Schwarzschild scalar wigs: spectral analysis and late time behavior

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GR21, July 14, 2016.

Outline

1. Why?

2. What are the scalar wigs?

3. Are they generic?

Why? Dark matter

- WIMP paradigm

- A different approach: The Scalar Field Dark Matter model (SFDM)
  The Dark Matter is modeled by a scalar field with a ultra-light associated
  particle. ($m \sim 10^{-23}$eV)

  - At cosmological scales it behaves as cold dark matter
    T. Matos, L.A. Urena-Lopez, Class. Quant. Grav. **17** L75 (2000),
    V. Sahni and L.M. Wang, Phys. Rev D **62**, 103517 (2000).

  - At galactic scales, it does not have its problems: neither a cuspy
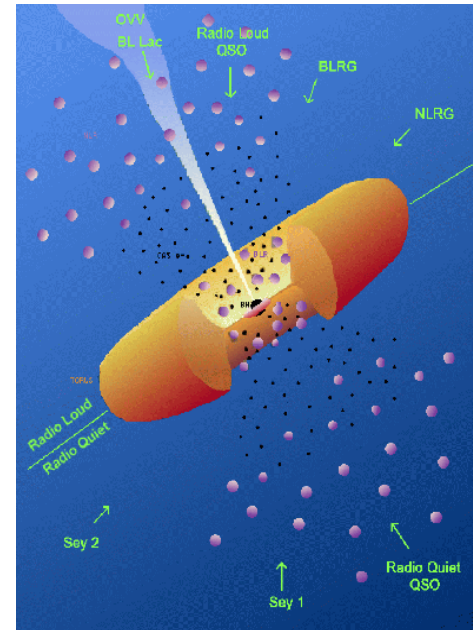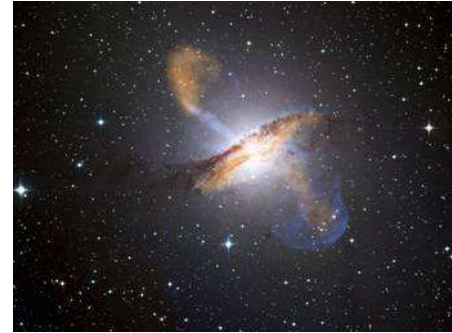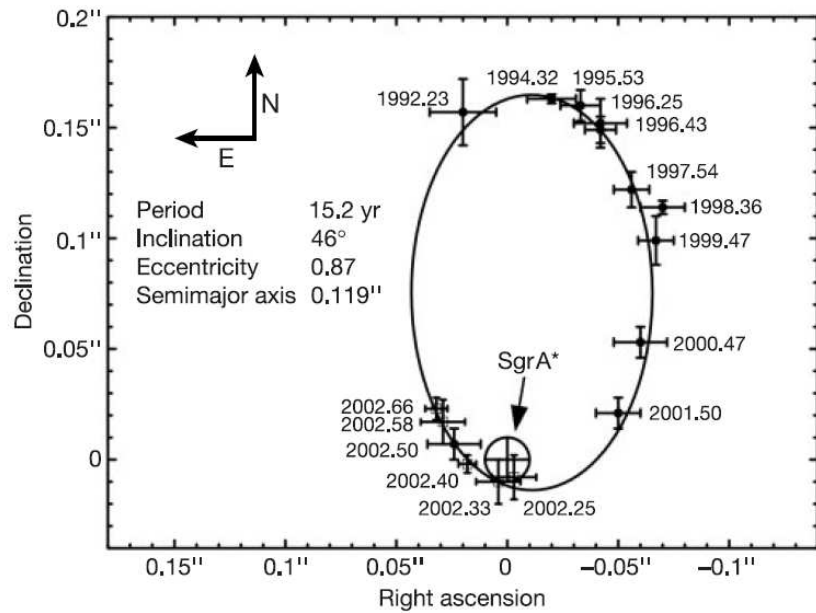   profile, nor a over-density of satellite galaxies.

How to model the DM halo within SFDM models?

A. Arbey, J. Lesgourgues, and P. Salati, *Phys. Rev. D* **64** (2001) 123528
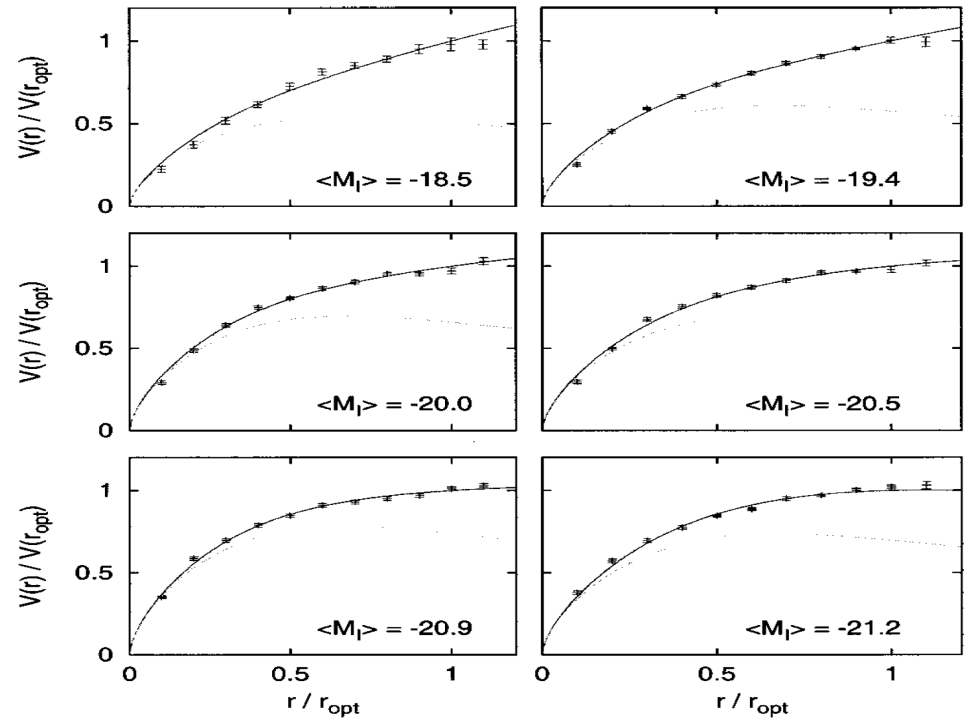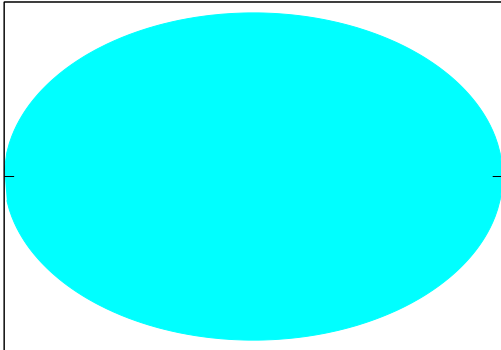
$$m \simeq 6 \times 10^{-24} \text{eV}$$

Astrophysical evidence of SMBH in galaxies

$$M_{BH} = 4{,}3 \times 10^6 M_\odot$$

$$M_{BH} \sim 10^9 M_\odot$$

2. What are the scalar wigs?

The problem: No-hair theorems

- **Theorem:**

  A static, spherically symmetric, asymptotically flat black hole spacetime, with regular (non-extreme) horizon, which satisfies Einstein's equations with the matter fields fulfilling $\rho := T^t_t > 0$ and $T^\theta_\theta \leq T^r_r$, vanishes identically and the spacetime corresponds to the Schwarzschild solution.

- A scalar wig is a non-static configuration that can last even more time that the age of the universe
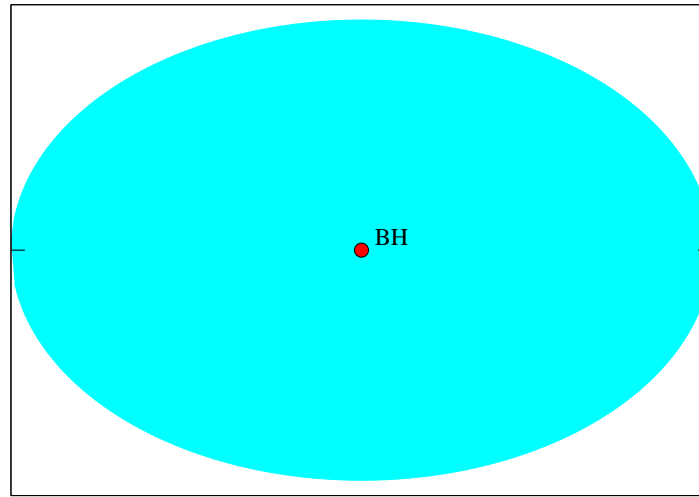
Finding scalar wigs:

Start with the Klein-Gordon $(\Box - \mu^2)\phi = 0$, with $\Box := (1/\sqrt{-g})\, \partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu)$ and

$$ds^2 = -N(r)dt^2 + \frac{dr^2}{N(r)} + r^2 d\Omega^2 \;, \qquad N(r) := 1 - 2M/r \;,$$

Look for stationary solutions $\psi_{\ell m}(t,r) = e^{i\omega_{\ell m}t}u_{\ell m}(r)$

$$\left[-N(r)\frac{\partial}{\partial r}\left(N(r)\frac{\partial}{\partial r}\right) + N(r)\,\mathcal{U}_\ell(\mu, M; r)\right]u(r) = \omega^2 u(r) \;, \qquad 2M < r < \infty \;.$$
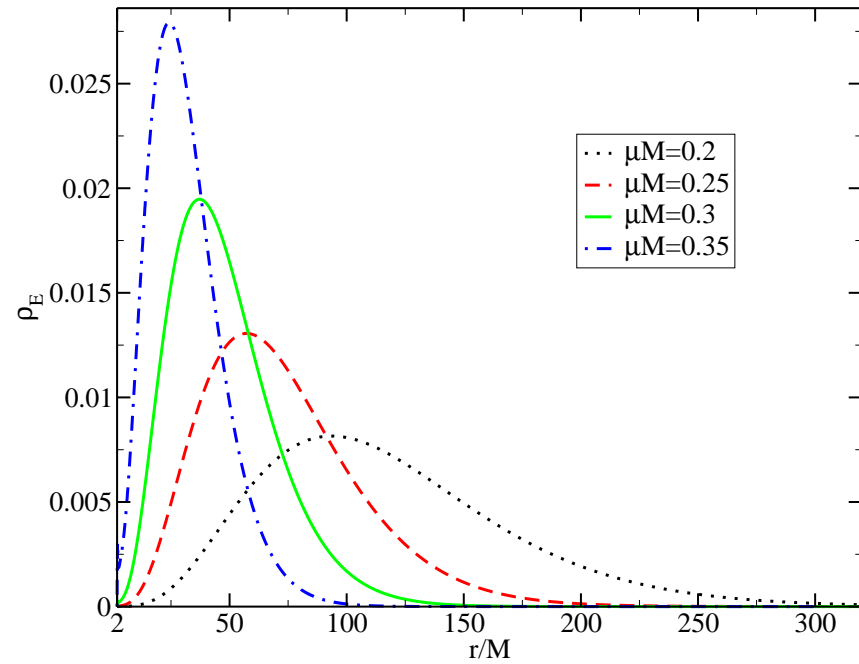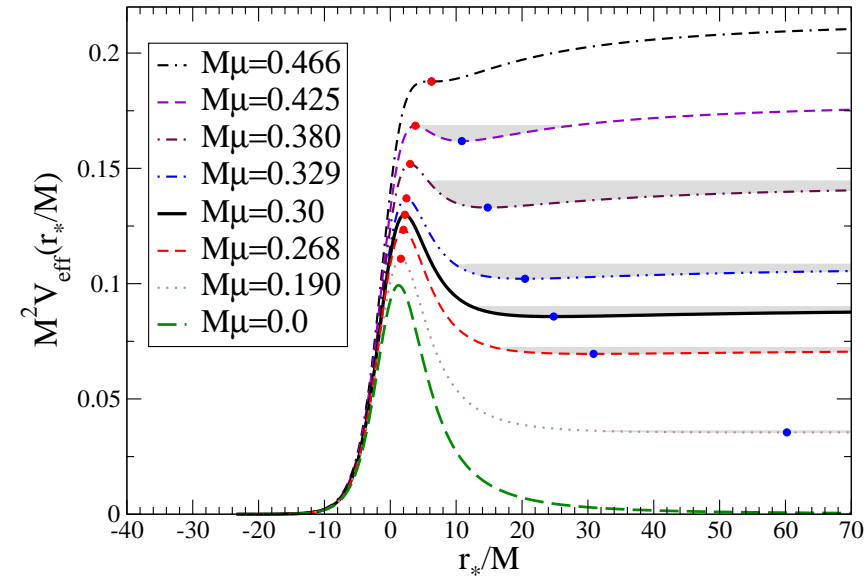
Clearer in Regge-Wheeler coordinates, $r^* := r + 2M\ln(r/2M - 1)$:

$$\left[-\frac{\partial^2}{\partial r^{*2}} + V_{\text{eff}}(r^*)\right]u(r^*) = \omega^2 u(r^*) \;, \qquad -\infty < r^* < \infty,$$
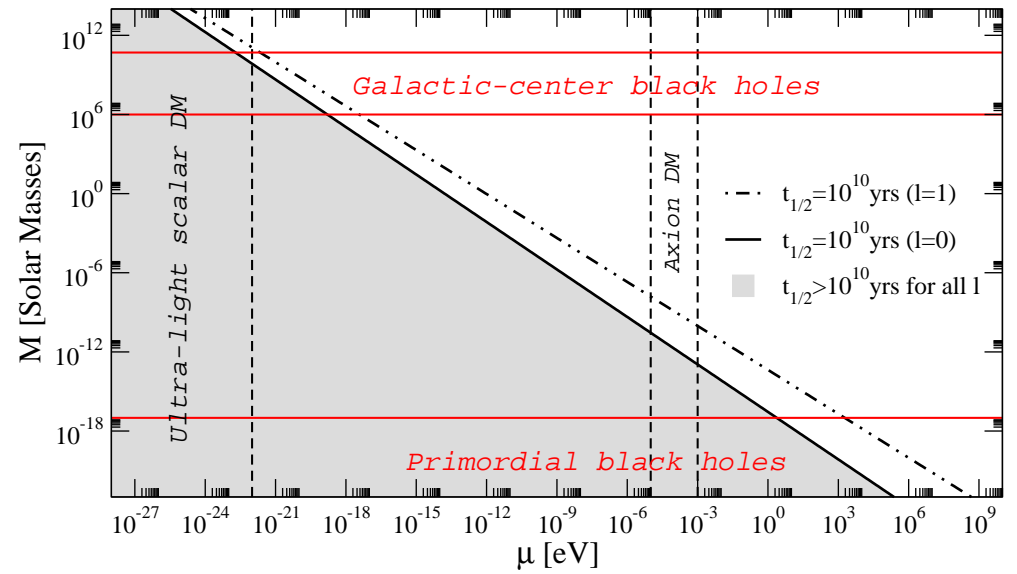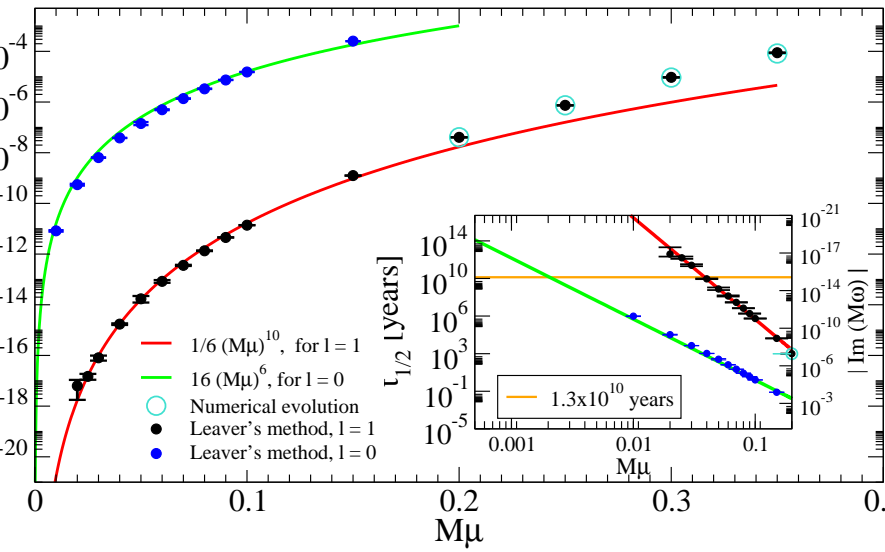
with the effective potential:

$$V_{\text{eff}}(r^*) := N(r)\,\mathcal{U}_\ell(\mu, M; r) \;, \qquad r = r(r^*).$$

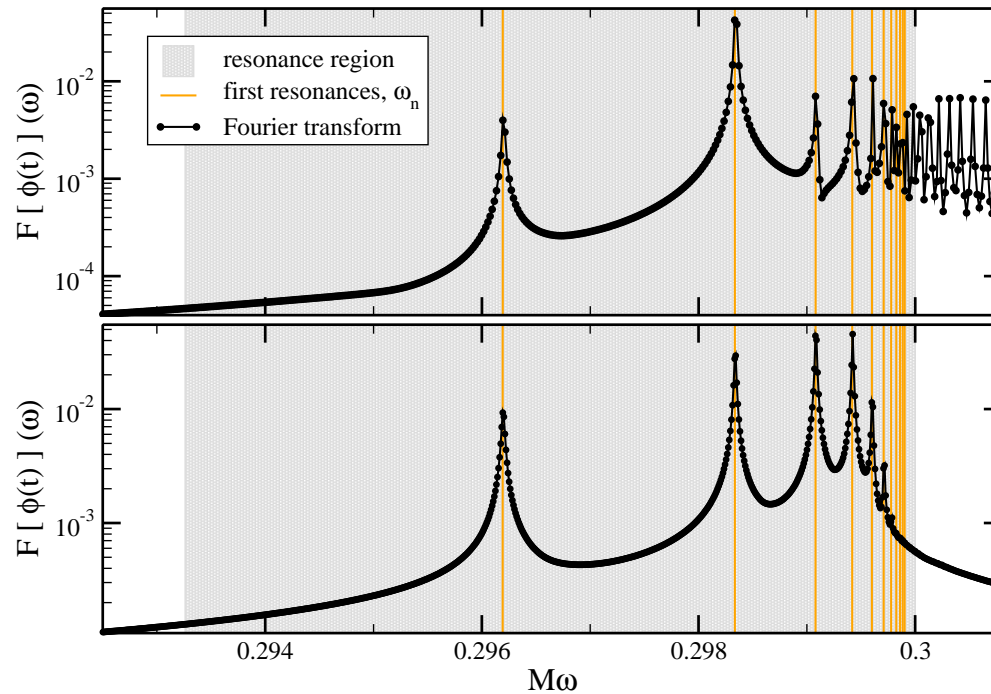# Finding scalar wigs (2)

Scalar wigs:

2. Are they generic?

$$u_0(r) = \begin{cases} N(r - R_1)^4(r - R_2)^4 & \text{for } R_1 \leq r \leq R_2 \\ 0 & \text{otherwise} \end{cases},$$

Analytical calculation:

1. Consider the Cauchy problem $\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} + V(x)\phi = 0$, with $t > 0$, $x \in \mathbb{R}$

2. Initial data $\phi(0, x) = \phi_0(x)$, $\quad \frac{\partial \phi}{\partial t}(0, x) = \pi_0(x)$

3. The solution: $\phi(t, x) = \int\limits_{-\infty}^{\infty} \frac{\partial k}{\partial t}(t, x, y)\phi_0(y)dy + \int\limits_{-\infty}^{\infty} k(t, x, y)\pi_0(y)dy$,

$k(t, x, y) = \frac{1}{2\pi i} \int\limits_{s=\eta-i\infty}^{\eta+i\infty} e^{st} G(s, x, y)ds$, $\qquad \eta > 0$, with $G(s, x, y) = G(s, y, x)$ the

Green's function,

$$G(s, x, y) := \frac{1}{W(s)} \left\{ \begin{array}{ll} f_-(s, y)f_+(s, x), & y \leq x \\ f_-(s, x)f_+(s, y), & y > x \end{array} \right\}.$$

$W(s)$ the Wronski determinant defined as $W(s) := \det \begin{pmatrix} f_+(s, x) & f_-(s, x) \\ f'_+(s, x) & f'_-(s, x) \end{pmatrix}$.

Here $f_+(s, x)$ and $f_-(s, x)$ are two nontrivial solutions of the mode equation $s^2 f - f'' + V(x)f = 0$ with exponential decay at $x \to +\infty$ and $x \to -\infty$.

Analytical calculation (2)

Finally: 4: The kernel typically splits into a sum of three different contributions,

$$k(t, x, y) = k_{\text{modes}}(t, x, y) + k_{\text{tail}}(t, x, y) + k_{\text{hfa}}(t, x, y).$$

Here, k_{modes} is a sum over the residua of the poles of the analytic continuation of the Green's function

The mode contribution to the solution from the n'th mode s_n is given by
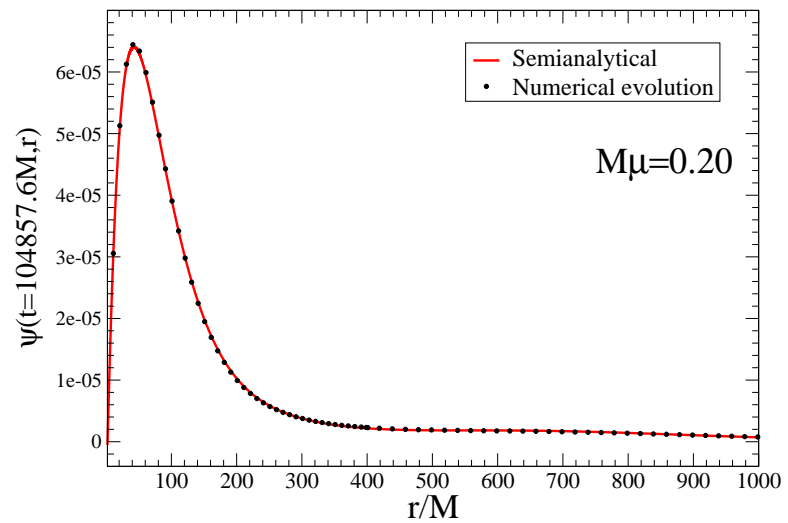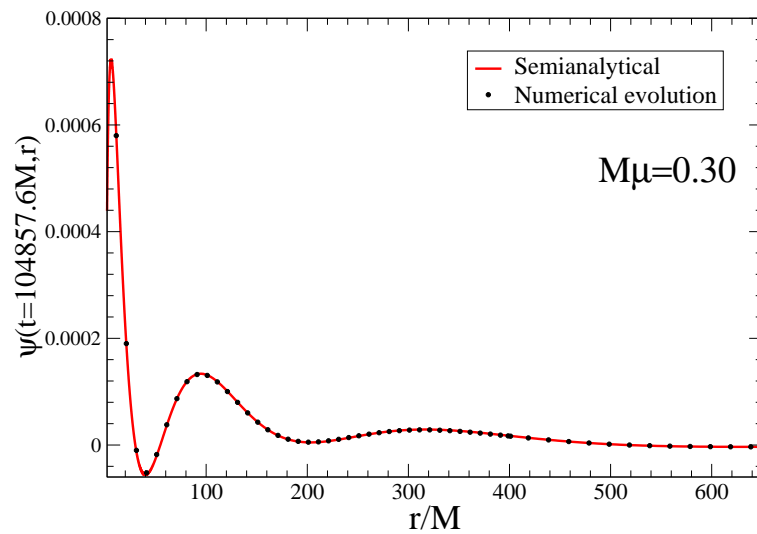
$$\phi_{\text{modes},n}(t, x) = A_n e^{s_n t} f_+(s_n, x),$$

with amplitude

$$A_n := \left[\frac{d}{ds} W(s) \right]^{-1} \Bigg|_{s=s_n} \times \int_{-\infty}^{\infty} f_-(s_n, y)[s_n \phi_0(y) + \pi_0(y)]dy.$$

$$\psi(t, r) = \sum_n \psi_n(t, r)$$

Comparison numerical vs. analytical

Conclusions

1. Why? Motivated by SFDM models

2. What are the scalar wigs? Configurations that survive around BH for cosmological times

3. Are they generic? Yes