^2}$$

$f_{\mathrm{NL}} \lesssim 1$

$$\frac{H}{M} \lesssim \epsilon^{1/2} \ll 1$$

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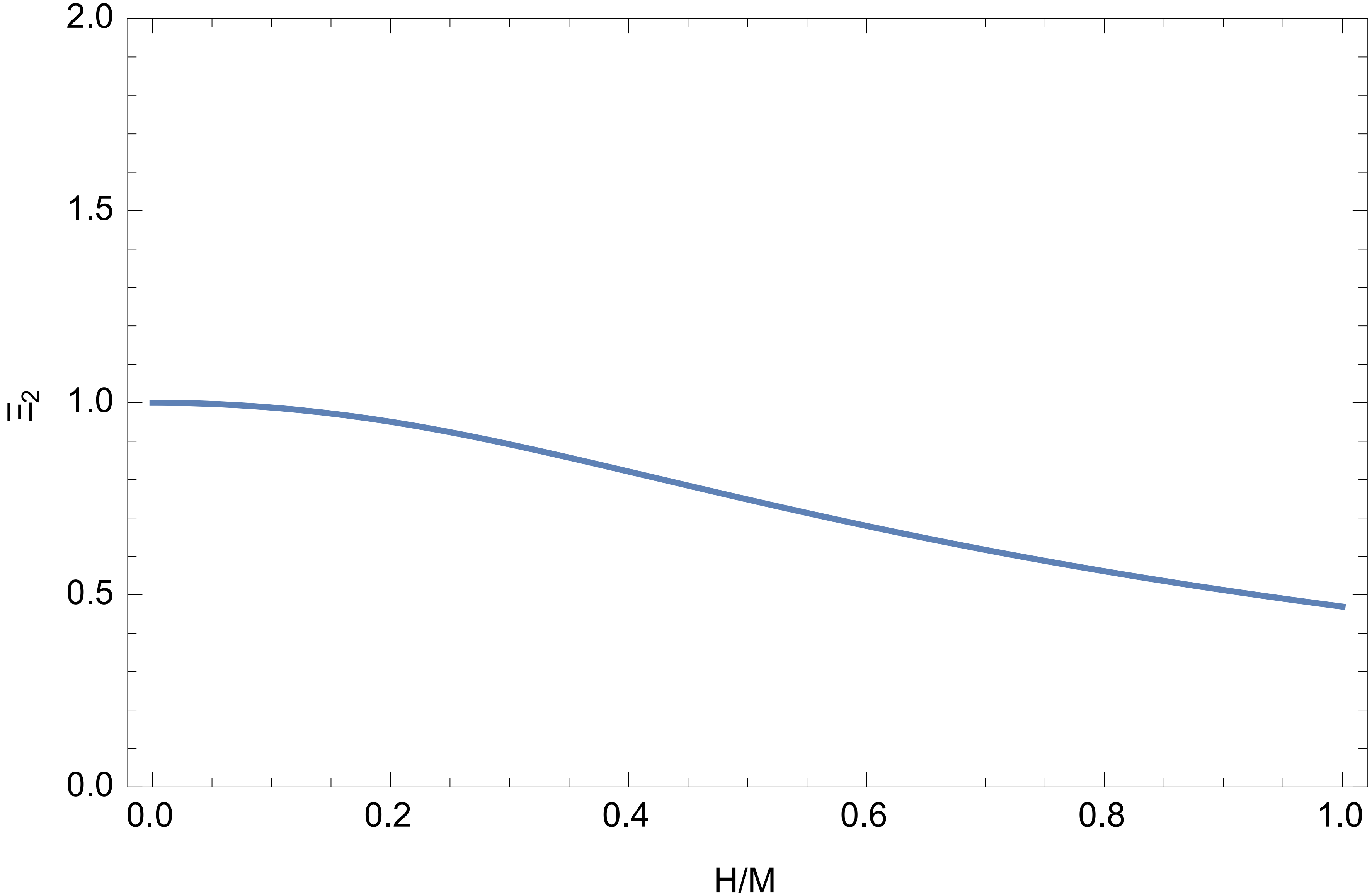
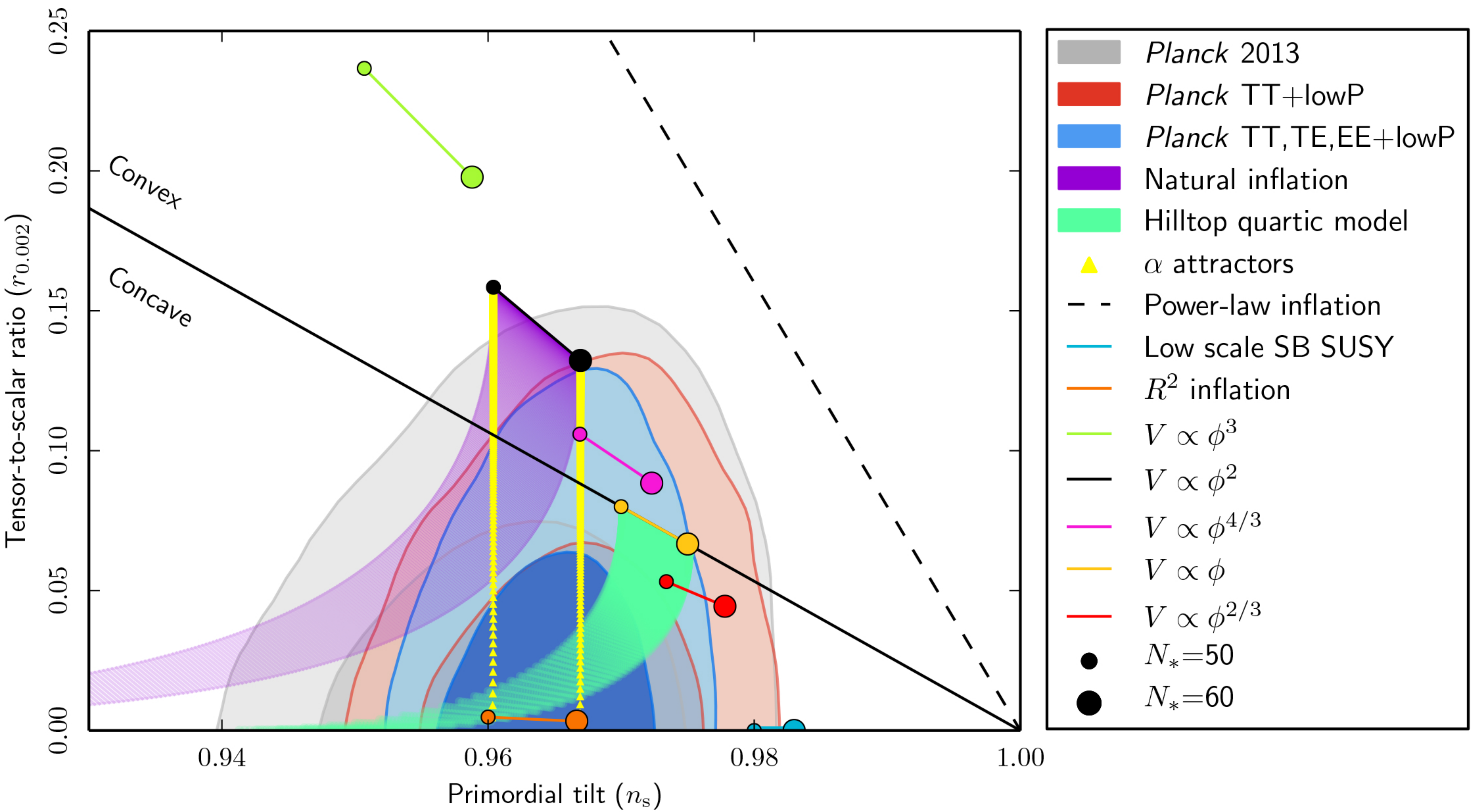
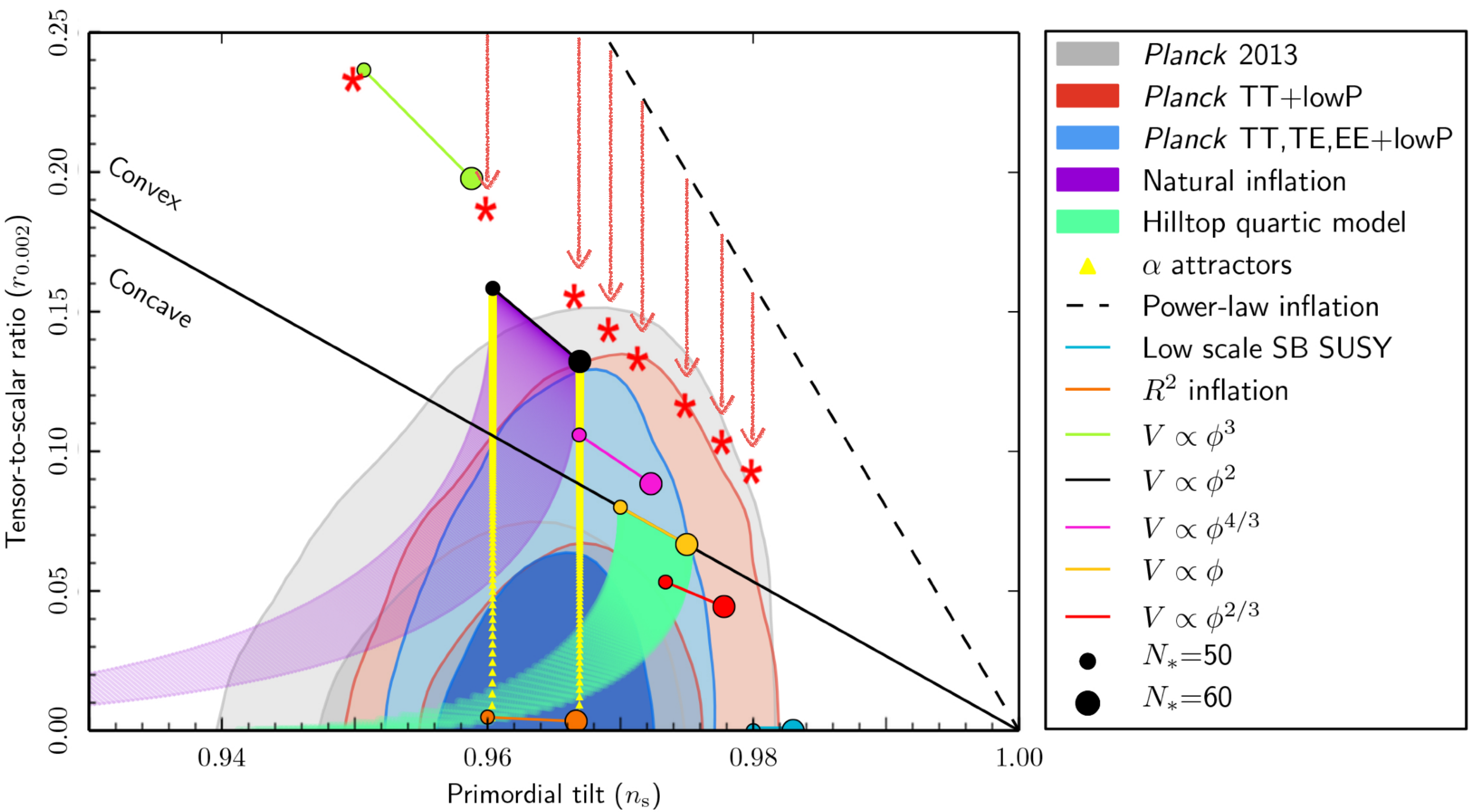


Suppression with $\mathcal{L}_1$ model



Suppression with $\mathcal{L}_1$ model

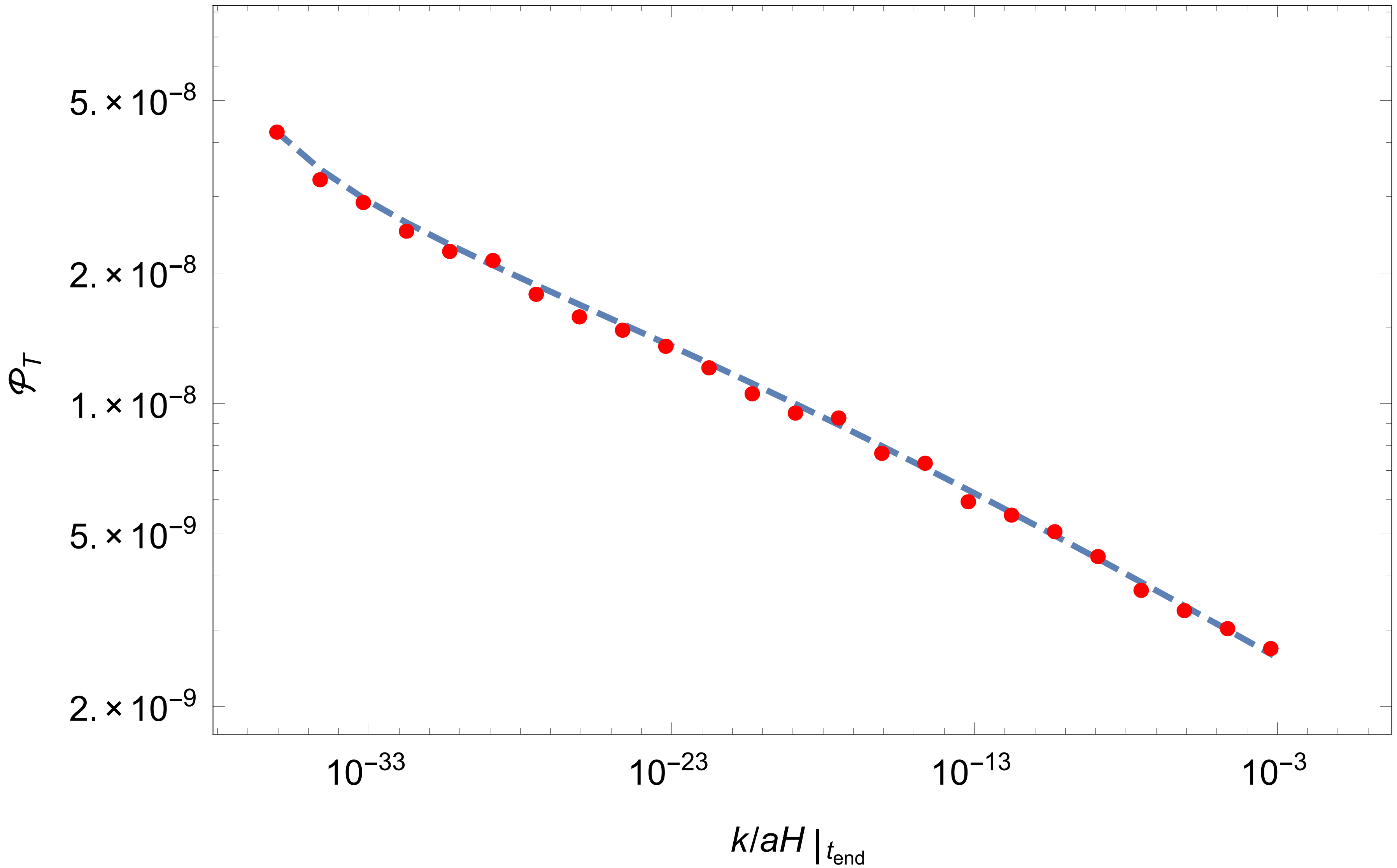


Summary

- We construct two possible theories which change only the dynamics of tensor perturbations without changing scalar sector.
- One of the theories, $\mathcal{L}_1$, can decrease the tensor amplitude up to 65%.
- We can put some inflation models which are out of the observational constraints into the 2σ contour with this suppression effect.

$\mathcal{L}_1$ model

$$\mathcal{P}_T(k) = \frac{2\kappa H^2}{\pi^2}\Xi_1(H/M)\Big|_{k=aH}$$



Blue dashed line: analytic

Red points: numerical

Tensor to scalar ratio $r = 16\epsilon\Xi_1$

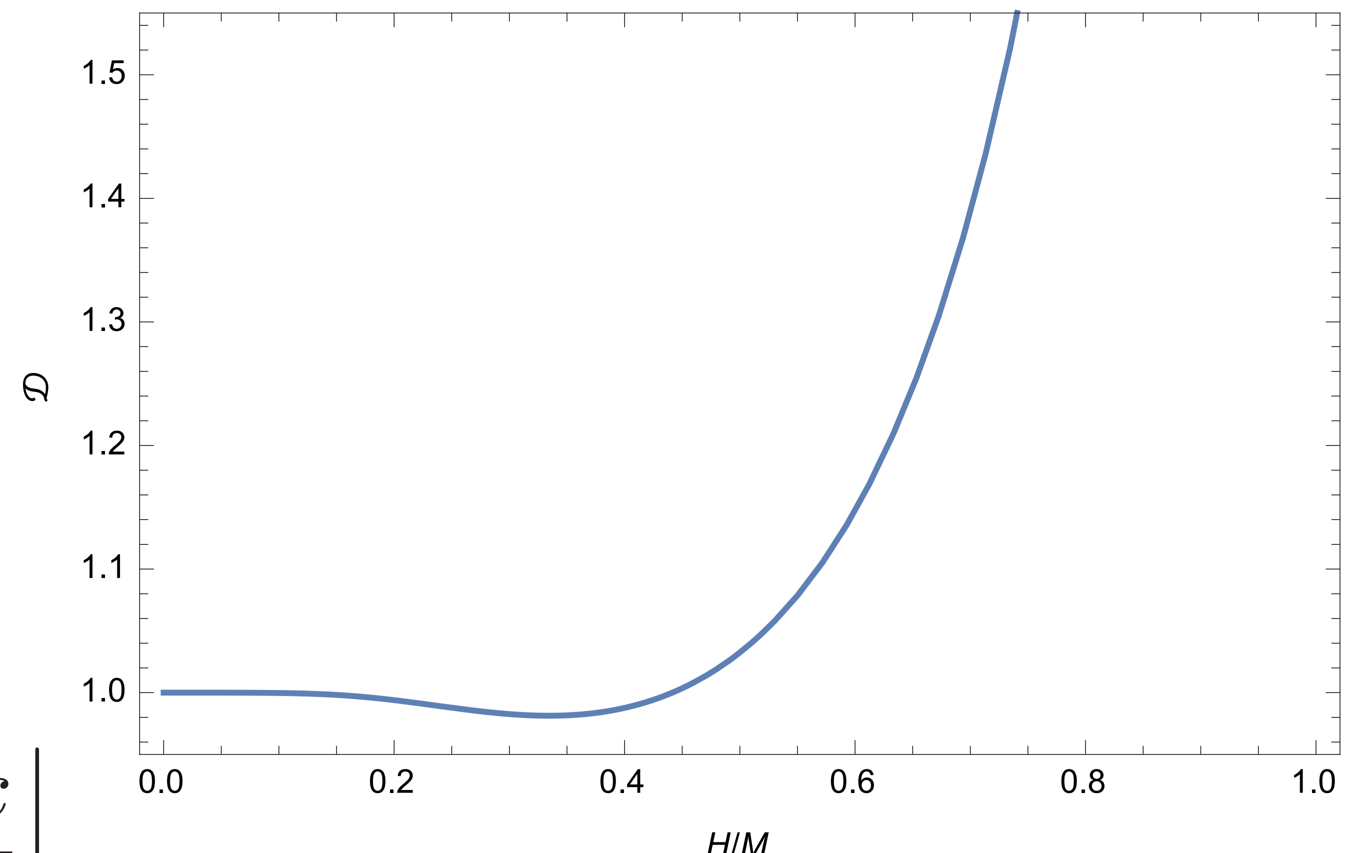
Tensor tilt $n_T := \mathrm{d}\ln\mathcal{P}_T/\mathrm{d}\ln k$

$$n_T = -\frac{2\epsilon}{1-\epsilon}\left[1+\frac{1}{2}\frac{\mathrm{d}\ln\Xi_1}{\mathrm{d}\ln(H/M)}\right]\Bigg|_{k=aH}$$

Consistency relation

$$-8n_T/r \simeq \mathcal{D}|_{k=aH}$$

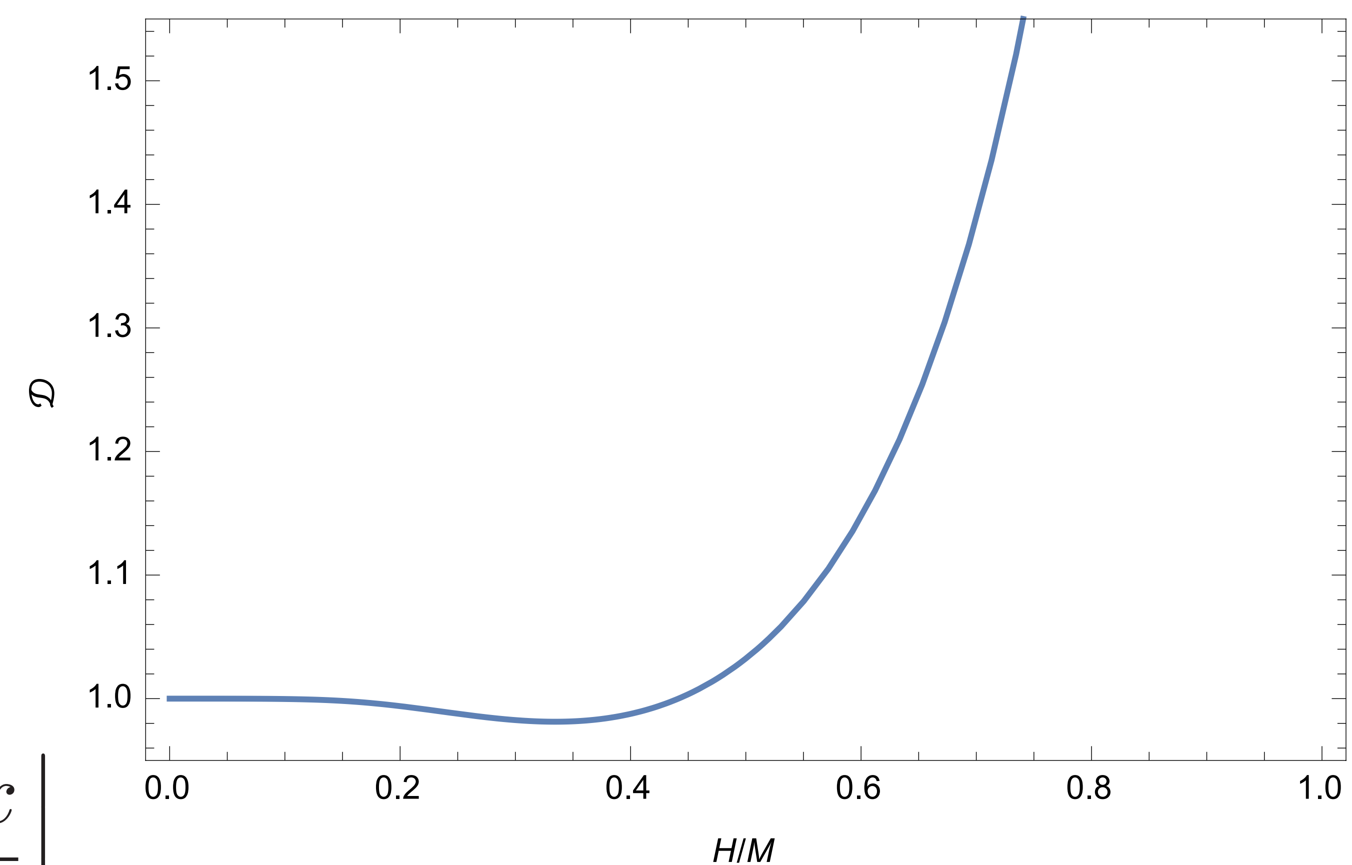
$$\mathcal{D} := \frac{1+(1/2)\mathrm{d}\ln\Xi_1/\mathrm{d}\ln x}{\Xi_1}\Bigg|_{x=H/M}$$



Tensor to scalar ratio $\quad r = 16\epsilon\Xi_1$

Tensor tilt $\quad n_T := \mathrm{d}\ln\mathcal{P}_T/\mathrm{d}\ln k$

$$n_T = -\frac{2\epsilon}{1-\epsilon}\left[1+\frac{1}{2}\frac{\mathrm{d}\ln\Xi_1}{\mathrm{d}\ln(H/M)}\right]\Bigg|_{k=aH} < 0$$



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