Bigravitons as dark matter and gravitational waves

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Katsuki Aoki,

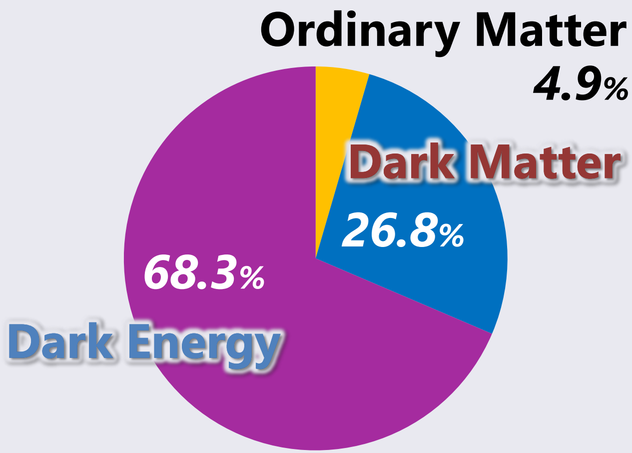
Waseda University (Japan)

Based on KA and Shinji Mukohyama,
“Massive gravitons as dark matter and gravitational waves”
PRD 94, 024001 (2016) [arXiv:1604.06704]

Introduction

GR can describe our Universe
if we introduce unknown matters.

We may need beyond standard model!!



Standard model of gravity = GR, but

What is graviton?

- It should be spin-2 field.
- Massless field or Massive field? How many gravitons?

There could be several gravitons as other gauge bosons.

For simplicity, we focus on the case of two gravitons

→ Bigravitons = massless graviton and massive graviton

Non-linear bigravity theory (Hassan,Rosen, '11)

Two dynamical metrics: $g_{\mu\nu}$ and $f_{\mu\nu}$ (Hassan, and Rosen, 2011)

$$S = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\mathcal{R}(f)$$

$$-\frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\sum_{i=0}^{4} b_i \mathscr{U}_i(g,f) + S^{[\mathrm{m}]}(g,\psi) \qquad \kappa^2 = \kappa_g^2 + \kappa_f^2$$

$$\mathscr{U}_n(g,f) = -\frac{1}{n!(4-n)!}\epsilon^{\cdots\cdot}\epsilon_{\cdots\cdot}(\gamma^\mu{}_\nu)^n \qquad \gamma^\mu{}_\alpha\gamma^\alpha{}_\nu = g^{\mu\alpha}f_{\alpha\nu}$$

Interaction is restricted to be free from ghost.

Particle aspect of massive graviton? = It may be just a massive particle.

We derive the energy-momentum (psudo-)tensor of massive graviton from nonlinear bigravity theory.

→ Massive graviton can be a dark matter.

Linearization of bigravity

Consider the perturbation $\delta g_{\mu\nu} := g_{\mu\nu} - \eta_{\mu\nu}\,, \quad \delta f_{\mu\nu} := f_{\mu\nu} - \eta_{\mu\nu}\,.$

Mass eigenstates with mass dimension one

$$h_{\mu\nu} := \frac{\kappa_f}{\kappa_g \kappa}\delta g_{\mu\nu} + \frac{\kappa_g}{\kappa_f \kappa}\delta f_{\mu\nu}\,, \quad \varphi_{\mu\nu} := \frac{1}{\kappa}\left(\delta g_{\mu\nu} - \delta f_{\mu\nu}\right)\,,$$

Linearized bigravity theory
= General Relativity (massless spin-2) + Fierz-Pauli theory (massive spin-2)

$$S_2 = \int d^4x \left[\frac{1}{\kappa_g^2}\mathcal{L}_{\rm EH}[\delta g] + \frac{1}{\kappa_f^2}\mathcal{L}_{\rm EH}[\delta f] + \mathcal{L}_{\rm int}[\delta g, \delta f] + \frac{1}{2}\delta g_{\mu\nu} T_{\rm m}^{\mu\nu}\right]$$

$$= \int d^4x \left[\underbrace{\mathcal{L}_{\rm EH}[h] + \frac{1}{2M_{\rm pl}} h_{\mu\nu} T_{\rm m}^{\mu\nu}}_{\text{GR (massless)}} + \underbrace{\mathcal{L}_{\rm EH}[\varphi] + \mathcal{L}_{\rm FP}[\varphi] + \frac{1}{2M_G}\varphi_{\mu\nu} T_{\rm m}^{\mu\nu}}_{\text{FP (massive)}}\right]\,,$$

GR (massless) $M_{\rm pl} := \frac{\kappa}{\kappa_g \kappa_f}$ FP (massive) $M_G := \frac{\kappa}{\kappa_g^2}$

Massive graviton as dark matter

We shall discuss gravitons perturbatively $\leftarrow T_{gw}^{\mu\nu}, T_G^{\mu\nu}$ = 2nd order of EOM

We focus on sub-horizon scale
→ background can be Minkowski spacetime

Perturbative expansion can used when

Vainshtein radius is much smaller than scales of DM
$\rightarrow \; p \ll \Lambda_3 \quad (\Lambda_3 := (mM_G)^{1/3})$

The universe is free from the Higuchi type instability
$\rightarrow H \lesssim m$

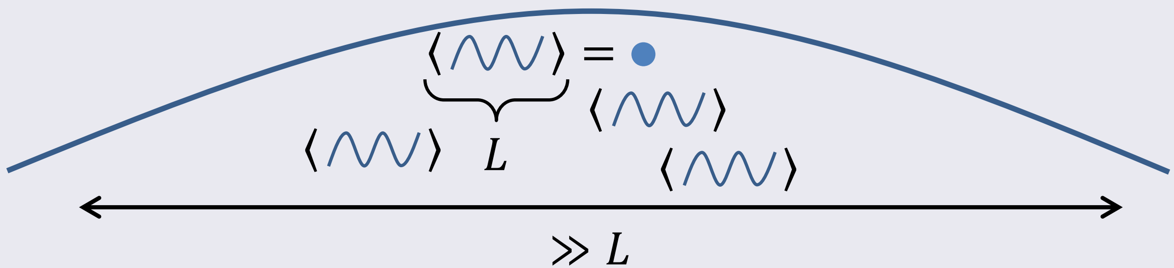
Energy-momentum tensor of graviton

In GR, $T^{\mu\nu}_{gw}$ can be obtained from the nonlinear part of EOM. (Isaacson, 1968)

$$T^{\mu\nu}_{\rm gw} := -\langle M_{\rm pl}^2 \overset{(2)}{\delta G^{\mu\nu}} \rangle = \frac{1}{4}\langle \partial^\mu h^{\alpha\beta} \partial^\nu h_{\alpha\beta} \rangle$$ in transverse-traceless gauge

→ (High-frequency) gravitons are sources of (low-frequency) graviton via nonlinear terms of Einstein equation in GR.

$$\mathcal{E}^{\mu\nu,\alpha\beta} h_{\alpha\beta} = \frac{1}{M_{\rm pl}}(T^{\mu\nu}_{\rm m} + T^{\mu\nu}_{\rm gw})$$



Bigravity? There are two Einstein equations → Source of which equation?

Energy-momentum of gravitons in bigravity

We find the energy-momentum tensors (both massless and massive) are sources of the massless graviton.

The fully nonlinear Einstein equations in bigravity

$$G^{\mu\nu}(g) = \kappa_g^2 \left(T_g^{(\mathrm{int})\mu\nu} + T_{\mathrm{m}}^{\mu\nu} \right) , \quad G^{\mu\nu}(f) = \kappa_f^2 T_f^{(\mathrm{int})\mu\nu}$$

EOM of massless graviton including nonlinear terms:

$$\mathcal{E}^{\mu\nu,\alpha\beta} h_{\alpha\beta} = \frac{1}{M_{\mathrm{pl}}} (T_{\mathrm{m}}^{\mu\nu} + T_{\mathrm{gw}}^{\mu\nu}(h) + T_G^{\mu\nu}(\varphi))$$

$$T_{\mathrm{gw}}^{\mu\nu} := \left\langle -M_{\mathrm{pl}}^2 \delta \overset{(2)}{G}{}^{\mu\nu}(h) \right\rangle = \frac{1}{4} \langle \partial^\mu h^{\alpha\beta} \partial^\nu h_{\alpha\beta} \rangle$$

$$T_G^{\mu\nu} := \left\langle -\frac{1}{\kappa^2} \delta \overset{(2)}{G}{}^{\mu\nu}(\varphi) + \delta \overset{(2)}{T}{}_g^{(\mathrm{int})\mu\nu}(h,\varphi) + \delta \overset{(2)}{T}{}_f^{(\mathrm{int})\mu\nu}(h,\varphi) \right\rangle = \frac{1}{4} \langle \partial^\mu \varphi^{\alpha\beta} \partial^\nu \varphi_{\alpha\beta} \rangle$$

Massive graviton as dark matter in bigravity

When we focus on the scales $\gg m^{-1}$,
the gravity mediated by the massive graviton is suppressed.

$$\Phi := -\delta g_{00}/2 = -\frac{GM}{r}\left(1+\alpha e^{-mr}\right) \approx -\frac{GM}{r} \qquad G := 1/8\pi M_{\rm pl}^2$$

→ Only the Einstein equation of massless graviton is relevant.

$$\mathcal{E}^{\mu\nu,\alpha\beta}h_{\alpha\beta} = \frac{1}{M_{\rm pl}}(T^{\mu\nu}_{\rm m} + T^{\mu\nu}_{\rm gw} + T^{\mu\nu}_{G}) \qquad T^{\mu\nu}_{G} = \frac{1}{4}\langle \partial^\mu \varphi^{\alpha\beta}\partial^\nu \varphi_{\alpha\beta}\rangle$$

In the scales $\gg m^{-1}$,
the massive gravitons does not modify the gravity as a mediator,
however they can modify the gravity as a gravitational source.

When $m \gg p \gg \text{galaxy scales}^{-1}$ $\quad (\Leftrightarrow m \gtrsim 10^{-23}\ \mathrm{eV})$

$$T^{\mu\nu}_{G} = \frac{m^2}{4}\mathrm{diag}[\langle\varphi^{\alpha\beta}\varphi_{\alpha\beta}\rangle, 0, 0, 0]$$ = Dark matter

Other constraint on massive graviton DM

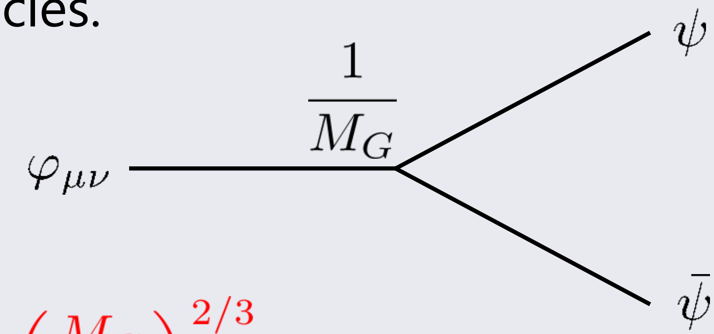
Lower bound on graviton mass: $m \gtrsim 10^{-23}\ \mathrm{eV}$

$$T_G^{\mu\nu} = \frac{m^2}{4}\mathrm{diag}[\langle \varphi^{\alpha\beta}\varphi_{\alpha\beta}\rangle, 0, 0, 0]$$ = Dark matter

We can obtain upper bound on graviton mass from the decay rate.

$$S_{\mathrm{FP}} = \int d^4x \left[\mathcal{L}_{\mathrm{EH}}[\varphi] + \mathcal{L}_{\mathrm{FP}}[\varphi] + \frac{1}{2M_G}\varphi_{\mu\nu}T_{\mathrm{m}}^{\mu\nu}\right]$$

Massive gravitons can decay to light particles.



Total decay rate of massive graviton

$$\Gamma_G \sim 0.1\frac{m^3}{M_G^2} \ll H_0$$

Upper bound on graviton mass: $m \lesssim 0.01 \left(\frac{M_G}{M_{\mathrm{pl}}}\right)^{2/3}\ \mathrm{GeV}$

Massive graviton production

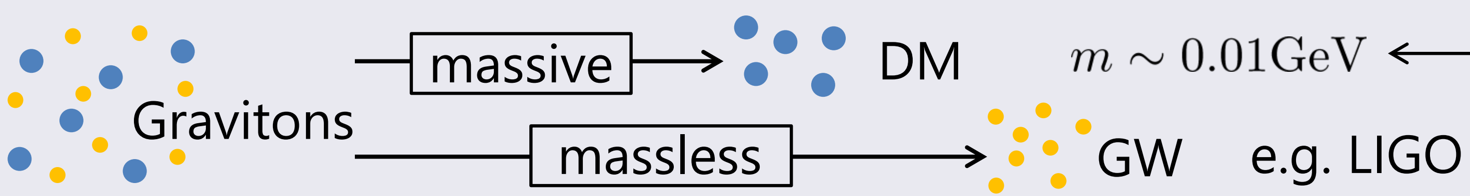
The massive graviton can be dark matter when

$$10^{-23}\ \mathrm{eV} \lesssim m \lesssim 0.01 \left(\frac{M_G}{M_{\rm pl}}\right)^{2/3}\ \mathrm{GeV}\,,$$

We assume the production of massive graviton happens when $H_* \lesssim m$

How to generate massive gravitons?

$$\to \frac{1}{2M_{\rm pl}} h_{\mu\nu} T^{\mu\nu}_{\rm m} + \frac{1}{2M_G} \varphi_{\mu\nu} T^{\mu\nu}_{\rm m}$$



When GWs are generated, MGs (=DM) are also generated.

→ Bigravitons as dark matter and gravitational waves

Massive gravitons from preheating

As an example, we consider generation through the preheating.

GW is generated by collisions of field bubbles (= large inhomogeneity) in sub-horizon scales.

GW momentum and energy $k_* \sim 1/R_* \,, \quad \rho^*_{\rm gw} \sim 0.1\,(R_* H_*)^2 \rho_*$

R_* : typical size of field bubble
ρ_* : background energy density

Dufaux et al. JCAP 0903, 001 (2009)

Produced massive gravitons are relativistic ($\Lambda_3 \gg p^\mu \gg m$)

$$\langle \varphi_{\alpha\beta}{}^{,\mu} \varphi^{\alpha\beta,\nu} \rangle \sim \frac{M_{\rm pl}^2}{M_G^2} \langle h_{\alpha\beta}{}^{,\mu} h^{\alpha\beta,\nu} \rangle$$

After the cosmic expansion, the present abundance is

$$\Omega_G \sim \frac{M_{\rm pl}^2}{M_G^2} \frac{m}{2\pi f} \Omega_{\rm gw}$$

f is the present frequency of GW

Massive gravitons from preheating

We focus on GW to be sensitive in the LIGO range.

e.g. $\rho_*^{1/4} \sim 10^8\,\mathrm{GeV}\,,\; R_*^{-1} \sim 0.1\,\mathrm{GeV} \Rightarrow f \sim 40\,\mathrm{Hz}\,,\; h^2\Omega_{\mathrm{gw}} \sim 10^{-9}$

If LIGO detects GW background and MG is dominant component of DM,

we obtain $\dfrac{M_{\mathrm{pl}}^2}{M_G^2} m \sim 10^{-14}\ \mathrm{GeV}$

Consistency of our assumptions: $R_*^{-1} > m > \sqrt{2}H_*$

A set of consistent parameters is $m \sim 0.01\ \mathrm{GeV}\,, \quad M_G \sim 10^6 M_{\mathrm{pl}}$

The corresponding free streaming scale ~ 10^{-7} pc → CDM

Even for our restricted case, we can construct a consistent model of DM.

Summary and Discussions

We have proposed a scenario in which the massive graviton is DM.

DM can be created when GM is created.
→ GW can carry information about DM in our scenario.
e.g. We can estimate the graviton mass from GW.

We construct a consistent model of DM originated by preheating.
→ $m \sim 0.01\ \mathrm{GeV},\ M_G \sim 10^6 M_{\mathrm{pl}}$ when LIGO observes GW from preheating.

Direct detection of graviton mass? → consistency relation of MG-DM.

Other scenario would be possible.

Babichev et al. arXiv: 1604.08564.

- ✓ Thermal production→ $\mathrm{TeV} \lesssim m \lesssim 10^{11}\ \mathrm{GeV}$ with $M_G \gg M_{\mathrm{pl}}$
- ✓ Other GW emission event? Inflation? First order transition?...