Gravitational waves from pulsars with measured braking index (n)

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Outline

- Modeling the spindown of the pulsars with known “n”
- GW amplitudes generate by these pulsares
- Detectability of these pulsares by aLIGO e ET

Recall that ...

The power of magnetic dipole is given by

$$\dot{E}_{\rm d} = \frac{16\pi^4}{3}\,\frac{B_0^2 R^6 \sin^2\phi}{P^4 c^3}$$

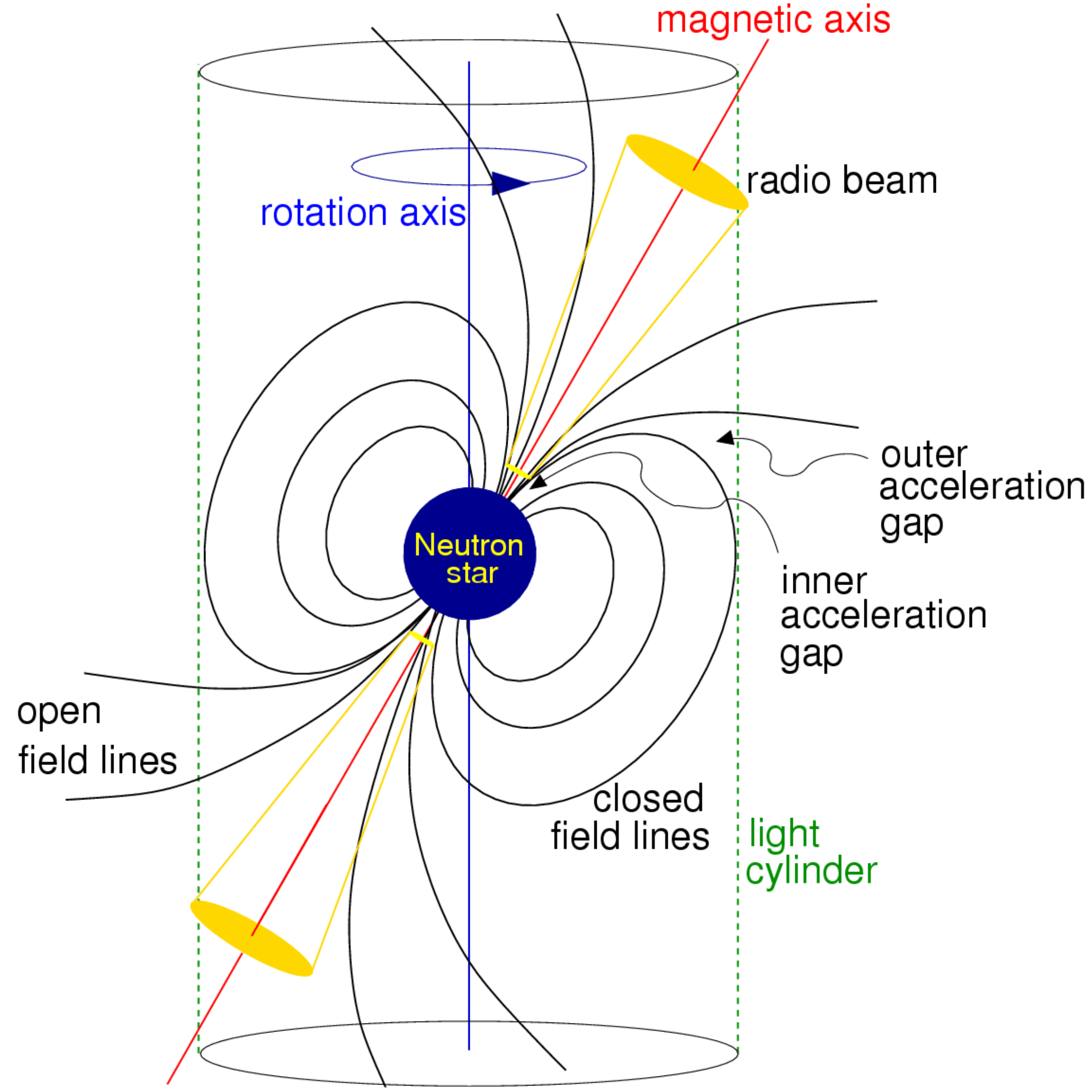
Where:

B_0 – magnetic field

R – radius

P – period of rotation

ϕ – angle between the rotating and the magnetic dipole axes



http://www.cv.nrao.edu/course/astr534/Pulsars.html

Recall that ...

The amplitude of GWs for pulsars :

$$h = \frac{16\pi^2 G}{c^4} \frac{I \epsilon f_{\rm rot}^2}{r}$$

Where:

$I = I_{zz}$ with respect to the rotation axis (z)

and

$$\epsilon = \frac{|I_{xx} - I_{yy}|}{I_{zz}}$$ (Ellipticity)

The corresponding power in GWs:

$$\dot{E}_{\rm GW} = \frac{2048\pi^6}{5} \frac{G}{c^5} \frac{I^2 \epsilon^2}{P^6}$$

What is the braking index (n)? What it is for?

Recall that “n” is given by

$$n \equiv \frac{\Omega_{rot}\,\ddot{\Omega}_{rot}}{\dot{\Omega}_{rot}^{2}}$$

As it is well known, this quantity gives us important information about the spindown of of pulsars.

What is n for a magnetic brake?

Starting from:

$$\dot{E}_{rot} = I\,\Omega_{rot}\,\dot{\Omega}_{rot}$$

$$\dot{E}_{rot} = \dot{E}_{d}$$

$$\dot{E}_{\mathrm{d}} = \frac{16\pi^4}{3}\frac{B_0^2 R^6 \sin^2\phi}{P^4 c^3}$$

One has

$$\dot{\Omega}_{rot} = \frac{1}{3}\frac{B_0^2 R^6 \sin^2\varphi}{I\,c}\Omega_{rot}^3$$

It is easy to show that:

$$\ddot{\Omega}_{rot} = \frac{3\dot{\Omega}_{rot}^2}{\Omega_{rot}}$$

$$\boxed{\Rightarrow\ n = 3}$$

What is n for a GW brake?

Starting from:

$$\dot{E}_{rot} = I\,\Omega_{rot}\,\dot{\Omega}_{rot}$$

$$\dot{E}_{rot} = \dot{E}_{GW}$$

$$\dot{E}_{\mathrm{GW}} = \frac{2048\pi^6}{5}\frac{G}{c^5}\frac{I^2\epsilon^2}{P^6}$$

One has

$$\dot{\Omega}_{rot} = \frac{32}{5}\frac{G}{c^5}\,I\,\epsilon^2\,\Omega_{rot}^5$$

It is easy to show that:

$$\ddot{\Omega}_{rot} = \frac{5\,\dot{\Omega}_{rot}^2}{\Omega_{rot}}$$

$$\boxed{\Rightarrow\ n = 5}$$

What about the observations?

Pulsar	P (s)	$\dot{P}$ (10^{-13} s/s)	n
PSR J1734-3333	1.17	22.8	0.9 ± 0.2
PSR B0833-45 (Vela)	0.089	1.25	1.4 ± 0.2
PSR J1833-1034	0.062	2.02	1.8569 ± 0.0006
PSR B0540-69	0.050	4.79	2.140 ± 0.009
PSR J1846-0258	0.324	71	2.19 ± 0.03
PSR B0531+21 (Crab)	0.033	4.21	2.51 ± 0.01
PSR J1119-6127	0.408	40.2	2.684 ± 0.002
PSR B1509-58	0.151	15.3	2.839 ± 0.001
PSR J1640-4631	0.207	9.72	3.15 ± 0.03

Modeling pulsars’ braking indices

Modeling the braking index of PSR J1640-4631 (n =3.15)*

This value of n suggests that the spindown can be given by a combination of a magnetic dipole and a gravitational wave brakes

Starting from:

$$E_{rot} = \frac{1}{2} I \Omega_{rot}^2 \quad \Longrightarrow \quad \dot{E}_{rot} = I \, \Omega_{rot} \, \dot{\Omega}_{rot}$$

Since

$$\dot{E}_{rot} = \dot{E}_{GW} + \dot{E}_d$$

Recall that

$$\dot{E}_{\mathrm{d}} = \frac{16\pi^4}{3} \frac{B_0^2 R^6 \sin^2\phi}{P^4 c^3} \qquad \dot{E}_{\mathrm{GW}} = \frac{2048\pi^6}{5} \frac{G}{c^5} \frac{I^2 \epsilon^2}{P^6}$$

$$\Longrightarrow \qquad \dot{\Omega}_{\mathrm{rot}} = \frac{32}{5} \frac{G}{c^5} I \epsilon^2 \Omega_{\mathrm{rot}}^5 + \frac{1}{3} \frac{B_0^2 R^6 \sin^2\phi}{I c^3} \Omega_{\mathrm{rot}}^3$$

Since

$$n \equiv \frac{\Omega_{rot} \ddot{\Omega}_{rot}}{\dot{\Omega}_{rot}^2} \quad \Longrightarrow \quad n = \frac{5\dot{\Omega}_{GW} + 3\dot{\Omega}_d}{\dot{\Omega}_{GW} + \dot{\Omega}_d}$$

*See arXiv 1603.05975 (to appear briefly in JCAP)

The fraction of power in GWs can be defined by:

$$\eta = \frac{\dot{E}_{GW}}{\dot{E}_{rot}} \qquad \left(\Rightarrow \quad \eta = \frac{\dot{\Omega}_{GW}}{\dot{\Omega}_{rot}} \right)$$

Substituting into

$$n = \frac{5\,\dot{\Omega}_{GW} + 3\,\dot{\Omega}_{d}}{\dot{\Omega}_{GW} + \dot{\Omega}_{d}}$$

One has:

$$\eta = \frac{n-3}{2} \qquad \left(\text{for} \quad 3 \leqslant n \leqslant 5 \right)$$

For $n = 3.15 \implies \eta = 0.075$

Thus, 7.5% of the power in GWs!

What about the amplitude of GWs?

Let us start from:

$$h^2 = \frac{5}{2}\frac{G}{c^3}\frac{I}{r^2}\frac{|\dot{f}_{\rm rot}|}{f_{\rm rot}}$$

This equation implicitly considers that the spindown is exclusively given by a GW brake, i.e., n = 5.

…. it should be modified to take into account that $3 \leq n \leq 5$.

From the previous slide one sees that:

$$\dot{\Omega}_{GW} = \eta \dot{\Omega}_{rot} \qquad \Rightarrow \qquad \dot{\bar{f}}_{rot} = \eta \dot{f}_{rot}$$

Therefore, the amplitude reads

$$\bar{h}^2 = \frac{5}{2}\frac{G}{c^3}\frac{I}{r^2}\frac{|\dot{\bar{f}}_{\rm rot}|}{f_{\rm rot}} = \frac{(n-3)}{2} h^2$$

What would be the ellipticity?

Combining the equations for the amplitude of GWs:

$$\bar{h}^2 = \frac{5}{2}\frac{G}{c^3}\frac{I}{r^2}\frac{|\dot{\bar{f}}_{\rm rot}|}{f_{\rm rot}} = \frac{(n-3)}{2}h^2$$

$$h = \frac{16\pi^2 G}{c^4}\frac{I\epsilon f_{\rm rot}^2}{r}$$

One has:

$$\epsilon = \sqrt{\frac{5}{1024\pi^4}\frac{c^5}{G}\frac{\dot{P}P^3}{I}(n-3)}.$$

Substituting the appropriate quantities:

$$\epsilon = 4.8 \times 10^{-3}$$

… this is extremely large!

Modeling the spindown of the other 8 pulsars and revisiting PSR J1640-4631 (n =3.15)*

*to appear briefly in EPJC

To model pulsars with $n < 3$, one can consider, for example, that the magnetic field and the angle between the magnetic dipole and the rotating axes are time dependent.

…. after some algebra one obtains:

$$n = 3 + 2\eta - 2\frac{P}{\dot{P}}(1-\eta)\left[\frac{\dot{B}_0}{B_0} + \dot{\phi}\cot\phi\right]$$

Combining appropriately the ingredients of the above equations, it is possible to obtain $n < 3$.

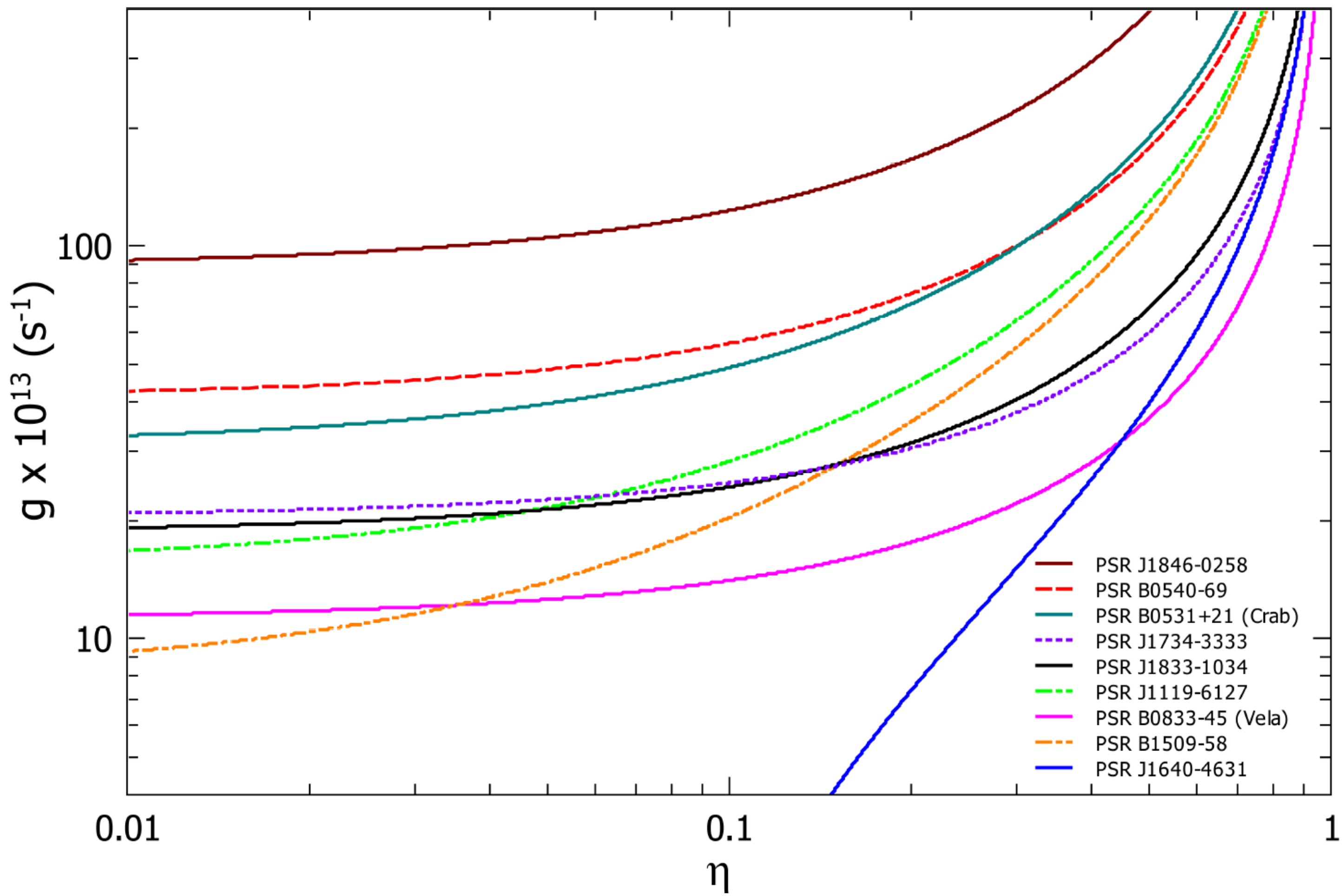
The term in brackets can be written in the following form:

$$g = g(B_0, \dot{B}_0, \phi, \dot{\phi}) \equiv \left[\frac{\dot{B}_0}{B_0} + \dot{\phi}\cot\phi\right]$$

This term as a function of η for a given pulsar reads

$$g = -\frac{(n-3-2\eta)}{2(1-\eta)}\frac{\dot{P}}{P}$$

(see the figure in the next slide)



For Crab, for example, it could be inferred from observations that:

$$\dot{\varphi} \simeq 3 \times 10^{-12} rads/s$$

(see Lune et al 2013, 2015; Yi & Zhang 2015). The relative variation of the magnetic field may be less important. Therefore

$$g \simeq 3 \times 10^{-12} s^{-1} \qquad \Rightarrow \qquad \eta < 0.1$$

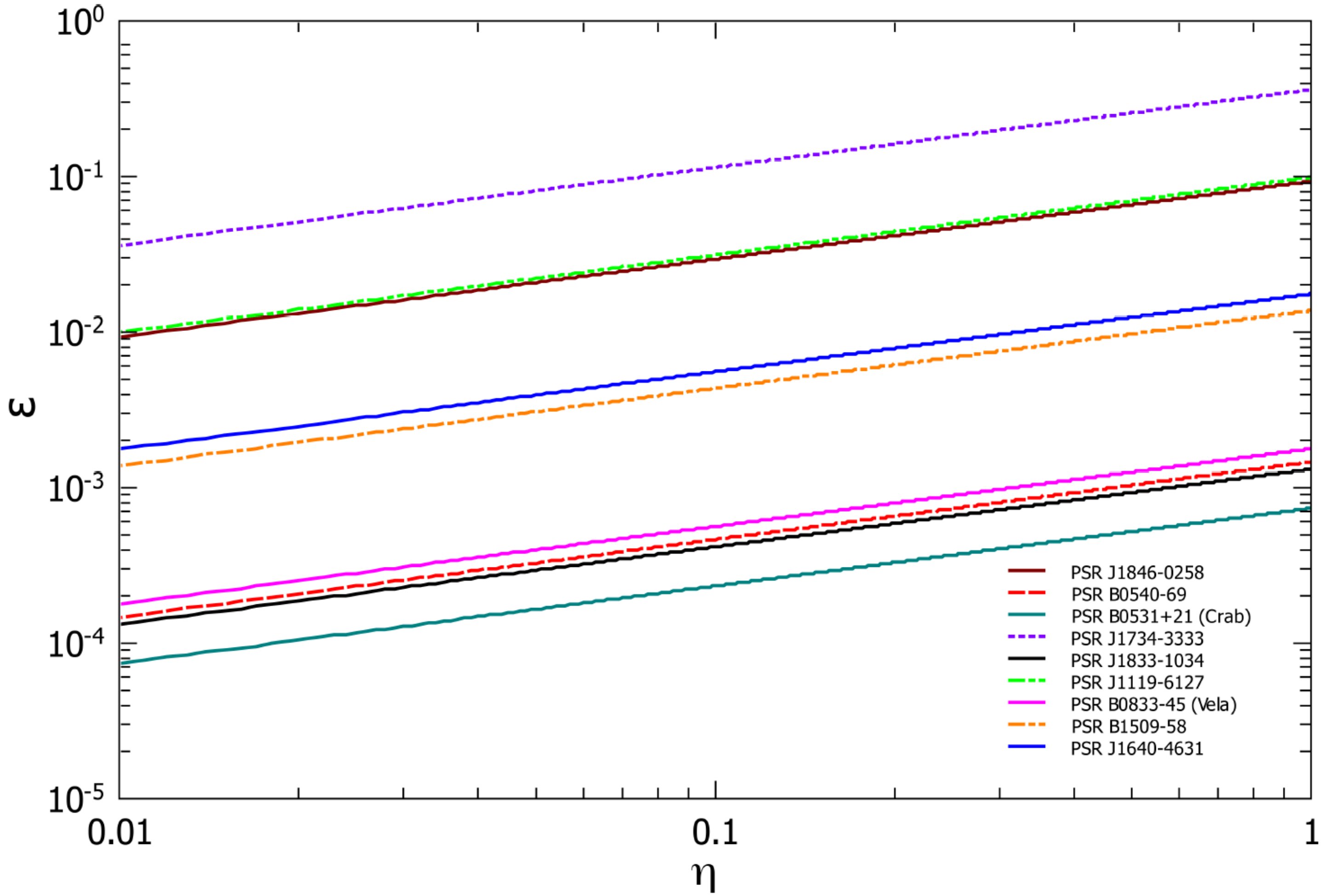
For the amplitude of GWs we follow the same prescription as for PSR J1640-4631.

Then, we have:

$$\bar{h}^2 = \frac{5}{2}\frac{G}{c^3}\frac{I}{r^2}\frac{\dot{\bar{f}}_{rot}}{f_{rot}} = \eta\, h^2$$

….. and for the ellipticity we have

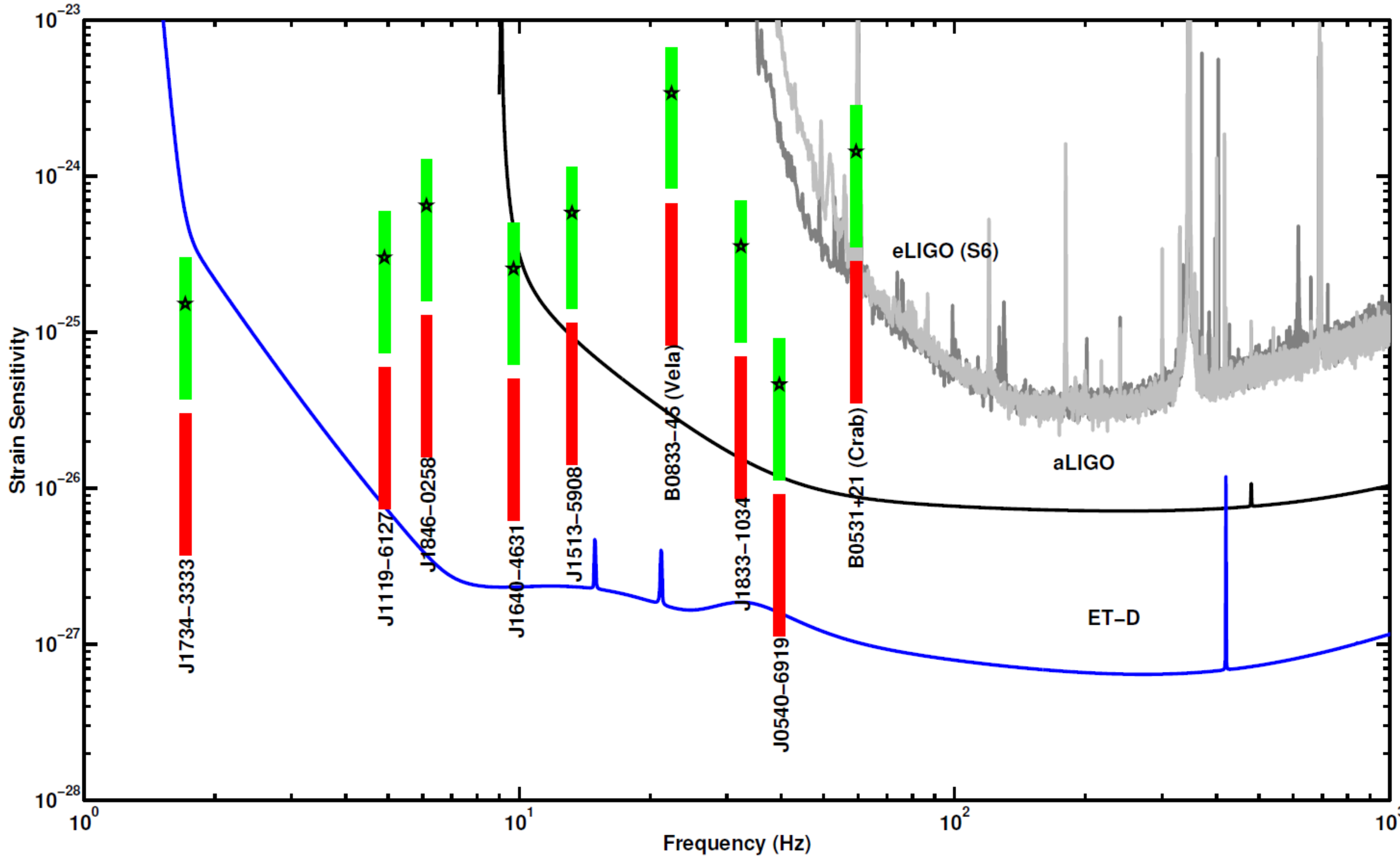
$$\epsilon = \sqrt{\frac{5}{512\pi^4}\frac{c^5}{G}\frac{\dot{P}P^3}{I}\eta}$$



Note that, for example

$$\varepsilon \ll 10^{-4} \;\Rightarrow\; \eta \ll 0.01$$

Strain sensitivities for eLIGO S6, aLIGO e ET-D for one year integration time.



GW amplitude for $\eta = 1$ (green) and $\eta = 0.01$ (red) for $7\times10^{36} < I < 1\times10^{38}$ kgm^2 (EoS GM1; Glendennig & Moszkowski 1991)

"Black star": $I = 10^{38}$ kgm^2 and $\eta = 1$.

Conclusions

It is possible to model the braking index of the Pulsar with $n = 3.15$ considering that the spindown is a combination of magnetic dipole and GW brakes. Although the ellipticity obtained is extremely large.

For the other 8 pulsares, which has $n < 3$, it is possible to model them by considering that the magnetic field and the angle between the rotating and the magnetic dipole axes are time dependent. This model can be extended to the pulsar with $n = 3.15$. As a result, its predicted ellipticity could well be much smaller ($\ll 10^{-3}$).

Our results also show that aLIGO, and more probably ET, could well be able to detect GWs from some pulsars considered here.