ISCO frequencies for rotating neutron stars - simple and accurate formulae

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11.7.2016

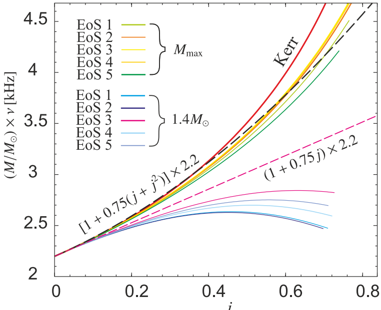
Linear approximation

Linear approximation (*Kluzniak & Wagoner 1985*)

$$\nu_{ISCO} = \frac{2200}{M}\,(1 + 0.75j)\,.$$

Orbital frequency of free test particle and ISCO radius (*Abramowicz et al. 2003*)

$$\nu_K = \frac{\mathcal{F}(M)}{r_{ISCO}^{3/2}}\left[1 - \frac{j}{r_{ISCO}^{3/2}} + j^2 F_1(r_{ISCO}) + qF_2(r_{ISCO})\right],\; r_{ISCO} = 6\,\left(1 - ja_1 + j^2 a_2 + qa_3\right).$$



Török et al. 2010

Equations of state

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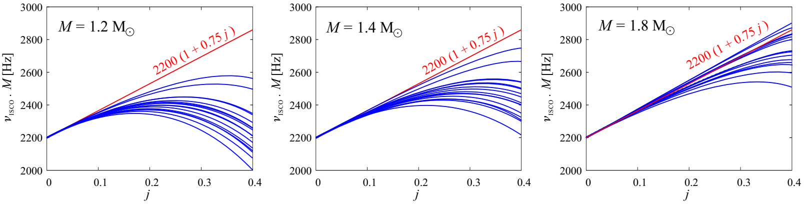
- *SkT5, SkO, SkO', SLy4, Gs, SkI2, SkI5, SGI* and *SV* (*Rikovska Stone et al. 2003*),
- *UBS, nocross* (Urbanec et al. 2010),
- *APR* (*Akmal et al. 1998*),
- *AU* (*Wiringa et al. 1988*),
- *L, N, A* (*Arnett & Bowers 1977*).

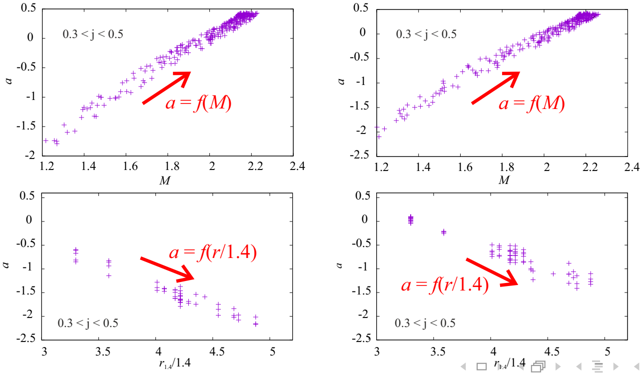


Finding dependency

Quadratic relation

$$\nu_{ISCO} = \frac{2200}{M}\left(1 + 0.75j + aj^2\right), \qquad a = \frac{\nu_{ISCO}(M, j, q)M}{2200j^2} - 1 - 0.75j.$$

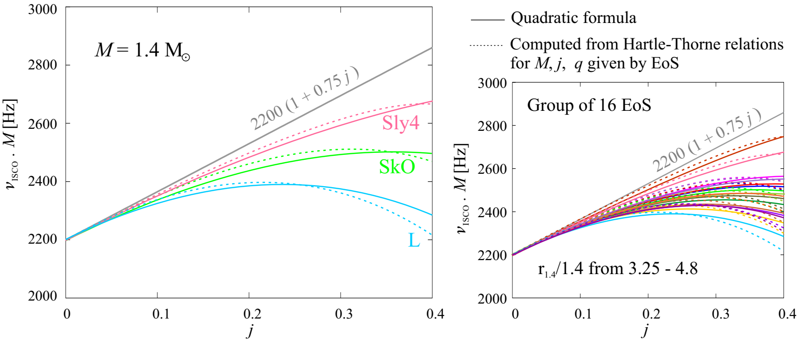
Dependency of a on NS mass (M) and EoS parameter ($r_{1.4}/1.4$, where $r_{1.4}$ is radius of nonrotating NS with $M = 1.4$ given by the EoS)



Quadratic formula for the ISCO frequency

Approximative quadratic formula (where $r_{1.4}$ is radius of nonrotating NS with $M = 1.4$ given by the EoS):

$$\nu_{ISCO} = \frac{2200}{M}\left(1 + 0.75j + (4.62\sqrt{M} - 0.73\frac{r_{1.4}}{1.4} - 3.37)j^2\right).$$



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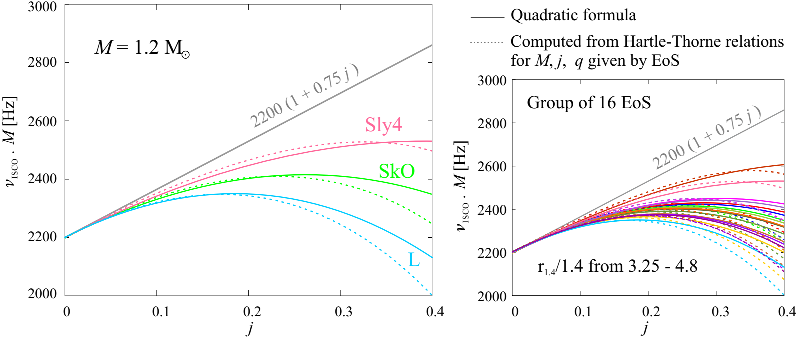

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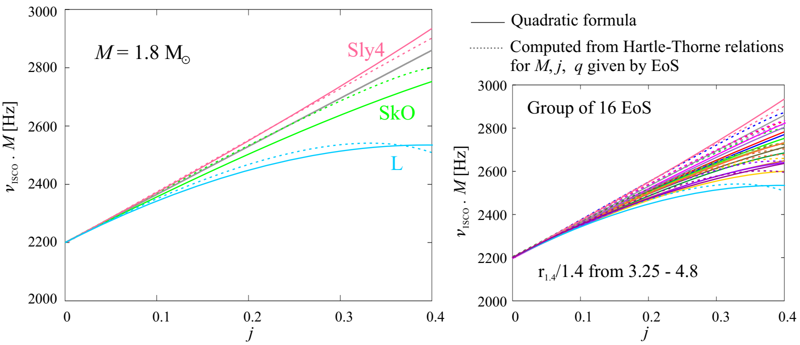

Dependency on compactness, mass and spin

Orbital frequency of free test particle and ISCO radius (*Abramowicz et. al 2003*)

$$\nu_K = \frac{\mathcal{F}(M)}{r_{ISCO}^{3/2}}\left[1 - \frac{j}{r_{ISCO}^{3/2}} + j^2 F_1(r_{ISCO}) + qF_2(r_{ISCO})\right], r_{ISCO} = 6\left(1 - ja_1 + j^2 a_2 + qa_3\right).$$

ISCO frequency up to the second order

$$\nu_{ISCO} = \frac{2200}{M}\left(1 + c_1 j + c_2 q + c_3 j^2\right),$$

q and j relations (*Urbanec et. al 2013*, $x_0 = 2.46$, $x = \frac{R}{2M_0}$):

$$x > x_0: \; q = \left(d_1 \frac{R}{M} - d_2\right) j^2, \quad x \leq x_0: \; q = \left(d_3\left(\frac{R}{M}\right)^2 - d_4 \frac{R}{M} + d_5\right) j^2.$$

Implied quadratic formula(s)

$$\nu_{ISCO} = \frac{2200}{M}\left[1 + \frac{11}{6\sqrt{6}} j + \left(-0.425\frac{R}{M} + 2\right) j^2\right], \qquad x > x_0.$$

$$\nu_{ISCO} = \frac{2200}{M}\left[1 + \frac{11}{6\sqrt{6}} j + \left(-0.07\left(\frac{R}{M}\right)^2 + 0.29\frac{R}{M} + 0.25\right) j^2\right], \quad x \leq x_0.$$

Quadratic formula for the ISCO frequency

Approximative quadratic formula ($x_0 = 2.46$, $x = \frac{R}{2M_0}$):

$$\nu_{ISCO} = \frac{2200}{M}\left[1 + \frac{11}{6\sqrt{6}}j + \left(-0.425\frac{R}{M} + 2\right)j^2\right], \qquad x > x_0.$$

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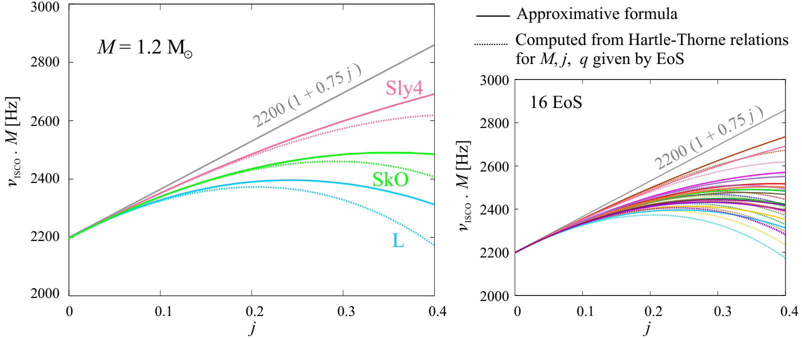

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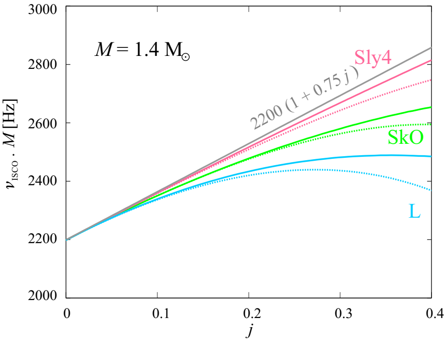

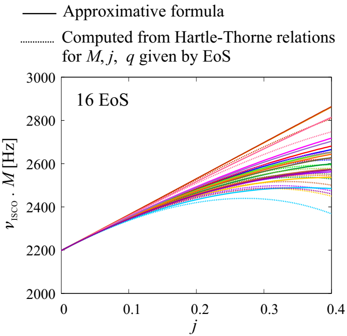

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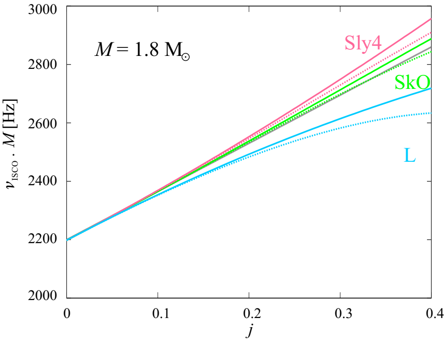

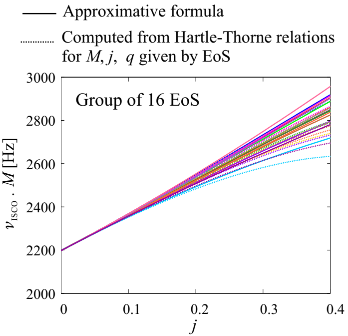

Summary

Approximative quadratic formulas:

- Dependency on equation of state, mass and spin Approximative quadratic formula (where $r_{1.4}$ is radius of nonrotating NS with $M = 1.4$ given by the EoS):

$$\nu_{ISCO} = \frac{2200}{M}\left(1 + 0.75j + (4.62\sqrt{M} - 0.73\frac{r_{1.4}}{1.4} - 3.37)j^2\right).$$

- Dependency on compactness, mass and spin ($x_0 = 2.46$, $x = \frac{R}{2M}$):

$$\nu_{ISCO} = \tfrac{2200}{M}\left[1 + \tfrac{11}{6\sqrt{6}}j + \left(-0.425\tfrac{R}{M} + 2\right)j^2\right], \qquad x > x_0.$$
$$\nu_{ISCO} = \tfrac{2200}{M}\left[1 + \tfrac{11}{6\sqrt{6}}j + \left(-0.07\left(\tfrac{R}{M}\right)^2 + 0.29\tfrac{R}{M} + 0.25\right)j^2\right], \quad x \le x_0.$$

Thank you for your attention