The Kaluza Ansatz in Eddington inspired Born-Infeld Gravity

Karan Fernandes
w/ Amitabha Lahiri

S. N. Bose National Centre for Basic Sciences, Kolkata, India

GR 21; July 10-15, 2016; New York

Based on **PRD 91.044014** (arXiv:1405.2172[gr-qc])

July 14, 2016

Motivation

- Geometric deduction of EM coupling with gravity
- Non-minimally coupled actions in higher derivative theories.
- EiBI: Metric-affine theory; Determinantal action
- Infinite number of higher derivative terms.
- Smooths out singularities

The Eddington inspired Born-Infeld action

$$S_{EiBI} = \frac{1}{8\pi\kappa}\int d^4x \left[\sqrt{|\mathbf{g} + \kappa\mathbf{R}(\Gamma)|} - \lambda\sqrt{|g|}\right] + S_M$$

- Einstein-Hilbert action with c.c. and matter when

  i) $\kappa R_{\mu\nu} << g_{\mu\nu}$, ii) $\lambda = \kappa\Lambda + 1$

- $\tilde{q}_{\mu\nu} = g_{\mu\nu} + \kappa R(\Gamma)_{\mu\nu}$

$$\sqrt{|\tilde{q}|}\tilde{q}^{\mu\nu} - (\kappa\Lambda + 1)\sqrt{|g|}g^{\mu\nu} = -8\pi\kappa\sqrt{|g|}\,T^{\mu\nu} \ \ \text{(metric)},$$

$$\nabla_\alpha(\sqrt{|\tilde{q}|}\tilde{q}^{\mu\nu}) = 0 \ \ \text{(connection)}\,.$$

The Eddington inspired Born-Infeld action

$$S_{EiBI} = \frac{1}{8\pi\kappa} \int d^4x \left[\sqrt{|\mathbf{g} + \kappa \mathbf{R}(\Gamma)|} - \lambda\sqrt{|g|} \right] + S_M$$

- $$R(\Gamma)_{\mu\nu} = \tau_{\mu\nu} + 64\pi^2\kappa \left[S_{\mu\nu} - \tfrac{1}{4} S g_{\mu\nu} \right] + \mathcal{O}(\kappa^2)\,;$$

  where $\quad S_{\mu\nu} = T_{\mu\alpha} T^{\alpha}{}_{\nu} - \tfrac{1}{2} T T_{\mu\nu}\,, \quad \tau_{\mu\nu} = \Lambda g_{\mu\nu} + 8\pi \left[T_{\mu\nu} - \tfrac{1}{2} T g_{\mu\nu} \right].$

- $$\Gamma^{\alpha}_{\beta\gamma} = \left\{ {\alpha \atop \beta\gamma} \right\} + \frac{1}{2}\kappa q^{\alpha\delta} \left(R(\Gamma)_{\delta\beta;\gamma} + R(\Gamma)_{\gamma\delta;\beta} - R(\Gamma)_{\beta\gamma;\delta} \right)$$

$$\begin{aligned} \Rightarrow R(g)_{\mu\nu} = \tau_{\mu\nu} + 64\pi^2\kappa \left[S_{\mu\nu} - \tfrac{1}{4} S g_{\mu\nu} \right] \\ + \tfrac{1}{2}\kappa \left[\nabla_\mu \nabla_\nu \tau - 2\nabla^\alpha \nabla_{(\mu}\tau_{\nu)\alpha} + \Box \tau_{\mu\nu} \right] + \mathcal{O}(\kappa^2); \end{aligned}$$

The Eddington inspired Born-Infeld action : Metric

$$S_{EiBI} = \frac{1}{8\pi\kappa}\int d^4x\left[\sqrt{|\mathbf{g}+\kappa\mathbf{R}(g)|} - \lambda\sqrt{|g|}\right] + S_M$$

- $R_{\mu\nu} = \tau_{\mu\nu} + 64\pi^2\kappa\left(S_{\mu\nu} - \frac{1}{4}Sg_{\mu\nu}\right)$
  $+\frac{1}{2}\kappa\left[2R_{\alpha\mu\beta\nu}R^{\alpha\beta} - 2R_{\mu\beta}R^{\beta}_{\nu} + \Box R_{\mu\nu}\right] + \mathcal{O}(\kappa^2)\,.$

- $R(\Gamma)_{\mu\nu} = R(g)_{\mu\nu}$ (to lowest order)

  The metric-affine theory and the metric theory agree to $\mathcal{O}(\kappa)$.

- Involves third order derivatives of matter fields (and above)

  $\Rightarrow$ Surface singularities [Pani,Sotiriou (2012)]

The Eddington inspired Born-Infeld action : Metric

$$S_{EiBI} = \frac{1}{8\pi\kappa}\int d^4x \left[\sqrt{|\mathbf{g}+\kappa\mathbf{R}(g)|} - \lambda\sqrt{|g|}\right] + S_M$$

- $R_{\mu\nu} = \tau_{\mu\nu} + 64\pi^2\kappa\left(S_{\mu\nu} - \frac{1}{4}Sg_{\mu\nu}\right)$
  $+\frac{1}{2}\kappa\left[2R_{\alpha\mu\beta\nu}R^{\alpha\beta} - 2R_{\mu\beta}R^{\beta}_{\nu} + \Box R_{\mu\nu}\right] + \mathcal{O}(\kappa^2)\,.$

- $R(\Gamma)_{\mu\nu} = R(g)_{\mu\nu}$ (to lowest order)

The metric-affine theory and the metric theory agree to $\mathcal{O}(\kappa)$.

- Involves third order derivatives of matter fields (and above)

$\Rightarrow$ Surface singularities [Pani,Sotiriou (2012)]

The Kaluza Ansatz

Geometrically determines the coupling to gravity

- $\hat{g}_{AB} = \begin{pmatrix} g_{\mu\nu} + \alpha^2 A_\mu A_\nu & \alpha A_\mu \\ \alpha A_\nu & 1 \end{pmatrix}, \quad A, B, ... = 0, 1, ..4\,; \quad \sqrt{|\hat{g}|} = \sqrt{|g|}$

- Quantities independent of the 5^{th} dimension.

- Need $\hat{q}_{AB} = \hat{g}_{AB} + \kappa \hat{R}_{AB}$. $\qquad (F^2 = F_{\mu\nu}F^{\mu\nu})$

$$\hat{q}_{\mu\nu} = g_{\mu\nu} + 4A_\mu A_\nu \left(1 + \kappa F^2\right) + \kappa \left[R_{\mu\nu} - 4A_{(\mu}\nabla_{|\beta|}{F^\beta}_{\nu)} - 2F_{\beta\mu}{F^\beta}_\nu\right],$$

$$\hat{q}_{5\nu} = 2A_\nu \left(1 + \kappa F^2\right) - \kappa \nabla_\beta {F^\beta}_\nu\,,$$

$$\hat{q}_{55} = 1 + \kappa F^2\,.$$

The Kaluza Ansatz : EiBI action

- $$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(D)\det(A - BD^{-1}C)$$

- $$\begin{aligned} |\hat{q}| &= q_{55}[q_{\mu\nu} - q_{\mu 5}(q_{55})^{-1}q_{5\nu}] \\ &= \left[(1+\kappa F^2)\left(g_{\mu\nu} + \kappa\left(R_{\mu\nu} + 2F_{\mu\beta}F^{\beta}{}_{\nu}\right)\right) - \kappa^2 \nabla_\delta F^{\delta}{}_{\mu} \nabla_\beta F^{\beta}{}_{\nu}\right] . \end{aligned}$$

- $$\hat{S}_{EiBI} = \frac{1}{8\pi\hat{G}_5\kappa}\int d^5x \left[\sqrt{|\hat{q}|} - (\kappa\Lambda + 1)\sqrt{|\hat{g}|}\right]$$

- $S_{EiBI+EM} =$
$$\frac{1}{8\pi\kappa}\int d^4x \left[\sqrt{\left|(1+\kappa F^2)\left(g_{\mu\nu} + \kappa\left(R_{\mu\nu} + 2F_{\mu\beta}F^{\beta}{}_{\nu}\right)\right) - \kappa^2 \nabla_\delta F^{\delta}{}_{\mu} \nabla_\beta F^{\beta}{}_{\nu}\right|} - (\kappa\Lambda + 1)\sqrt{|g|}\right] .$$

The Kaluza Ansatz : EiBI action

- $$\det\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(D)\det(A - BD^{-1}C)$$

- $$|\hat{q}| = q_{55}[q_{\mu\nu} - q_{\mu 5}(q_{55})^{-1}q_{5\nu}]$$
$$= \left[(1+\kappa F^2)\left(g_{\mu\nu} + \kappa\left(R_{\mu\nu} + 2F_{\mu\beta}F^{\beta}{}_{\nu}\right)\right) - \kappa^2\nabla_\delta F^{\delta}{}_{\mu}\nabla_\beta F^{\beta}{}_{\nu}\right] .$$

- $$\hat{S}_{EiBI} = \frac{1}{8\pi\hat{G}_5\kappa}\int d^5x\left[\sqrt{|\hat{q}|} - (\kappa\Lambda + 1)\sqrt{|\hat{g}|}\right]$$

- $S_{EiBI+EM} =$
$$\boxed{\frac{1}{8\pi\kappa}\int d^4x\left[\sqrt{\left|(1+\kappa F^2)\left(g_{\mu\nu} + \kappa\left(R_{\mu\nu} + 2F_{\mu\beta}F^{\beta}{}_{\nu}\right)\right) - \kappa^2\nabla_\delta F^{\delta}{}_{\mu}\nabla_\beta F^{\beta}{}_{\nu}\right|}\right.}$$
$$\left. -(\kappa\Lambda + 1)\sqrt{|g|}\right] .$$

Equations of motion

- $\mathcal{O}(\kappa^0)$: $G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi T_{\mu\nu}$, $\qquad \nabla_\alpha F^{\alpha\nu} = 0$.

- $\mathcal{O}(\kappa)$: $\quad G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi T_{\mu\nu} + \kappa P_{\mu\nu} + \kappa Q_{\mu\nu}$, where

$$
\begin{aligned}
P_{\mu\nu} =& R_{\mu\alpha} R^\alpha_\nu - \tfrac{1}{2} R R_{\mu\nu} - \tfrac{1}{4} R_{\alpha\beta} R^{\alpha\beta} g_{\mu\nu} + \tfrac{1}{8} R^2 g_{\mu\nu} + \tfrac{1}{2} \nabla_\mu \nabla_\nu R \\
&\qquad - \tfrac{1}{2} g_{\mu\nu} \Box R - \nabla_\alpha \nabla_{(\mu} R^\alpha_{\nu)} + \tfrac{1}{2} \Box R_{\mu\nu} + \tfrac{1}{2} g_{\mu\nu} \nabla_\alpha \nabla_\beta R^{\alpha\beta} , \\
Q_{\mu\nu} =& R F_{\mu\alpha} F_\nu{}^\alpha + \nabla_\alpha F^\alpha{}_\nu \nabla_\beta F^\beta{}_\mu + 4 R_{(\mu|\alpha|} F^{\alpha\beta} F_{|\beta|\nu)} + 2 F_{\mu\alpha} R^{\alpha\beta} F_{\beta\nu} \\
& + 8 F_{\mu\alpha} F^{\alpha\beta} F_{\beta\gamma} F^\gamma{}_\nu - 2 \nabla_{(\mu} (F_{\nu)\beta} \nabla_\alpha F^{\alpha\beta}) - \tfrac{1}{8} F^4 g_{\mu\nu} - \tfrac{1}{4} g_{\mu\nu} R F^2 \\
& - 2 \nabla_\alpha (F^\alpha{}_{(\mu} \nabla_{|\beta|} F^\beta{}_{\nu)}) + F^2 F_{\mu\beta} F_\nu{}^\beta + 2 \left(\nabla_{(\mu} F_{\nu)\beta} \right) \nabla_\alpha F^{\alpha\beta} \\
& - 2 \nabla_\alpha \nabla_{(\mu} \left(F_{\nu)\beta} F^{\beta\alpha} \right) + \Box \left(F_{\mu\beta} F^\beta{}_\nu \right) + g_{\mu\nu} \nabla_\alpha \nabla_\beta \left(F^{\alpha\gamma} F_\gamma{}^\beta \right) \\
& - \tfrac{1}{2} \nabla_\mu \nabla_\nu F^2 + \tfrac{1}{2} g_{\mu\nu} \Box F^2 + \tfrac{1}{2} F^2 R_{\mu\nu} - F_{\alpha\beta} F^{\beta\gamma} F_{\gamma\delta} F^{\delta\alpha} g_{\mu\nu} \\
& - F_{\alpha\beta} F^{\beta\gamma} R^\alpha_\gamma g_{\mu\nu} + \nabla_\alpha (F^\alpha{}_\beta \nabla_\gamma F^{\gamma\beta}) g_{\mu\nu} - \tfrac{1}{2} \left(\nabla_\alpha F^\alpha{}_\beta \right) \nabla_\gamma F^{\gamma\beta} g_{\mu\nu}
\end{aligned}
$$

Ricci scalar and Electromagnetic EOM

$$R = 4\Lambda - \kappa\left[\tfrac{1}{2}F^4 + \tfrac{1}{2}RF^2 + 2R^{\alpha\beta}F_{\beta\gamma}F^{\gamma}{}_{\alpha} + 4F^{\alpha\beta}F_{\beta\gamma}F^{\gamma\delta}F_{\delta\alpha} \right.$$
$$\left. + \nabla_\alpha\nabla_\beta\left(R^{\alpha\beta} + 2F^{\alpha\delta}F_{\delta}{}^{\beta}\right) - \Box\left(R - \tfrac{1}{2}F^2\right) + \nabla_\beta F^{\beta\gamma}\nabla^\alpha F_{\alpha\gamma}\right] .$$

$$\nabla_\alpha F^{\alpha\nu} = -\kappa\left[\nabla_\alpha\left(F^{\alpha\nu}\left(\frac{1}{2}R + \frac{1}{2}F^2\right)\right) - 4\nabla_\alpha\left(F^{\alpha\beta}F_{\beta\gamma}F^{\gamma\nu}\right) - \frac{1}{2}\Box\left(\nabla_\alpha F^{\alpha\nu}\right) \right.$$
$$\left. -\nabla_\alpha\left(R^{\alpha\beta}F_{\beta}{}^{\nu} - R^{\nu\beta}F_{\beta}{}^{\alpha}\right) + \frac{1}{2}\nabla_\alpha\nabla^\nu\left(\nabla_\beta F^{\beta\alpha}\right)\right] .$$

- Regardless of appearances, the solutions are rather simple.

Solutions : Iterative procedure

$$\Lambda g_{\mu\nu} = -G_{\mu\nu} + 8\pi T_{\mu\nu} - \kappa C_{\mu\nu} \, ,$$
$$\nabla_\mu F^{\mu\nu} = -\kappa D^\nu \, .$$

- Solve via $g_{\mu\nu} = g^0_{\mu\nu} + g^1_{\mu\nu}$ and $A_\mu = A^0_\mu + A^1_\mu$
- $g^0_{\mu\nu} = (f(r), f(r)^{-1}, r^2, r^2 sin\theta^2)$; $\qquad f(r) = 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3} + \frac{q^2}{r^2}$

$$A^0_\mu = \left(\frac{q}{r}, 0, 0, 0\right)$$

- Next order can now be solved :

$$\Lambda g^1_{\mu\nu} + G^1_{\mu\nu} - 8\pi T^1_{\mu\nu} = -\kappa C_{\mu\nu} \, ,$$
$$\nabla^0_\mu F^{1\mu\nu} + \nabla^1_\mu F^{0\mu\nu} = -\kappa D^\nu \, ,$$

Solutions : Iterative procedure

$$\kappa C_{\mu\nu} = \begin{pmatrix} -f(r)\alpha(r) & 0 & 0 & 0 \\ 0 & \alpha(r)(f(r))^{-1} & 0 & 0 \\ 0 & 0 & r^2\beta(r) & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta\beta(r) \end{pmatrix},$$

$$\kappa D^{\nu} = \left(\frac{12\kappa\, q^3}{r^7}, 0, 0, 0\right) ;$$

$$\text{where } \alpha(r) = -\frac{9\,\kappa\, q^4}{2\, r^8} - \frac{\kappa\,\Lambda\, q^2}{r^4} \text{ and } \beta(r) = \frac{3\,\kappa\, q^4}{2\,\Lambda\, r^8} + \frac{\kappa\, q^2}{r^4}$$

- General solution for $g_{\mu\nu}$ and A_μ

$$g_{\mu\nu} = (-(f(r) + \kappa G(r)), (f(r) + \kappa G(r))^{-1}, 0, 0)$$

$$A_\mu = \left(\frac{q}{r} + \kappa B(r), 0, 0, 0\right)$$

Solutions : Iterative procedure

- RNdS correction:

$$G(r) = \frac{3\,q^4}{10\,r^6} - \frac{\Lambda\,q^2}{r^2} \quad , \quad B(r) = \frac{3q^3}{5r^5}$$

- Trace : $R = 4\Lambda + \dfrac{6\kappa q^4}{r^8}$; finite for all finite r.
- No surface singularities present in the derived solution.
- Procedure can be repeated for higher order solutions.

Conclusions

- Derived a 4-d action of EM coupled to EiBI.
- Iterative procedure for solutions about a fixed background

  $\rightarrow \mathcal{O}(\kappa)$ correction of Reissner-Nördstrom de Sitter.
- Questions remain : Stability of solutions

  EM wave propagation

  Behaviour of geodesics.