First Law for fields with Internal Gauge

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First law (Iyer-Wald)

Lagrangian $L(g_{\mu\nu}, R, \nabla R, \ldots, \varphi, \nabla\varphi, \ldots)$ on spacetime.
For *stationary axisymmetric* black hole solution

$$T_H \delta S = \delta E_{can} - \Omega_H \delta J_{can}$$

- E_{can} canonical energy; J_{can} angular momenta at spatial infinity

- $T_H = \kappa/2\pi$ where κ is the surface gravity.

- δS depends on $\delta L/\delta R_{\mu\nu\rho}{}^{\lambda}$

Iyer-Wald assume all dynamical fields ψ

- are *smooth* tensor fields on spacetime

- have a well-defined group action of diffeomorphisms e.g. to decide stationarity $\pounds_t \psi = 0$

Problem 1: smooth tensor fields

In general, gauge fields $A^I_\mu\big|$ *cannot* be chosen to be smooth everywhere

- E.g. magnetic monopole in Electrodynamics *(Dirac string singularity)*

would be nice to have a first law *without gauge-fixing*

Problem 2: diffeomorphisms

Charged fields have internal gauge transformations $g \in G$

$$\Psi(x) \mapsto g^{-1}(x)\Psi(x)$$

$$A(x) \mapsto g^{-1}(x)A(x)g(x) + g^{-1}(x)dg(x)$$

- just a gauge transformation *at fixed x* is well-defined

- but diffeomorphism is only defined up to an *arbitrary* gauge!

- stationarity $\pounds_t \psi = gauge$

Goal

Derive the *First Law* of Black Hole Mechanics for Einstein-Yang-Mills

- also covers *Tetrad GR, Einstein-Dirac*, Lovelock, B-F gravity, higher derivative gravity, arbitrary charged tensor-spinor fields, **all of Standard Model**...

Solution: work on Principal Bundle
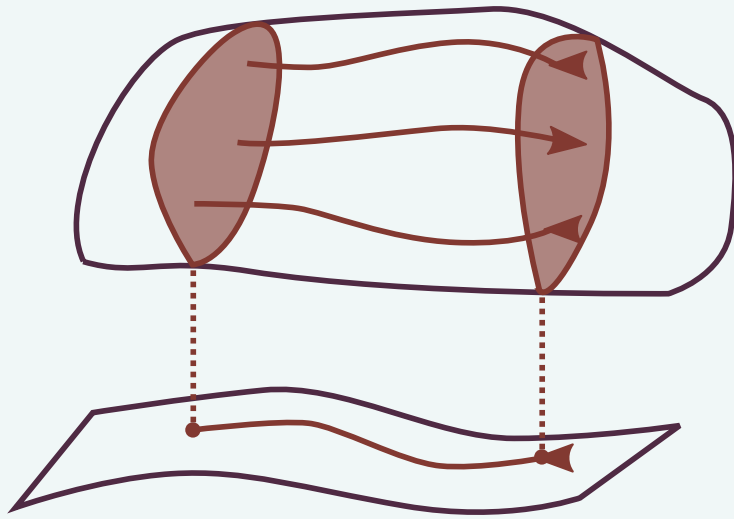
- $\pi : P \to M$; $\pi^{-1}(x) \cong G$

- all fields smooth on P; gauge fields ≡ connection

- $f : P \to P$ automorphism of P ≡ combined diffeo & gauge

- stationary $\pounds_X \psi = 0$ where $X \in TP$ and $\pi_* X = t$

- apply Iyer-Wald procedure but on P

First law — Einstein-Yang Mills

Gauge field as *connection* $A^I_\mu\big|$ on bundle with
$$L = L_{EH} + \star F \wedge F + \theta(F \wedge F)\big|$$

$$T_H \delta S + (\mathscr{V}^\Lambda \delta \mathcal{Q}_\Lambda)|_B = \delta E_{can} - \Omega_H \delta J_{can}\big|$$

- $\delta E_{can} = \delta M_{ADM} + (\mathscr{V}^\Lambda \delta \mathcal{Q}_\Lambda)|_\infty\big|$

- $\mathscr{V}^\Lambda\big|$ potentials; depend explicity only on connection $A^I_\mu\big|$

- charges $\mathcal{Q}_\Lambda\big|$ depend only on $\delta L/\delta F^I_{\mu\nu}\big|$

Yang-Mills charges

- $\mathcal{Q}_\Lambda = \int *F_I h_\Lambda^I\big|$ and $\widetilde{\mathcal{Q}}_\Lambda = \theta \int F_I h_\Lambda^I\big|$ electric and magnetic charges

- e.g. n independent charges for $U(1)^n\big|$ or $SU(n+1)\big|$

- Sudarsky-Wald get *zero* potential at horizon due to assuming $\pounds_t A = 0\big|$, in general horizon potential is *not zero*.

- Magnetic charge is topological and does not contribute to first law

Temperature & Entropy

Gravity: *tetrads* $e_\mu^a\big|$ and *spin connection* $\omega_\mu{}^a{}_b\big|$ on the bundle. Compute $\mathscr{V}^\Lambda\big|$ for spin connection for *any* covariant Lagrangian.

- Only *one* non-zero potential ≡ boosts along the horizon

- $\mathscr{V}_{grav} = \kappa \implies T_H = \frac{1}{2\pi}\,\mathscr{V}_{grav}\big|$

- and perturbed entropy $\delta S = 2\pi\delta\mathcal{Q}_{grav}\big|$

- for Einstein-Hilbert Lagrangian: $\mathcal{Q}_{grav} = \frac{1}{8\pi}\,\mathrm{Area}\big|$

Einstein-Dirac

For spinor fields Ψ with Dirac Lagrangian on bundle

- no contribution at the horizon

- no contribution at infinity due to fall-off conditions

- usual form of first law!

Not Covered

- p-form gauge fields with magnetic charge

- Chern-Simons Lagrangians *(coming soon)*