Stochastic Cosmological Lensing

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Julien Larena

Department of Mathematics

Rhodes University

South Africa

RHODES UNIVERSITY
Where leaders learn

Based on arXiv:1508.07903 (JCAP 11 (2015) 022)

with P. Fleury and J.-P. Uzan (IAP, Paris)

Context

Standard cosmology relies (mostly) on distances measured along our lightcone.

- Some sources are extended: CMB, BAO;
- Some are 'point' sources (narrow light beam): Supernovæ.
- A simple question: Can we describe all these observations with the same model?

  Well, yes; mostly: Concordance cosmology.
- But for SN, fluid approx. along line-of-sight might be misguided [Clarkson et al, 2011].
- Small corrections may be important [Fleury et al, 2013] for precision cosmology.

Aim

Here we propose a new approach to this problem:

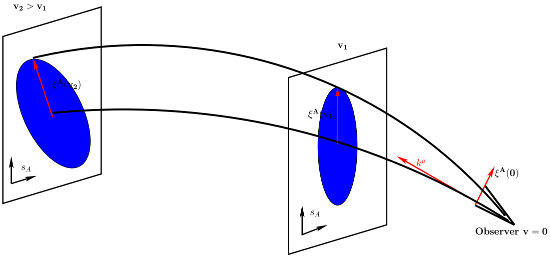
- Describe propagation light from point sources using stochastic description for lenses along the line of sight.
- Coherent description of multi-scale lensing:
  - Fluid = average lensing: Smooth lensing
  - non-fluid = noise: Clumpiness.
- Effects on distance-redshift relation.

The Jacobi matrix

Jacobi matrix $\boldsymbol{\mathcal{D}}$ relates shape of light beam to its angular shape at observation.

- ξ^A: shape of image.
- Jacobi matrix (Linearity of NGDE):

$$\boxed{\xi^A(v) = \boldsymbol{\mathcal{D}}^A{}_B \xi^B(0)}$$



Distances: $D_A = \sqrt{\det \boldsymbol{\mathcal{D}}}$ and $D_L = (1+z)^2 D_A$

The Sachs equation

- Sachs equation:

$$\boxed{\frac{d^2\boldsymbol{\mathcal{D}}^A{}_B}{dv^2} = \mathcal{R}^A{}_C\boldsymbol{\mathcal{D}}^C{}_B}$$

- Optical tidal matrix $\mathcal{R}_{AB} = R^a{}_{bcd}s^a_A k^b k^c s^d_B$:

$$\mathcal{R}_{AB} = \mathcal{R}I_2 + \mathcal{W}_{AB}$$

- With:
  - $\mathcal{R} = -\frac{1}{2}R_{ab}k^a k^b$: Ricci focussing;
  - Weyl distortions:

$$\mathcal{W}_{AB} = C^a{}_{bcd}s^a_A k^b k^c s^d_B = \begin{pmatrix} -\mathcal{W}_1 & \mathcal{W}_2 \\ \mathcal{W}_2 & \mathcal{W}_1 \end{pmatrix}.$$

The Sachs-Langevin equation

$$\frac{d^2\boldsymbol{\mathcal{D}}}{dv^2} = \langle\mathcal{R}\rangle\boldsymbol{\mathcal{D}} + \delta\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{D}}$$

- $\langle\boldsymbol{\mathcal{R}}\rangle = \langle\mathcal{R}\rangle I_2$: Slow varying: Deterministic;
- $\delta\boldsymbol{\mathcal{R}} = \delta\mathcal{R}\boldsymbol{I}_2 + \boldsymbol{\mathcal{W}}$: Rapidly varying: Stochastic noise.
- Statistical homogeneity and isotropy:

$$\langle\boldsymbol{\mathcal{W}}\rangle = \langle\delta\mathcal{R}(w)\mathcal{W}_A(v)\rangle = \langle\mathcal{W}_1(w)\mathcal{W}_2(v)\rangle = 0$$

- White noises:

$$\begin{aligned} \langle\delta\mathcal{R}(v)\delta\mathcal{R}(w)\rangle &= C_{\mathcal{R}}(v)\delta(v-w) \\ \langle\mathcal{W}_A(v)\mathcal{W}_B(w)\rangle &= C_{\mathcal{W}}(v)\delta(v-w)\delta_{AB} \end{aligned}$$

Fokker-Planck-Kolmogorov equation for the S-L equation

Assuming Noise is a Gaussian white noise:

$$\frac{\partial p\left(v, \boldsymbol{\mathcal{D}}, \dot{\boldsymbol{\mathcal{D}}}\right)}{\partial v} = -\dot{\mathcal{D}}_{AB}\frac{\partial p}{\partial \mathcal{D}_{AB}} - \langle \mathcal{R} \rangle \mathcal{D}_{AB}\frac{\partial p}{\partial \dot{\mathcal{D}}_{AB}}$$

$$+\frac{1}{2}\left[C_{\mathcal{R}}\,\delta_{AE}\delta_{CF} + C_{\mathcal{W}}(\delta_{AC}\delta_{EF} - \varepsilon_{AC}\varepsilon_{EF})\right]\mathcal{D}_{EB}\mathcal{D}_{FD}\,\frac{\partial^2 p}{\partial \dot{\mathcal{D}}_{AB}\partial \dot{\mathcal{D}}_{CD}}$$

- Boundary condition fixed: $p\left(0, \boldsymbol{\mathcal{D}}, \dot{\boldsymbol{\mathcal{D}}}\right) = \delta(\boldsymbol{\mathcal{D}})\delta\left(\dot{\boldsymbol{\mathcal{D}}} - \boldsymbol{I}_2\right)$.
- Full statistical info on lensing: evolution for moments etc.

Some analytic results: first order corrections

FPK Eq. can be used to calculate moments of $p\left(v, \boldsymbol{\mathcal{D}}, \dot{\boldsymbol{\mathcal{D}}}\right)$.

- Expand $D_A = D_0 + D_1$ and $\theta = \theta_0 + \theta_1$ with 'background':

$$\ddot{D}_0 = \langle \mathcal{R} \rangle D_0 \text{ and } \theta_0 = \frac{\dot{D}_0}{D_0}.$$

- First order corrections to D_A (see also [Kantowski, 1969]):

$$\boxed{\delta^{(1)}_{D_A} \equiv \frac{\langle D_1 \rangle}{D_0} = -2 \int_0^v \frac{dv_1}{D_0^2(v_1)} \int_0^{v_1} \frac{dv_2}{D_0^2(v_2)} \int_0^{v_2} dv_3 \; D_0^4(v_3) C_{\mathcal{W}}(v_3) < 0}$$

A result on the variance of D_A

Pushing to second order, we get:

$$\frac{d^3}{dx^3}\left[\frac{\mathrm{var}\;[D_A]}{D_0^2}\right] \quad + \quad 2D_0^6(2C_{\mathcal{W}}-C_{\mathcal{R}})\frac{\mathrm{var}\;[D_A]}{D_0^2} = 2C_{\mathcal{R}}D_0^6$$

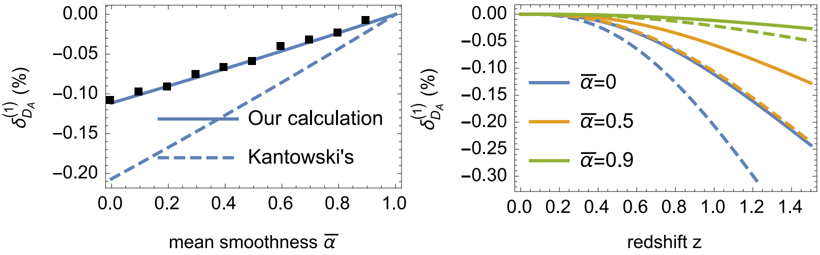
$$+6\int_o^x dx'\left[\frac{d^2\delta^{(1)}_{D_A}}{dx^2}\right]^2 + \mathcal{O}(C^3_{\mathcal{W}}).$$

where $D_0^2 dx = dv$.

- Valid at second order in C.
- Both $\mathcal{R}$ and $\mathcal{W}$ contribute to dispersion of distance.

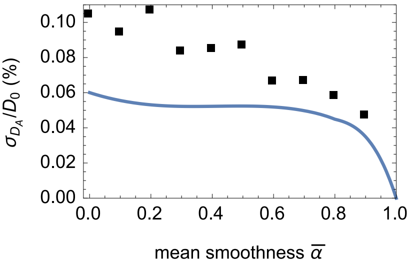
Results: Post-Kantowski-Dyer-Roeder corrections

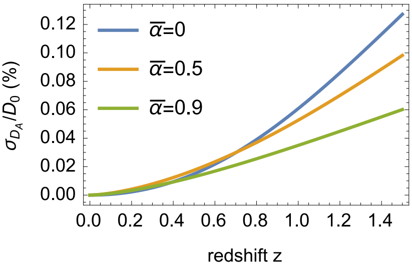
- Application to Swiss-Cheese: Deterministic background is Dyer-Roeder with $\bar{\alpha} = 1 - \lim_{V \to +\infty} \frac{V_{\text{holes}}}{V}$
- First order correction to D_A from Weyl focussing (source at $z = 1$) (see also [Kantowski, 1969] and [Gunn,1967]).



Results: Variance of D_A

We look at corrections to the variance of D_A (source at $z = 1$):





- For clumpy universes, our estimates are way-off.
- There seems to be a problem here!
- It has to do with Weyl focussing (It is getting worse at $\bar{\alpha}$ decreases)

A limitation: non-Gaussianity

- Use of FPK Eq. relies on Gaussianity of the noise.
- In Brownian motion: $N_{col} \sim 10^{20}$ s^{-1} (Central limit theorem).
- But in Cosmo lensing: $N_{holes} \sim 10^{3}$ between source and observer. Is it enough?
- $\mathcal{R}$ oscillates between 0 (holes) and $\mathcal{R}_{FRW}$ (cheese): compact support so sum of contributions converges quickly to Gaussian.
- But:

$$p(|\mathcal{W}|) = \frac{2}{3\mathcal{W}_{min}} \left(\frac{|\mathcal{W}|}{\mathcal{W}_{min}} - \frac{1}{3} \right)^{-2} \quad \text{for } \mathcal{W}_{min} \leq |\mathcal{W}| \leq \mathcal{W}_{max} \gg \mathcal{W}_{min}$$

- Long algebraic tail $\Rightarrow$ slow convergence to central limit.

Summary

- Stochastic lensing promising new, simple formalism to take into account effects of clumpiness on cosmo observables.
- Given some partial, statistical info on distribution of matter, one can infer generic properties of lensed observables (like distances).
- New, post-Dyer-Roeder approximation: shift in $D_A(z)$ due to stochastic noise (clumpiness).
- Issue with central limit theorem for Weyl lensing: Analytical estimates of variance break down for clumpy universe.
- Numerical integration of Sachs-Langevin Eq. allows one to go beyond the Gaussian approx.

What's next?

- Applying Stochastic lensing to more realistic models (FLRW + Pert: WIP).
- Extend formalism to include other observables; e.g. redshift (WIP).
- Use of stochastic lensing to avoid time-consuming ray-tracing in N-body codes.
- Problem of non-Gaussianity.

THANK YOU!