Local vs. global temperature for QFT in curved spacetimes

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Inst. f. Theoretische Physik

Global Temperature

Consider a scalar quantum field in a stationary space-time M:

$$(-\Box + \xi R + m^2)\phi = 0.$$

Definition

ω is in global thermal equilibrium at temperature $T \geq 0$

$$\Leftrightarrow$$

$\omega = \omega^{(\beta)}$ is a β-KMS state with $k_B T = \beta^{-1}$.

pros:

- Strong motivation by analogy with quantum statistical mechanics.

cons:

- T is a global property, not a local observable.
- Motivation can be questioned, e.g. for accelerated observers. (Earman, Stud. Hist. Philos. Mod. Phys. **42** (2011) 81–97.)

Local Temperature

In general (globally hyperbolic) M, for $m = 0$ and ω Hadamard:

Definition (Buchholz and Schlemmer, Class. Quantum Grav. **24**, F25–F31 (2007))

Whenever $\omega(:\phi^2:(x)) \geq 0$, the local temperature of ω at x is

$$k_B T_\omega(x) := \sqrt{12\, \omega(:\phi^2:(x))}.$$

pros:

- In Minkowski space: $k_B T_{\omega^{(\beta)}}(x) \equiv \beta^{-1} = k_B T$ (when $m = 0$).
- $:\phi^2:$ can be chosen local and generally covariant.

cons:

- $:\phi^2:$ has a renormalisation ambiguity $\sim R$ (Hollands and Wald (2001)).
- The choice of $:\phi^2:(x)$ as a local thermometer seems arbitrary.
- $T_\omega(x)$ is often ill-defined, even for ground states!

Properties of the Wick Square

Let M be stationary and assume $\omega^{(\beta)}$ exists for all $\beta \in (0, \infty]$.

Proposition

For all $x \in M$:

1. *the following map is continuous and monotonic:*

$$T \mapsto \omega^{(\beta)}(:\phi^2:(x)), \qquad \beta = (k_B T)^{-1}.$$

2. *for all stationary states ω we have: $\omega(:\phi^2:(x)) \geq \omega^{(\infty)}(:\phi^2:(x))$.*

Conclusions:

- When local and global temperature both make sense,

$$T \mapsto T_{\omega^{(\beta)}}(x)$$

  is monotonic and continuous.
- If $T_{\omega^{(\infty)}}(x)$ exists, so does $T_\omega(x)$ for all stationary ω.

Main Result

Sufficient conditions for the existence of $T_\omega(x)$ are:

Theorem

- *M is ultra-static with compact Cauchy surface Σ and non-trivial scalar curvature $R \geq 0$.*
- *ϕ is massless with scalar curvature coupling $\xi \in (0, \frac{1}{6})$.*
- *The Riemann curvature vanishes on an open set $O \subset \Sigma$.*

Then $T_\omega(x)$ exists for all $x \in O$ and all stationary ω.

Remarks:

- Space-times with such a geometry exist.
- M is Minkowski space near O, so no local physics enters.
- Proof uses: Wick rotation, positivity of Euclidean Green's function, conformal transformation and the positive mass theorem.
  (Schoen J. Differential Geom. **20** (1984) 479–495,
  Schoen and Yau Phys. Rev. Lett. **42**, 547–548 (1979).)

Ground States without Local Temperature

To what extent are these conditions necessary?

Some counterexamples:

- Accelerated observers:
  The Fulling vacuum in Rindler space-time has a (strictly) negative Wick square, making the local temperature ill defined.
  (Buchholz and Solveen Class. Quantum Grav. **30** (2013) 085011.)

- Violation of energy conditions:
  There are ultra-static, globally hyperbolic space-times with

$$M = \mathbb{R}^4, \qquad g = -dt^2 + h, \qquad h_{ij} = \Omega^2(\vec{x})\delta_{ij},$$

  such that $R \leq 0$ is non-trivial, a ground state $\omega^{(\infty)}$ exists, and M has a flat region with points where $\omega^{(\infty)}(:\phi^2:(x)) < 0$.

Interpretation

M has classical matter satisfying

$$T^{\mathrm{cl}}_{\mu\nu} = \frac{1}{8\pi} G_{\mu\nu} = \frac{1}{8\pi} \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right).$$

The quantum field ϕ is treated as a test-field, so in general

$$\omega(T^{\mathrm{ren}}_{\mu\nu}(\phi)) + T^{\mathrm{cl}}_{\mu\nu} \neq \frac{1}{8\pi} G_{\mu\nu}.$$

- $\omega(T^{\mathrm{ren}}_{\mu\nu}(\phi))$ is the stress, energy and momentum injected into ϕ by keeping $g_{\mu\nu}$ fixed.
- When $R < 0$ somewhere, $T^{\mathrm{cl}}_{\mu\nu}$ has a negative energy density, and ϕ can continually transfer energy to the metric/matter.
- Can we trust a thermodynamical interpretation of $\omega^{(\infty)}$?
- Does this explain the absence of a local temperature for $\omega^{(\infty)}$?

Conclusions and Outlook

Conclusions:

- pro: Under some physically reasonable circumstances, local and global temperature exist and have qualitatively similar behaviour.
- con: $T_\omega(x)$ may not exist for ground states, due to acceleration (Rindler spacetime) or violation of energy conditions ($R < 0$).

Possibly interesting extensions include:

- points x where space-time is not flat,
- non-compact Cauchy surfaces (perhaps with $R_{ab} \geq 0$?),
- other local observables, e.g. the renormalised stress tensor $T_{\mu\nu}$,
- massive theories.