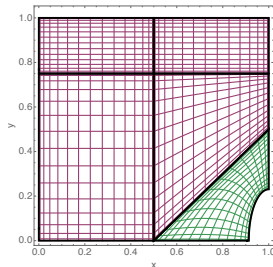# Localised AdS$_5$ × S^5 Black Holes

Jorge E. Santos

GR 21 - New York

In collaboration with
Óscar J. C. Dias and Benson Way

The original AdS/CFT - '97 Maldacena

Where it all began:

Type IIB supergravity theory on AdS$_5$ × S^5 with radius **L** and **N** units of $F_{(5)}$ flux on S^5

$\Updownarrow$

The large **N** limit and strong coupling limit of $\mathcal{N} = 4$ **Super Yang-Mills (SYM) theory** with t'Hooft coupling λ.

The original AdS/CFT - '97 Maldacena

Where it all began:

Type IIB supergravity theory on AdS$_5$ × S^5 with radius **L** and **N** units of $F_{(5)}$ flux on S^5

$\Updownarrow$

The large **N** limit and strong coupling limit of $\mathcal{N} = 4$ **Super Yang-Mills (SYM) theory** with t'Hooft coupling λ.
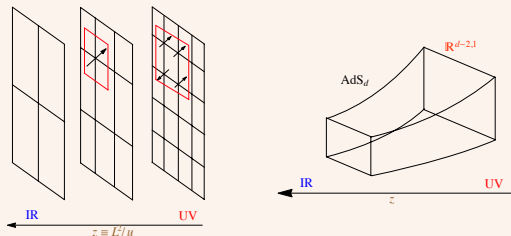
$\mathcal{N} = 4$ SYM on Einstein static universe - '98 Witten

The role of Black Holes:

- Black holes radiate at **temperature** T - '74 Hawking.

$\mathcal{N} = 4$ SYM on Einstein static universe - '98 Witten

The role of Black Holes:

- Black holes radiate at **temperature** T - '74 Hawking.
- Witten **conjectured** that black holes in global AdS$_5$ should be **dual to the hot** phase of $\mathcal{N} = 4$ SYM on $\mathbb{R}_t \times \mathrm{S}^3_{\mathbf{L}}$.

$\mathcal{N} = 4$ SYM on Einstein static universe - '98 Witten

The role of Black Holes:

- Black holes radiate at **temperature** T - '74 Hawking.
- Witten **conjectured** that black holes in global AdS$_5$ should be **dual to the hot** phase of $\mathcal{N} = 4$ SYM on $\mathbb{R}_t \times S^3_L$.
- Black holes should be the **the holographic dual to the end state of a real time** thermalisation process - **maximum entropy at fixed energy**.

$\mathcal{N} = 4$ SYM on Einstein static universe - '98 Witten

The role of Black Holes:

- Black holes radiate at **temperature T** - '74 Hawking.
- Witten **conjectured** that black holes in global AdS$_5$ should be **dual to the hot** phase of $\mathcal{N} = 4$ SYM on $\mathbb{R}_t \times \mathrm{S}^3_{\mathbf{L}}$.
- Black holes should be the **the holographic dual to the end state of a real time** thermalisation process - **maximum entropy at fixed energy**.

Question:

What is the phase diagram of static black holes
in global AdS$_5$ × S^5?

The equations of motion

Keeping it *simple*:

- Type IIB supergravity has many fields:

The equations of motion

Keeping it *simple*:

- Type IIB supergravity has many fields:
    - **one symmetric two tensor** - g
    - **one real self dual five-form** - F
    - **one complex three-form** - G_3
    - **one complex scalar** - B

The equations of motion

Keeping it *simple*:

- Type IIB supergravity has many fields:
    - **one symmetric two tensor** - g
    - **one real self dual five-form** - F
    - **one complex three-form** - G_3
    - **one complex scalar** - B
- Can consistently set $G_3 = 0$ and $B = 1$.

The equations of motion

Keeping it *simple*:

- Type IIB supergravity has many fields:
  - **one symmetric two tensor** - g
  - **one real self dual five-form** - F
  - **one complex three-form** - G_3
  - **one complex scalar** - B
- Can consistently set $G_3 = 0$ and $B = 1$.

Reduced equations of motion:

$$R_{ab} = \frac{1}{48} F_a{}^{cdef} F_{bcdef}, \quad \nabla_a F^{abcde} = 0 \quad \text{and} \quad \star F = F.$$

The smeared black hole

> ### A smeared family of solutions - '80 Freund & Rubin:
>
> **Any** solution to:
> $$R^{(5)}_{\mu\nu} = \frac{4}{\mathbf{L}^2} g^{(5)}_{\mu\nu}$$
> can be **oxidised** to **ten-dimensions** via
>
> $$\mathrm{d}s^2 = g^{(5)}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathbf{L}^2\mathrm{d}\Omega_5^2 \qquad \text{and} \qquad F = \mathrm{Vol}_{\mathrm{AdS}_5} + \mathrm{Vol}_{\mathrm{S}^5}\,.$$

The smeared black hole

<div>

A smeared Black Hole:

The Schwarschild-AdS$_5$ black hole solution lifts to:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2 + \mathbf{L}^2 d\Omega_5^2 \,,$$

where

$$f(r) = \frac{r^2}{\mathbf{L}^2} + 1 - \frac{r_+^2}{r^2}\left(\frac{r_+^2}{\mathbf{L}^2} + 1\right) \,,$$

with $y_+ \equiv r_+/\mathbf{L}$ measuring the **black hole radius** with respect to the **radius of the** S^5 (also the radius of the S^3 at ∂).

</div>

The smeared black hole

<div>

A smeared Black Hole:

The Schwarschild-AdS$_5$ black hole solution lifts to:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2 + \mathbf{L}^2 d\Omega_5^2\,,$$

where

$$f(r) = \frac{r^2}{\mathbf{L}^2} + 1 - \frac{r_+^2}{r^2}\left(\frac{r_+^2}{\mathbf{L}^2} + 1\right)\,,$$

with $y_+ \equiv r_+/\mathbf{L}$ measuring the **black hole radius** with respect to the **radius of the** S^5 (also the radius of the S^3 at ∂).

Thermodynamic Properties:

$$E = \frac{3\mathbf{N}^2}{4\,\mathbf{L}}y_+^2\,(1+y_+^2)\,, \quad S = \pi\,y_+^2\,\mathbf{N}^2 \quad \text{and} \quad T = \frac{1+2y_+^2}{2\pi y_+\,\mathbf{L}}\,.$$

</div>

Microcanonical Ensemble
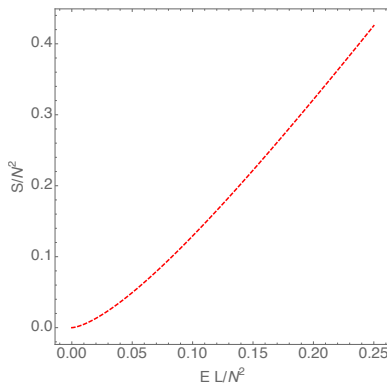
Key question:

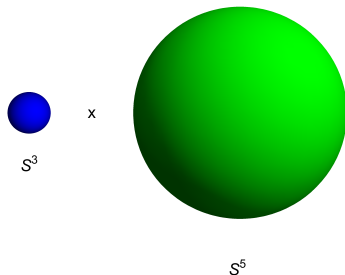Is this the unique static solution with the desired asymptotics?

Size Matters - '98 Banks *et al.*
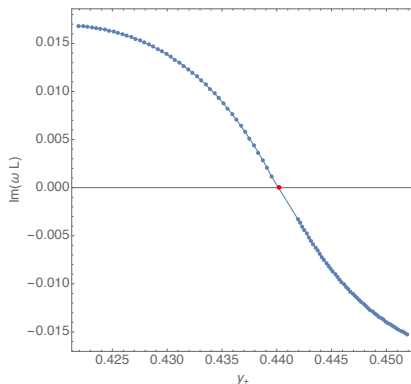
When black hole radius **is small**, *i.e.* $y_+ \ll 1$:

Size Matters - '98 Banks *et al.*

When black hole radius **is small**, *i.e.* $y_+ \ll 1$:

S^3

x

S^5

The separation of **horizon length scales** suggests a
Gregory-Laflamme instability:
heuristics confirmed '02 Hubeny & Rangamani.

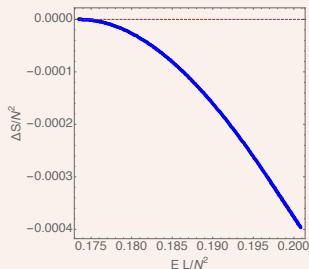# Instability Growth Rate - '15 Buchel, Lehner

Alike in **Gregory-Laflamme instability**, **zero-mode** - $y_+ \sim 0.44$ - connects to novel **non-uniform solutions** along the S^5 - with horizon topology S^3 × S^5.

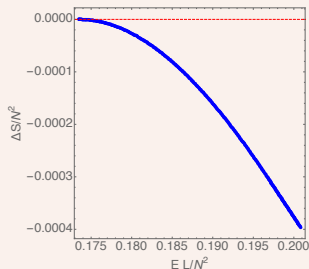# Lumpy holes in AdS$_5$ × S^5 - '15 Dias, Santos and Way

Lumpy black holes:

- **Lumpy holes** have a **lower entropy** than smeared Schwarzschild-AdS$_5$ at **fixed energy**.

Lumpy holes in AdS$_5$ × S^5 - '15 Dias, Santos and Way

Lumpy black holes:

- **Lumpy holes** have a **lower entropy** than smeared Schwarzschild-AdS$_5$ at **fixed energy**.

- They cannot be the **endpoint of the instability**!

Other candidate endpoints - '98 Banks *et al.*

Localised black holes:

- At least one other candidate endpoint exists: a **localised black hole in AdS$_5 \times$ S^5**.

Other candidate endpoints - '98 Banks *et al.*

Localised black holes:

- At least one other candidate endpoint exists: a **localised black hole in AdS$_5$ × S^5**.
- Must have the following property: **higher entropy** than smeared Schwarzschild-AdS$_5$ hole, at fixed energy.

Other candidate endpoints - '98 Banks *et al.*

Localised black holes:

- At least one other candidate endpoint exists: a **localised black hole in AdS$_5$ × S^5**.
- Must have the following property: **higher entropy** than smeared Schwarzschild-AdS$_5$ hole, at fixed energy.
- If true, this would imply a **violation of weak cosmic censorship** in AdS$_5$ × S^5.

Other candidate endpoints - '98 Banks *et al.*

Localised black holes:

- At least one other candidate endpoint exists: a **localised black hole in AdS$_5$ × S^5**.
- Must have the following property: **higher entropy** than smeared Schwarzschild-AdS$_5$ hole, at fixed energy.
- If true, this would imply a **violation of weak cosmic censorship** in AdS$_5$ × S^5.

Vanilla flavour Gregory-Laflamme instability:

Other candidate endpoints - '98 Banks *et al.*

Localised black holes:

- At least one other candidate endpoint exists: a **localised black hole in AdS$_5$ × S^5**.
- Must have the following property: **higher entropy** than smeared Schwarzschild-AdS$_5$ hole, at fixed energy.
- If true, this would imply a **violation of weak cosmic censorship** in AdS$_5$ × S^5.

Vanilla flavour Gregory-Laflamme instability:

- For the **original Gregory-Laflamme instability**, this is indeed what happens: '10 Pretorius and Lehner.

Other candidate endpoints - '98 Banks *et al.*

Localised black holes:

- At least one other candidate endpoint exists: a **localised black hole in AdS$_5$ $\times$ S^5**.
- Must have the following property: **higher entropy** than smeared Schwarzschild-AdS$_5$ hole, at fixed energy.
- If true, this would imply a **violation of weak cosmic censorship** in AdS$_5$ $\times$ S^5.

Vanilla flavour Gregory-Laflamme instability:

- For the **original Gregory-Laflamme instability**, this is indeed what happens: '10 Pretorius and Lehner.
- **Localised solutions** where first constructed in '05 by Kudoh and Wiseman.

Localised Black Holes in AdS$_5$ $\times$ S^5

- These **localised black holes** have **horizon topology** S^8 and asymptote to AdS$_5$ $\times$ S^5, whose **constant time and 'holographic' slices** have topology S^3 $\times$ S^5.

Localised Black Holes in AdS$_5$ $\times$ S^5

- These **localised black holes** have **horizon topology** S^8 and asymptote to AdS$_5$ $\times$ S^5, whose **constant time and 'holographic' slices** have topology S^3 $\times$ S^5.
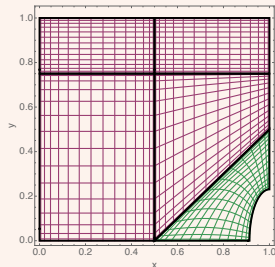- Can think of it as **'inverted' black rings**.

Localised Black Holes in AdS$_5$ × S^5

- These **localised black holes** have **horizon topology** S^8 and asymptote to AdS$_5$ × S^5, whose **constant time and 'holographic' slices** have topology S^3 × S^5.
- Can think of it as **'inverted' black rings**.

Why is it hard to solve for these solutions?

Localised Black Holes in AdS$_5$ × S^5

- These **localised black holes** have **horizon topology** S^8 and asymptote to AdS$_5$ × S^5, whose **constant time and 'holographic' slices** have topology S^3 × S^5.
- Can think of it as **'inverted' black rings**.

Why is it hard to solve for these solutions?
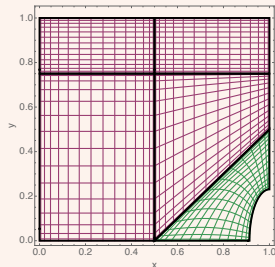
- Need to solve equations on a pentagon!

Localised Black Holes in AdS$_5$ × S^5

- These **localised black holes** have **horizon topology** S^8 and asymptote to AdS$_5$ × S^5, whose **constant time and 'holographic' slices** have topology S^3 × S^5.
- Can think of it as **'inverted' black rings**.

Why is it hard to solve for these solutions?

- Need to solve equations on a pentagon!
    - Horizon
   - S^3 axis
    - South pole
    - North pole

Localised Black Holes in AdS$_5$ × S^5

- These **localised black holes** have **horizon topology** S^8 and asymptote to AdS$_5$ × S^5, whose **constant time and 'holographic' slices** have topology S^3 × S^5.
- Can think of it as **'inverted' black rings**.

Why is it hard to solve for these solutions?

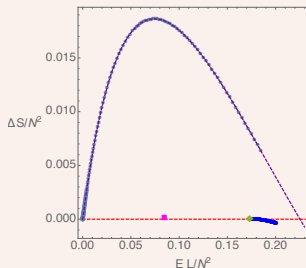- Need to solve equations on a pentagon!
    - Horizon
    - S^3 axis
    - South pole
    - North pole
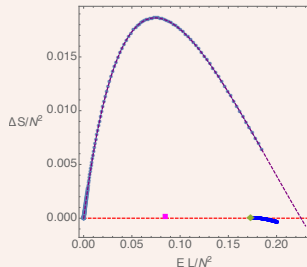- They live in 10d - strong decay.
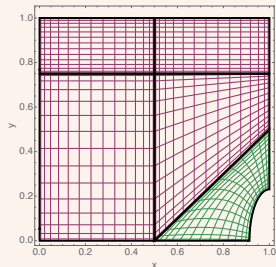
Lumpy holes in AdS$_5$ × S^5 - '15 Dias, Santos and Way

- **Localised holes** in AdS$_5$ × S^5 have a **higher entropy** than smeared Schwarzschild-AdS$_5$ at **fixed energy**.

Lumpy holes in AdS$_5$ $\times$ S^5 - '15 Dias, Santos and Way

- **Localised holes** in AdS$_5$ $\times$ S^5 have a **higher entropy** than smeared Schwarzschild-AdS$_5$ at **fixed energy**.

- They can be the **endpoint of the instability** - modulo violation of weak cosmic censorship!

Conclusions:

- We have constructed localised black holes in AdS$_5$ × S^5.
- They are likely to be the endpoint of the GL instability.
- New phase dominates microcanonical ensemble.

Conclusions:

- We have constructed localised black holes in AdS$_5$ × S^5.
- They are likely to be the endpoint of the GL instability.
- New phase dominates microcanonical ensemble.

What should you ask me now:

- Non-uniqueness for smeared Schwarzschild-AdS$_5$?
- What is the story in the canonical ensemble?
- Can you make predictions for field theory observables?

Conclusions:

- We have constructed localised black holes in $AdS_5 \times S^5$.
- They are likely to be the endpoint of the GL instability.
- New phase dominates microcanonical ensemble.

What should you ask me now:

- Non-uniqueness for smeared Schwarzschild-AdS_5?
- What is the story in the canonical ensemble?
- Can you make predictions for field theory observables?

Outlook:

- What is the field theory interpretation of this phenomenon?
- Can we run Monte-Carlo simulations of $\mathcal{N} = 4$ SYM in the microcanonical ensemble and confirm phase diagram?
- . . .

Thank You!