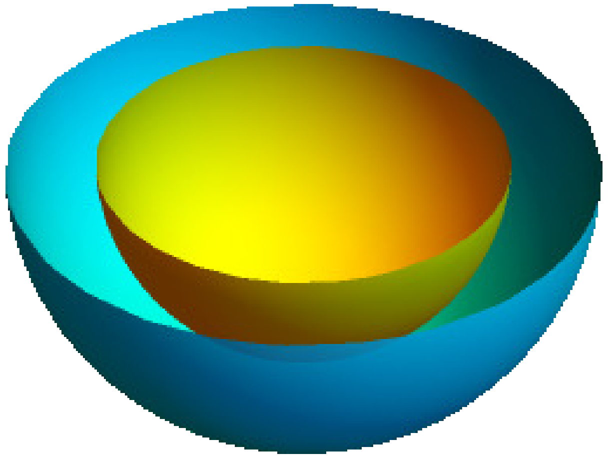
Dynamics of confined double-shells systems, critical behavior and chaos

Jorge V. Rocha (U. Barcelona)



GR21, Columbia U., New York

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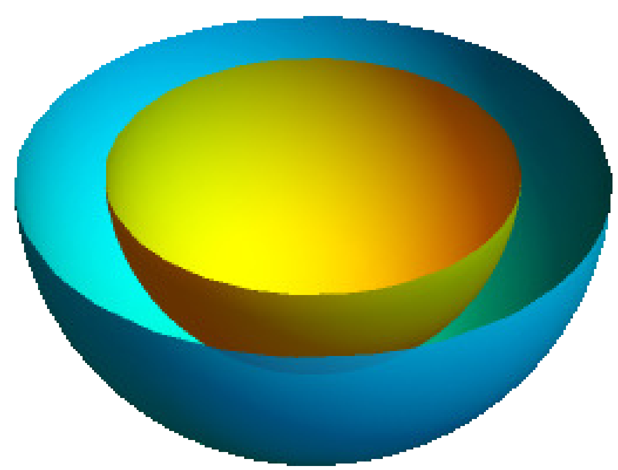

based on:

- 1601.07552 [gr-qc] w/ **Vitor Cardoso**
- 1602.03535 [hep-th] w/ **Richard Brito** and **Vitor Cardoso**

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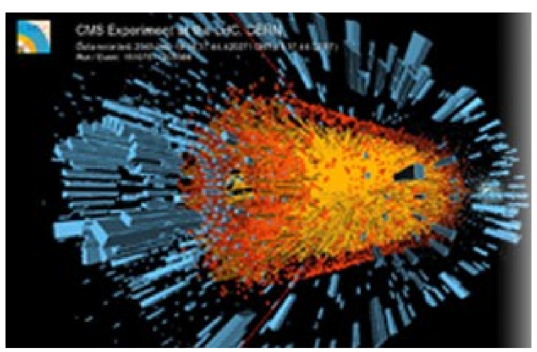
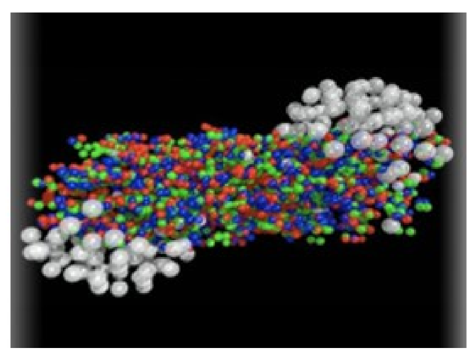

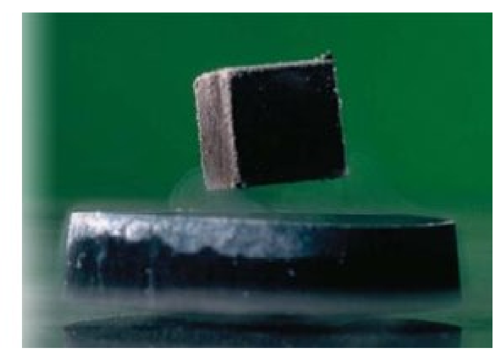

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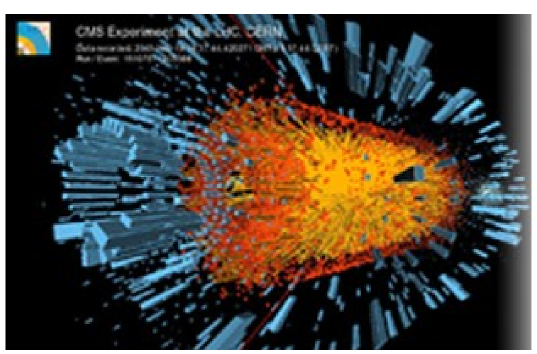
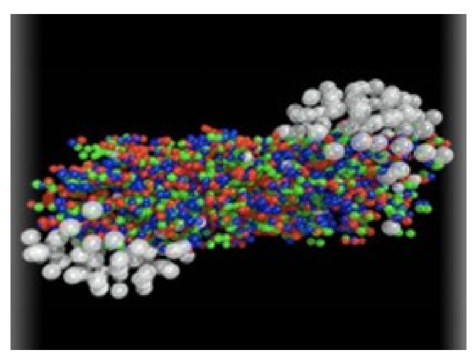

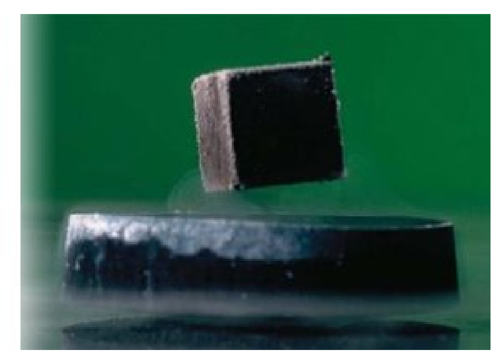

- But the subject of gravity in confined spaces has intrinsic interest.
  Even in the context of four-dimensional classical gravity.

4D classical gravity

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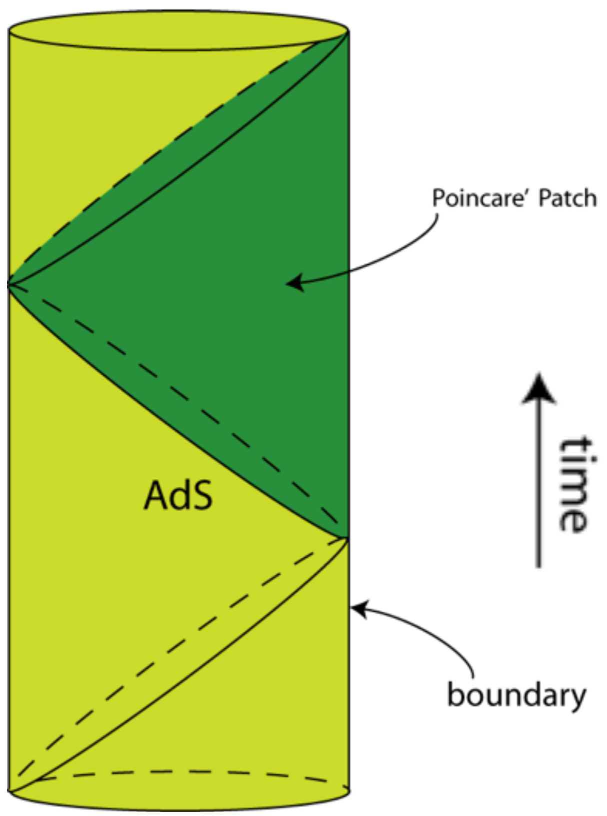

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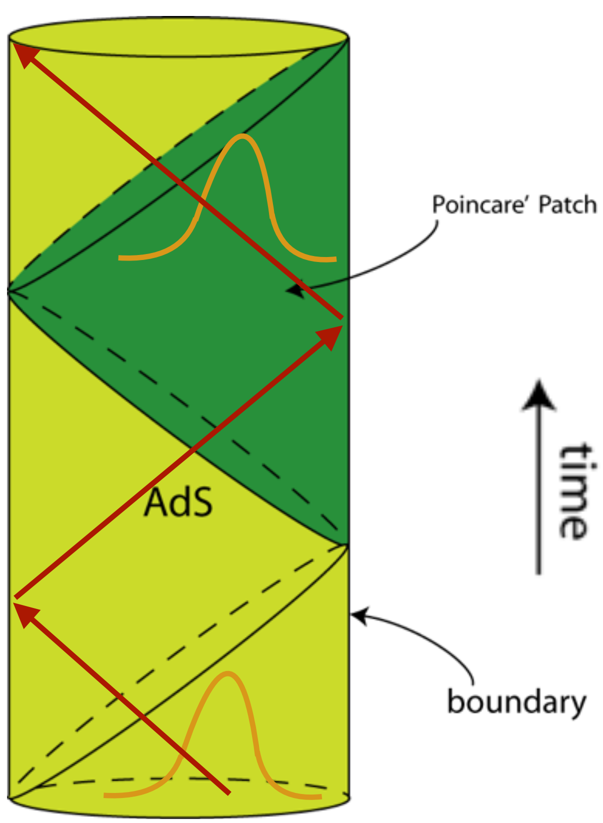

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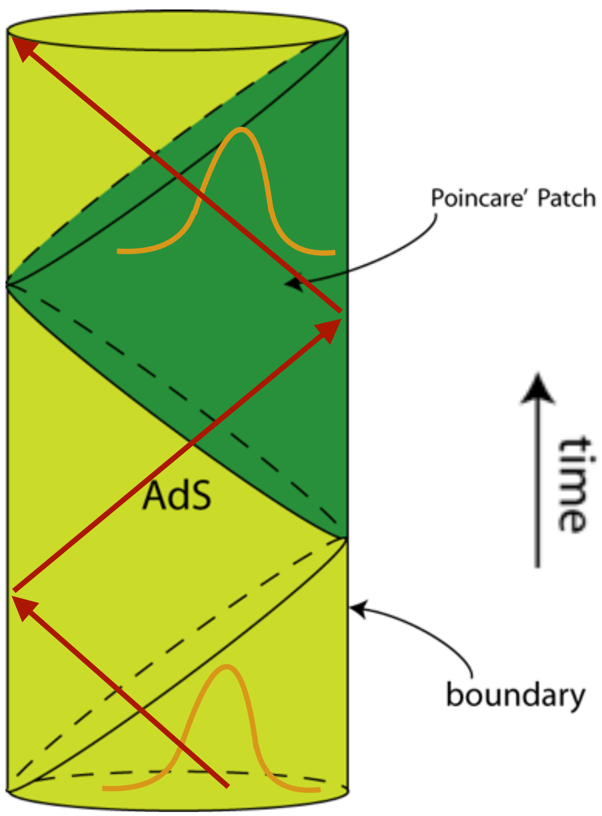

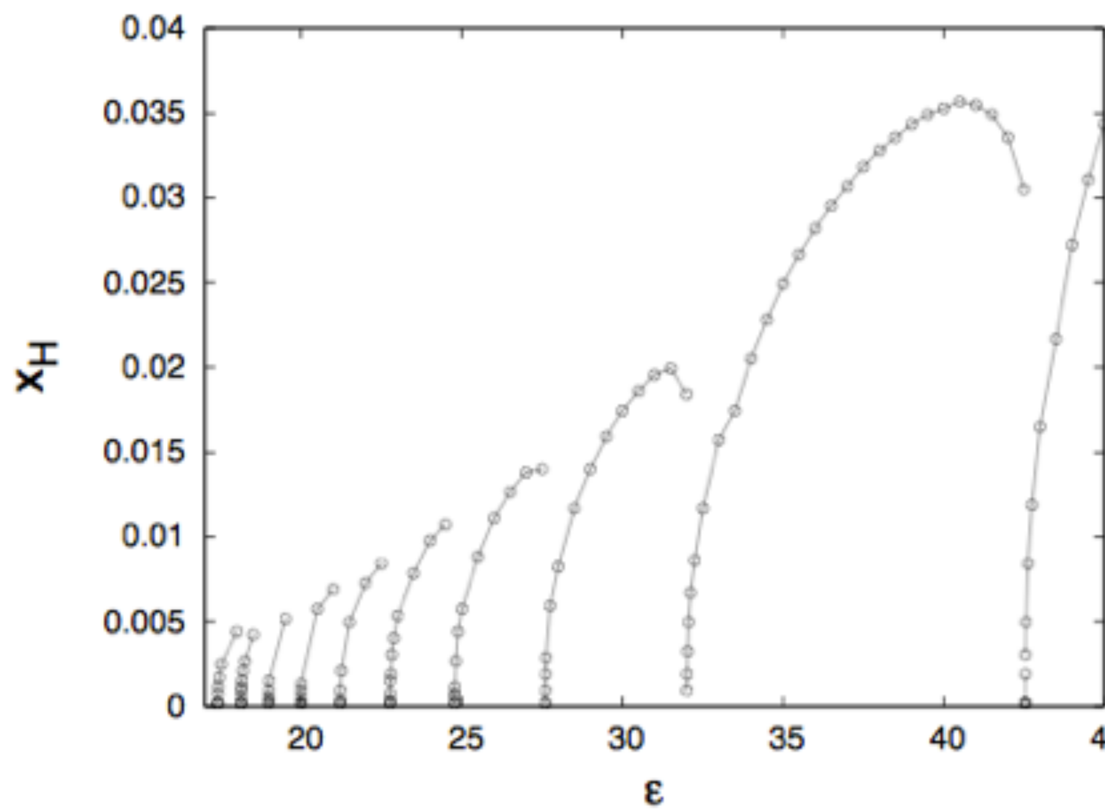

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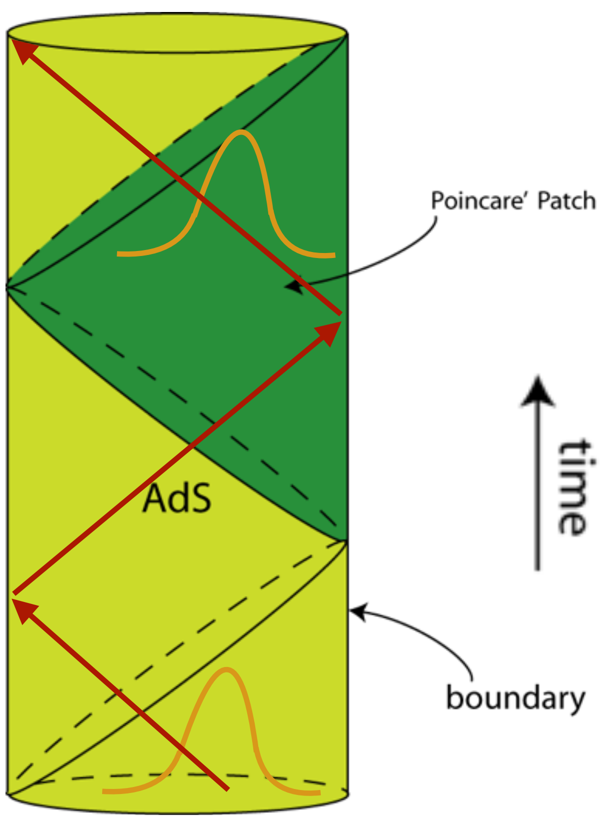

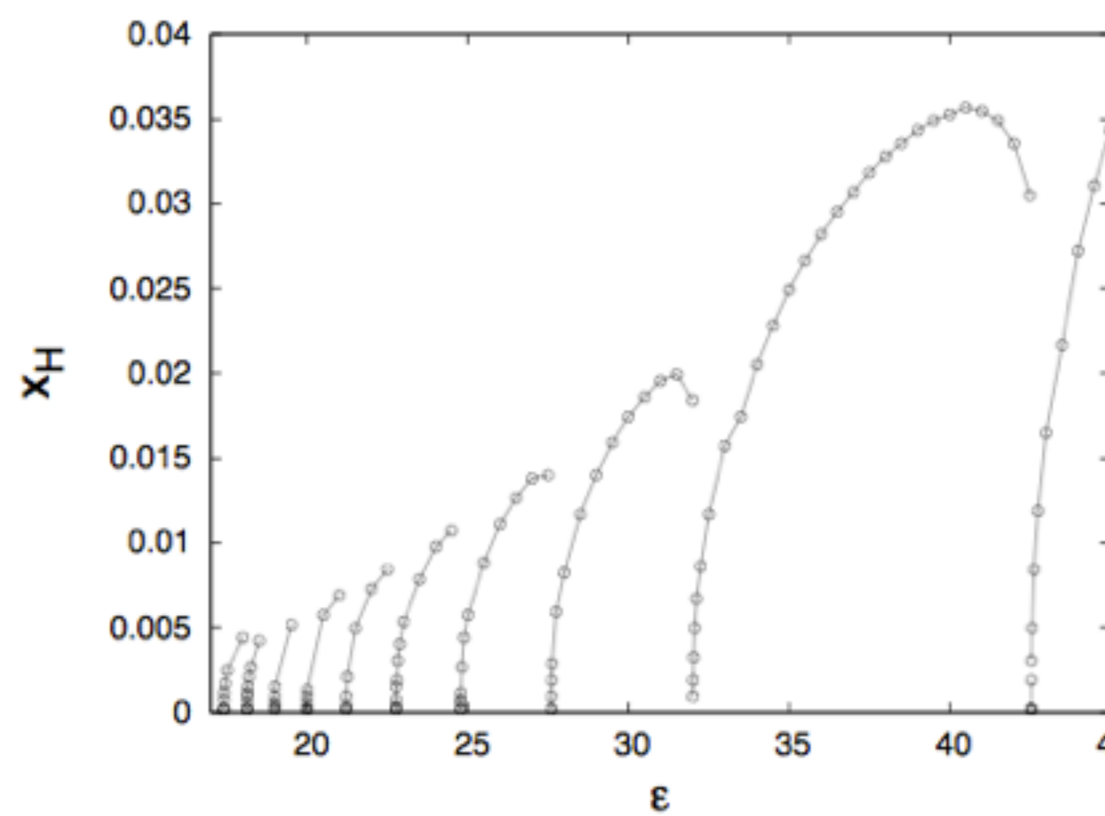

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- Arbitrarily (?) small perturbations of AdS typically (?) collapse after a sufficiently large number of reflections.

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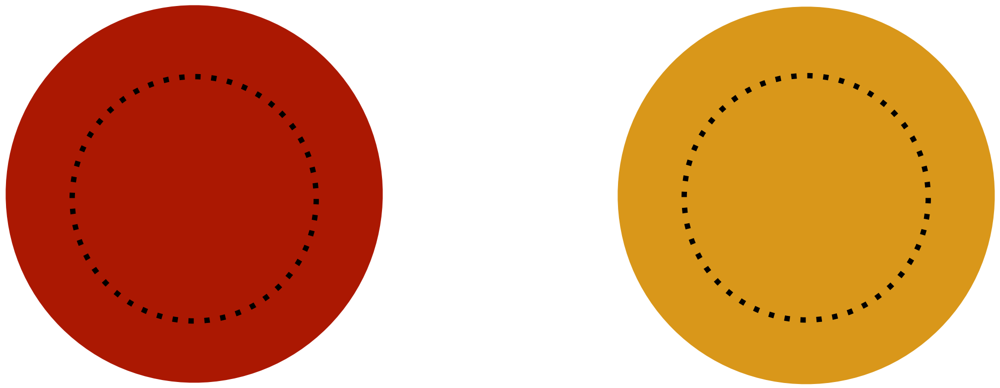

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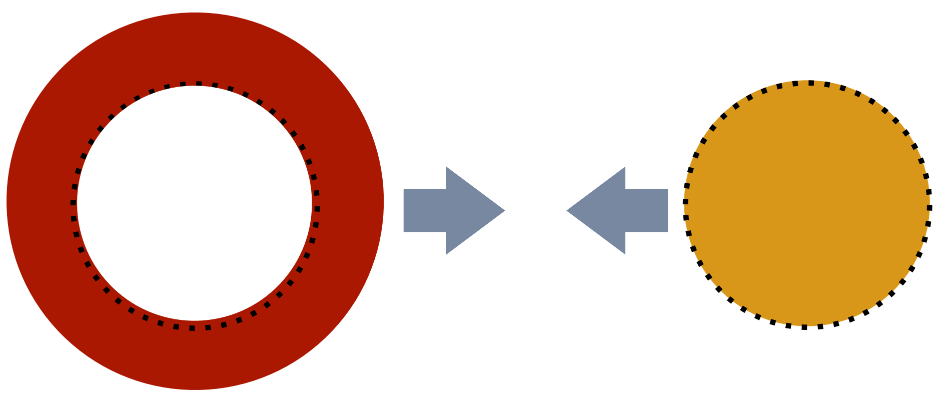

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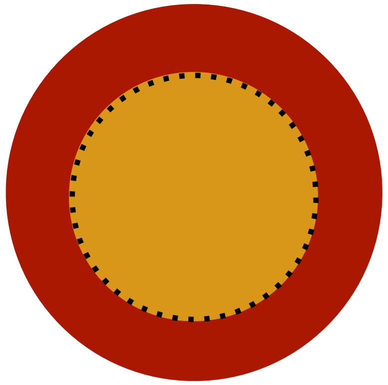

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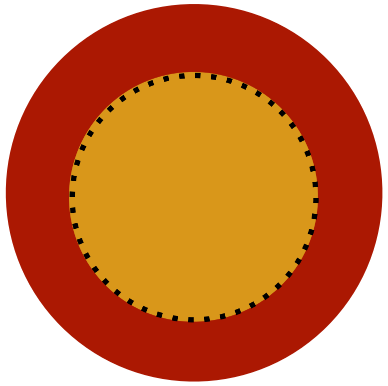

- Note: the thin shell may be *dynamical* even if the two spacetimes matched are static.

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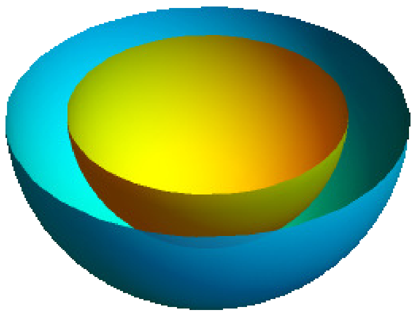

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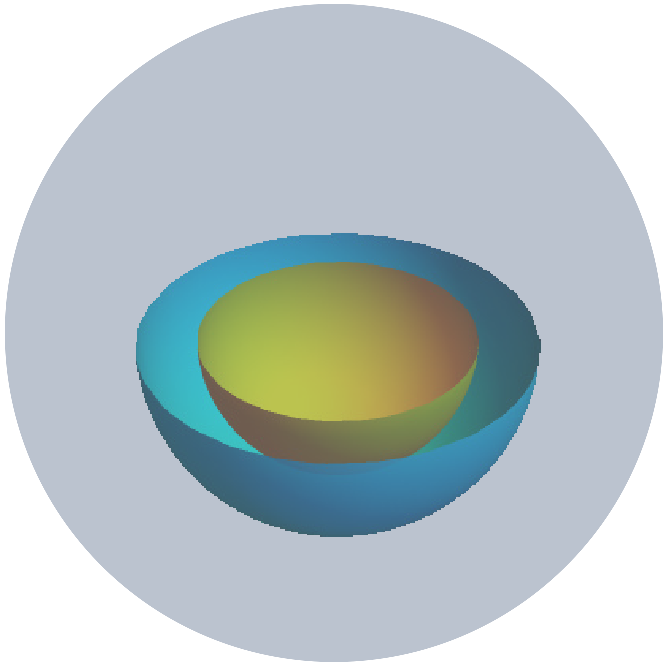

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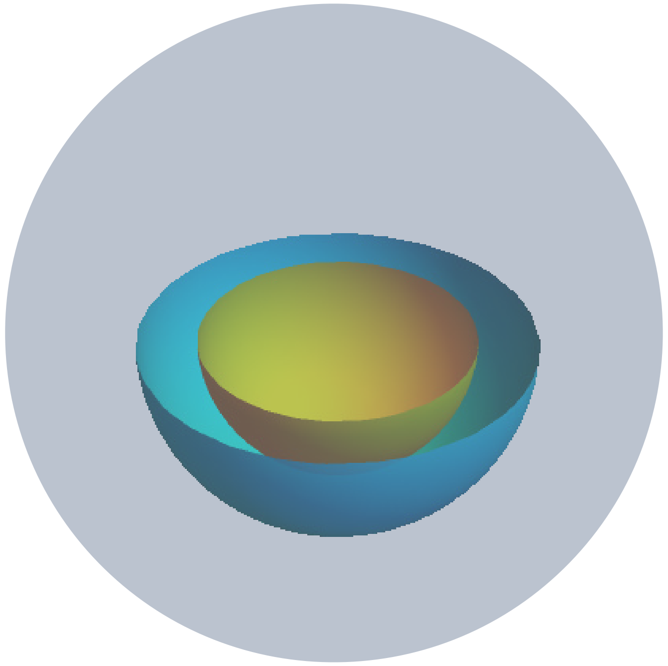

This basic setting displays ***astonishingly rich dynamics*** and provides ***easily tractable time evolution***.

Introduction: Earlier literature

- Previous studies with relativistic multiple-shells systems:

  — collision of two null shells [Dray, 't Hooft (1985)] [Redmount (1985)]

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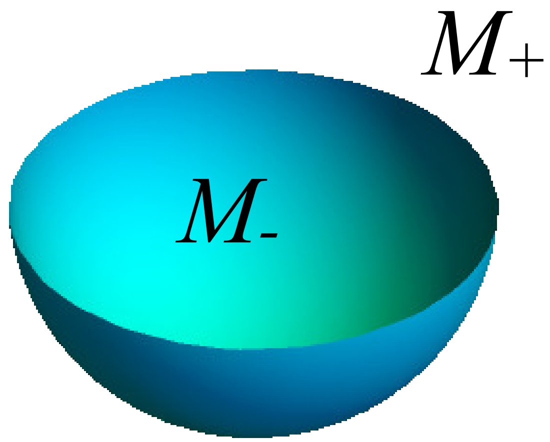
Crucial difference: **absence of a confining mechanism**

Dynamics of a spherical shell: Details of the setup

- For a single spherical shell: inner and outer geometries are both Schwarzschild-AdS.

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2\,, \qquad f(r) = \left(1 - \frac{2M}{r} + \frac{r^2}{l^2}\right)$$



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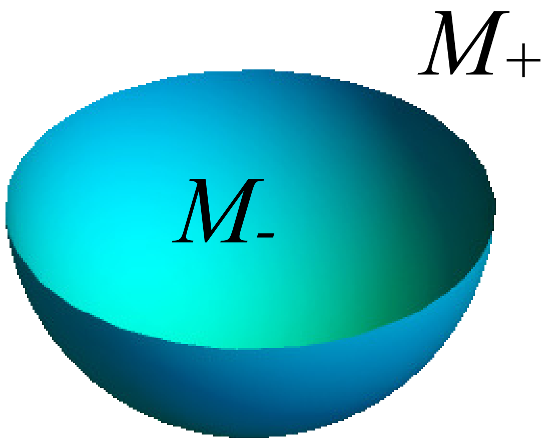

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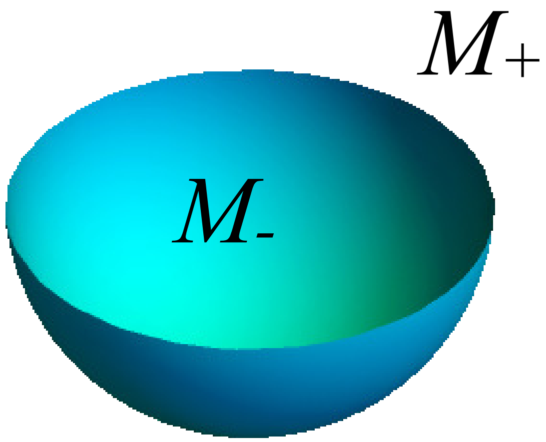

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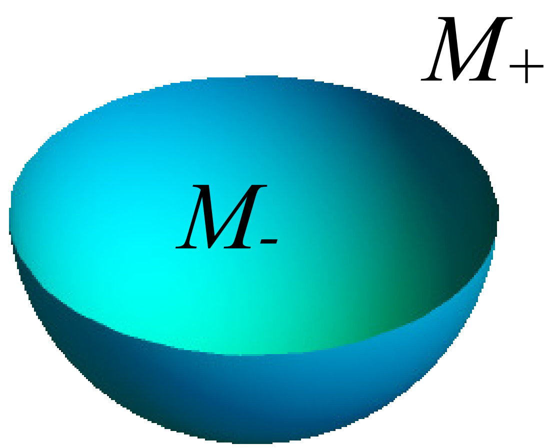

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integration constant (invariant mass of shell): $\rho = \dfrac{m\, l^{2w}}{4\pi G R^{2+2w}}$

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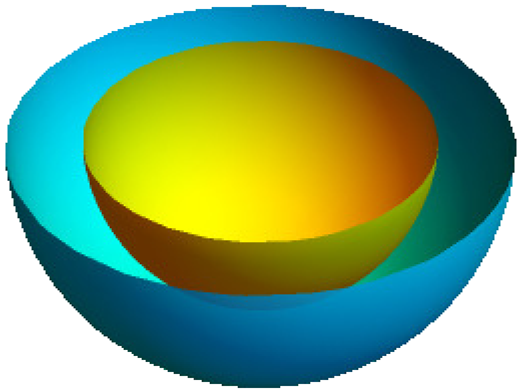

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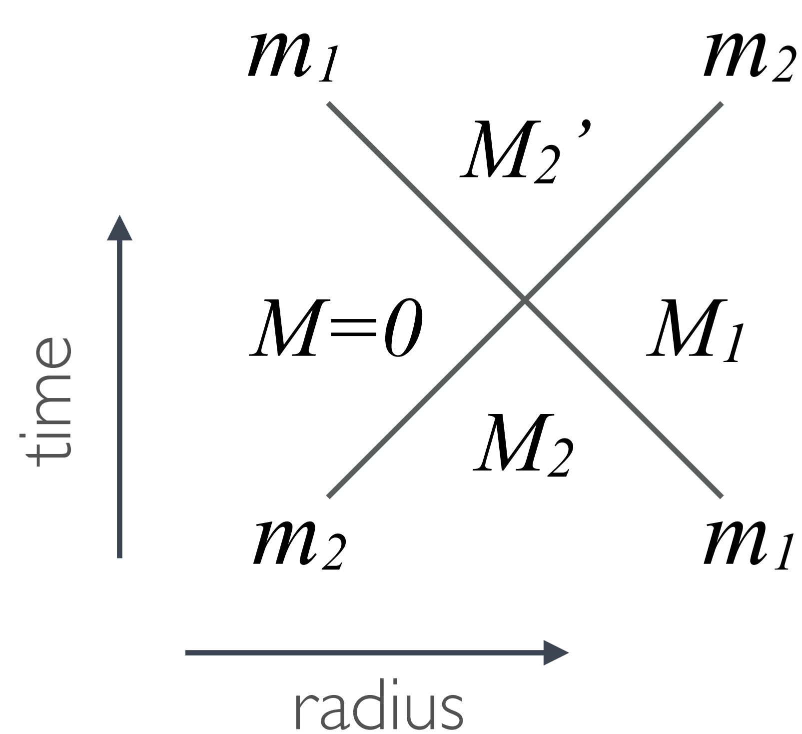

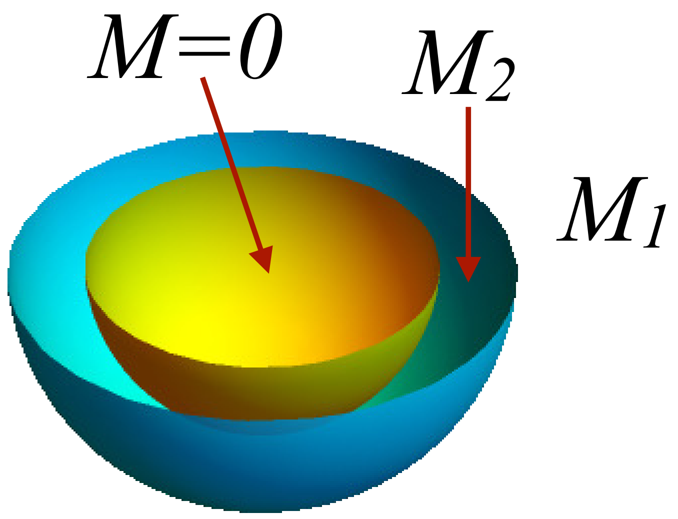

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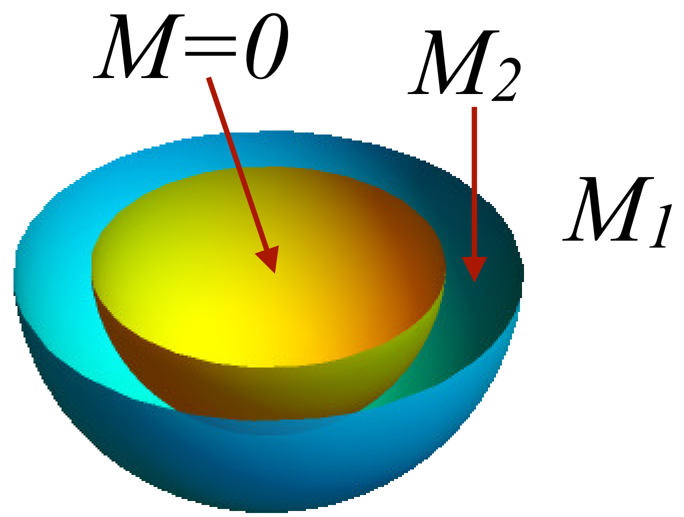

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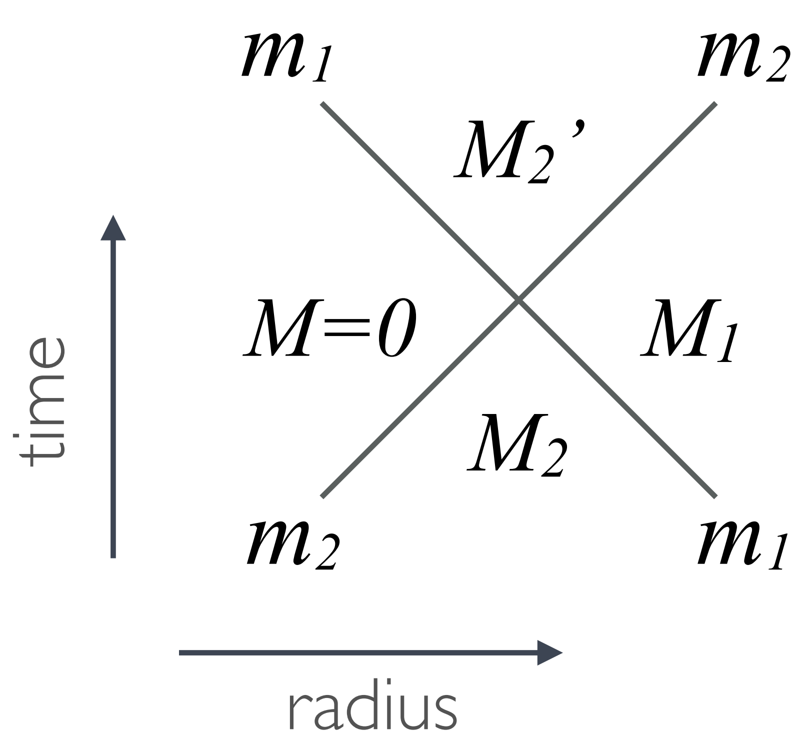

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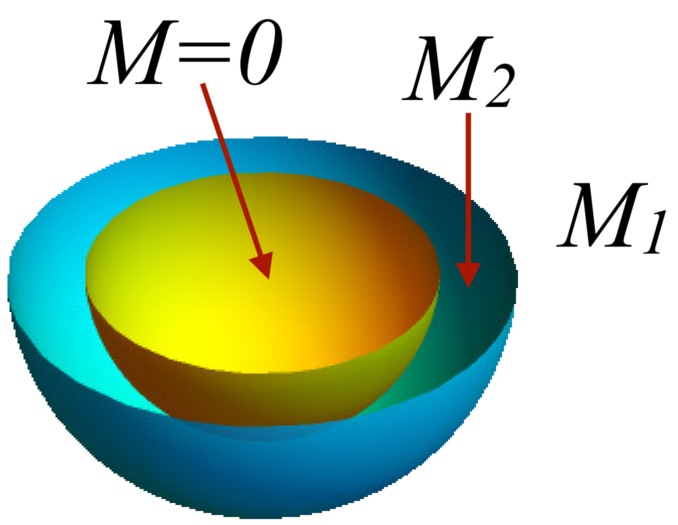

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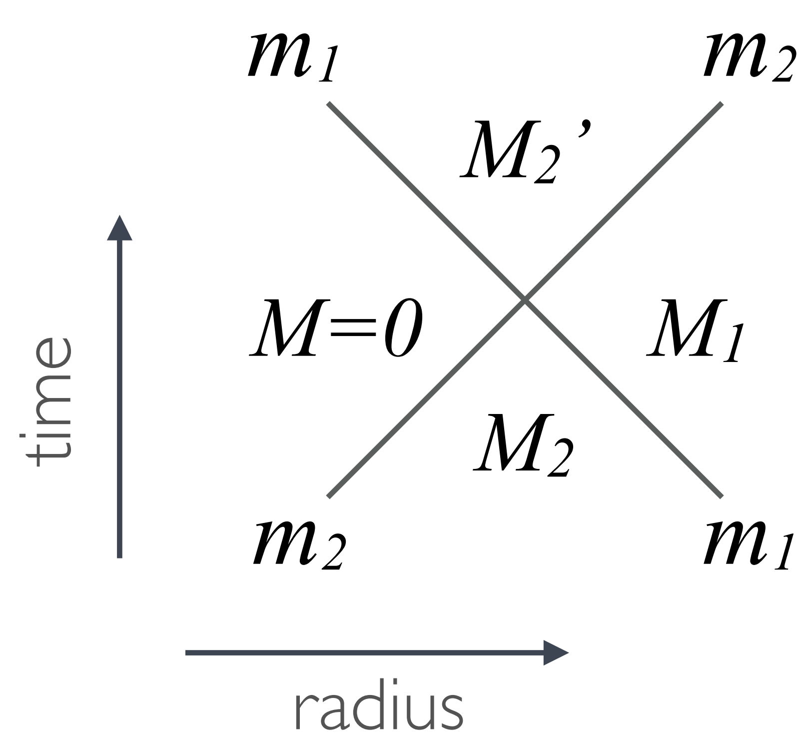

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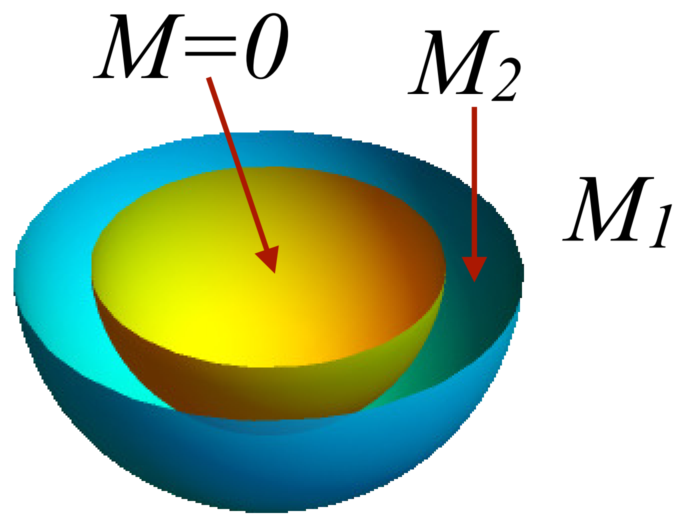

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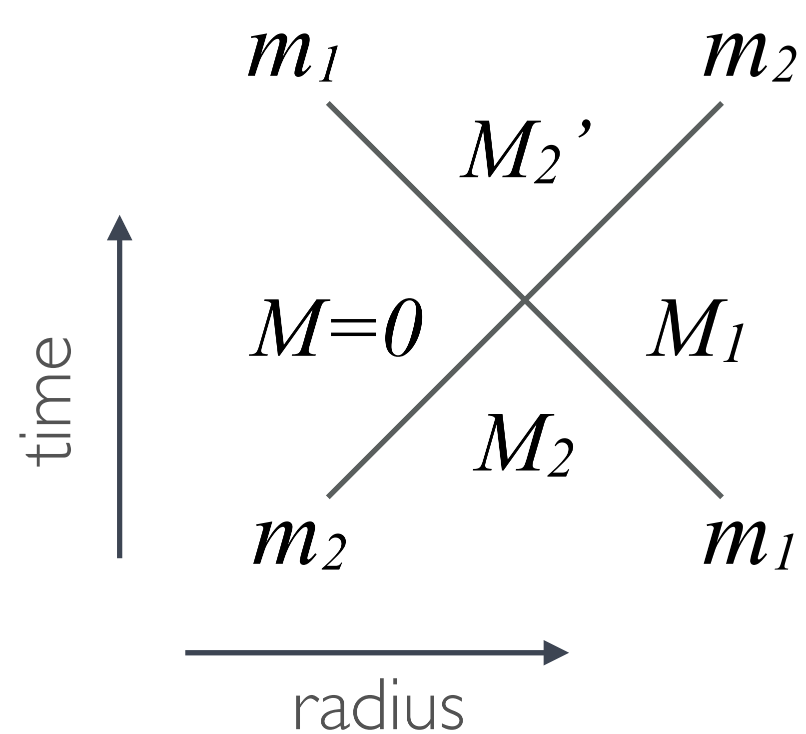

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Note: The outgoing shell always transfers energy to the ingoing shell.

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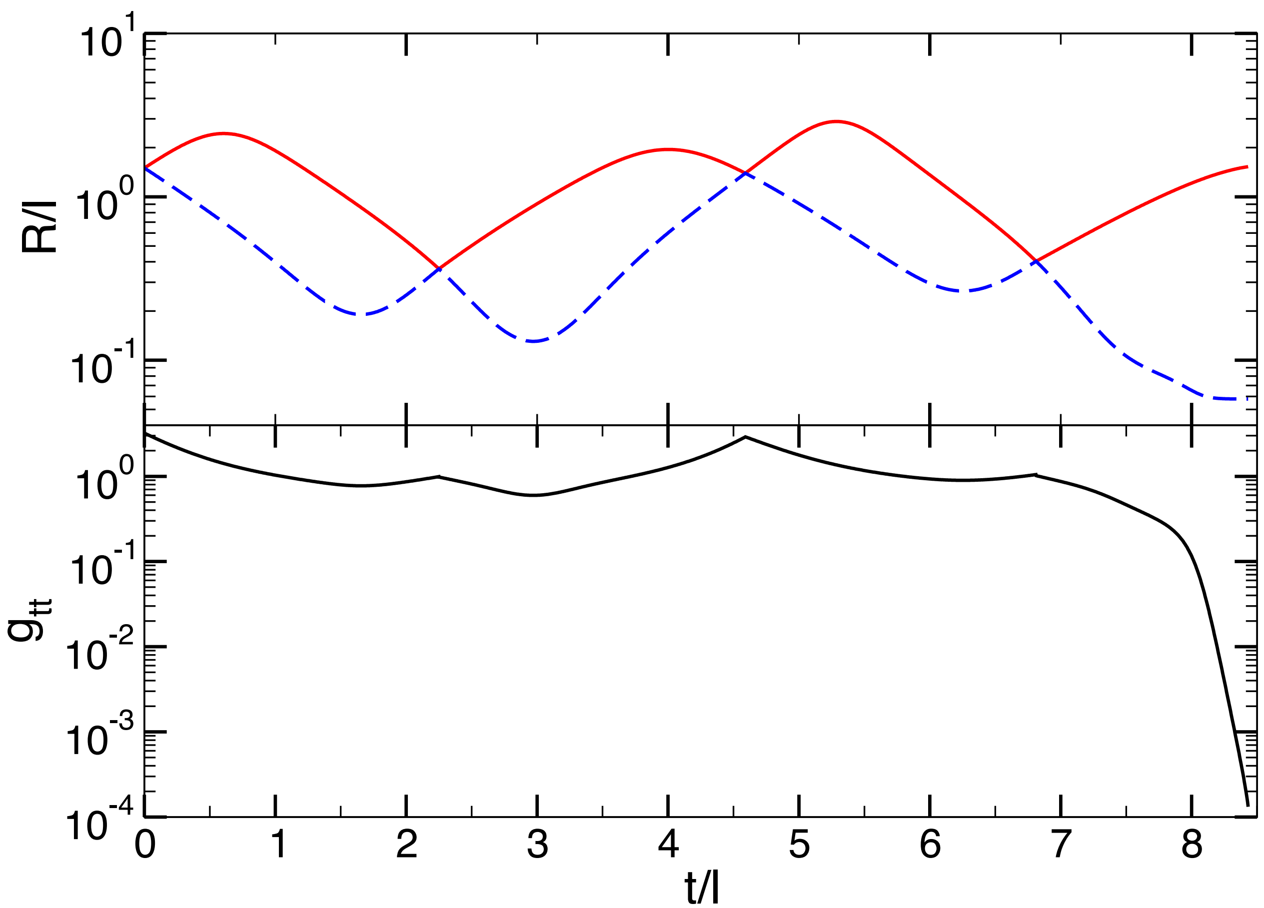
The confined double-shell system: Time evolution

double-shell in AdS



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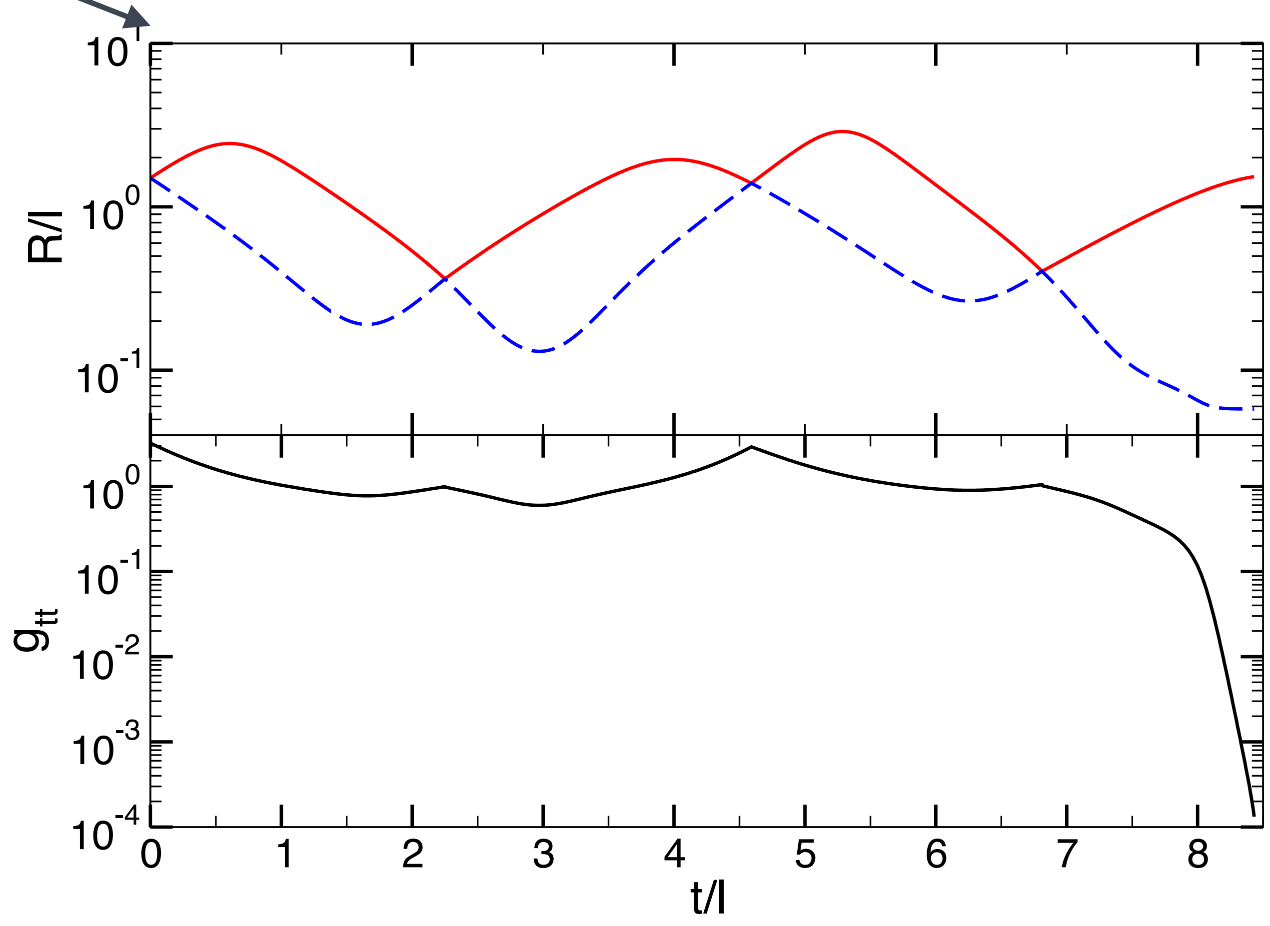

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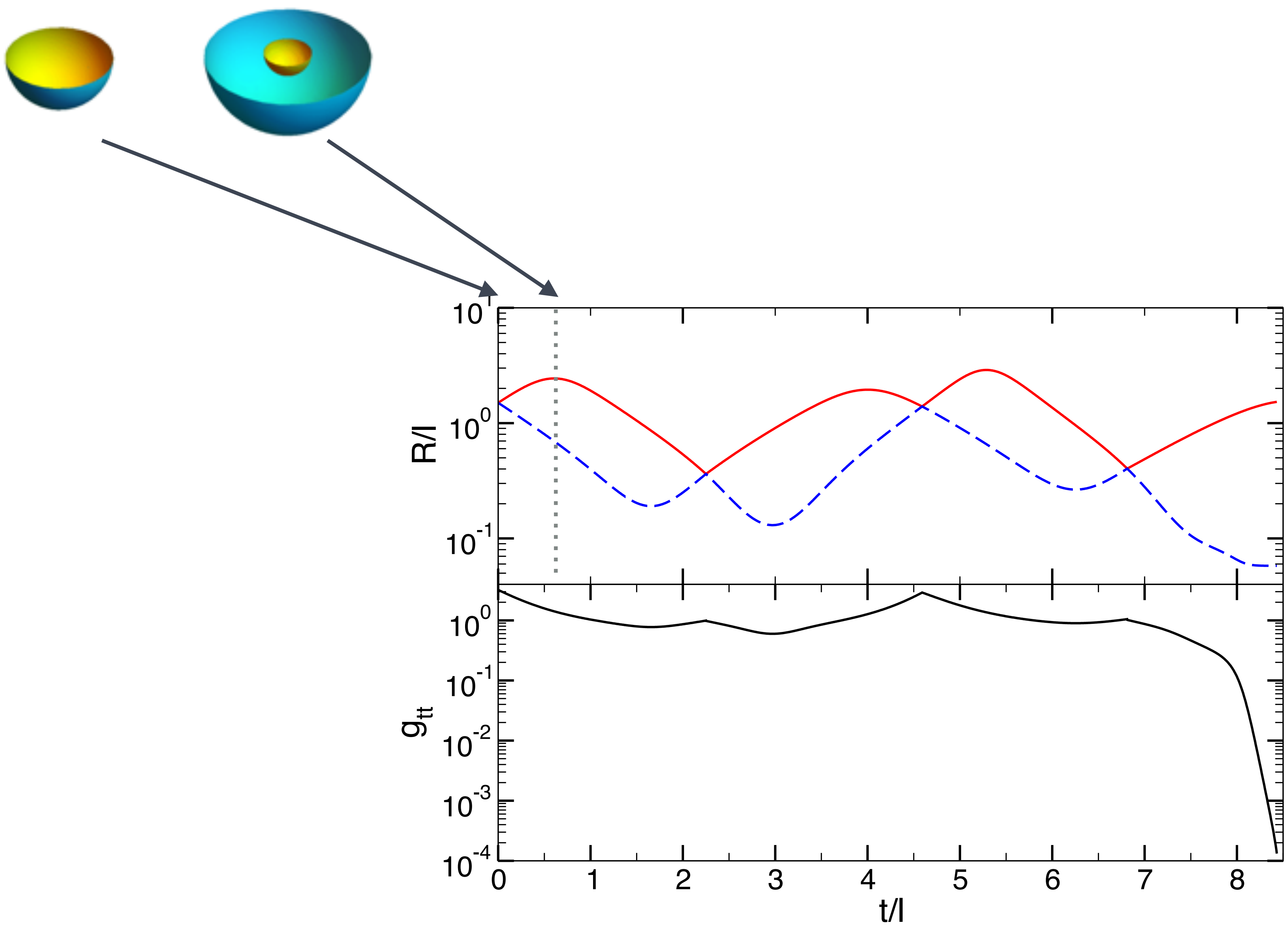

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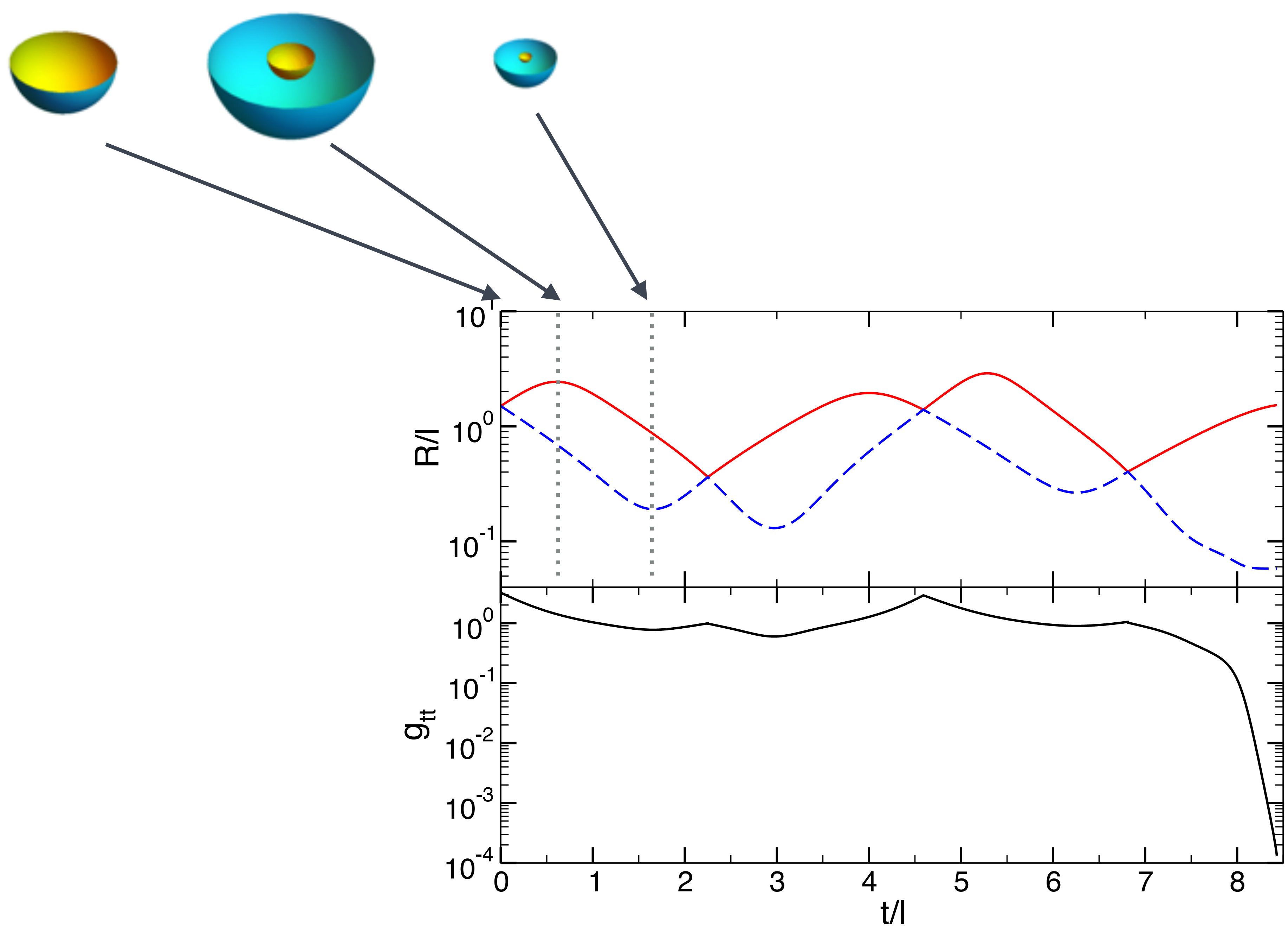

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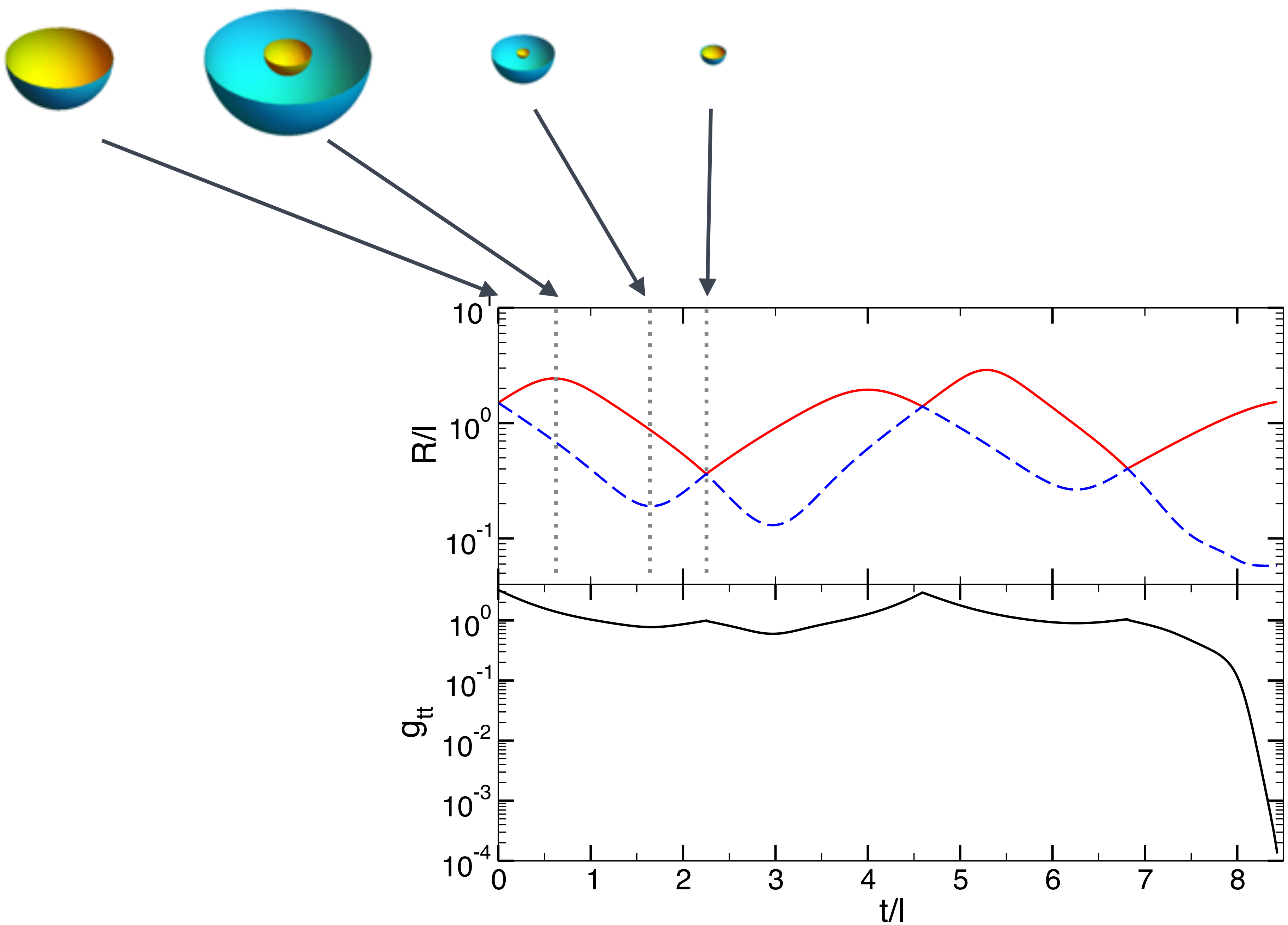

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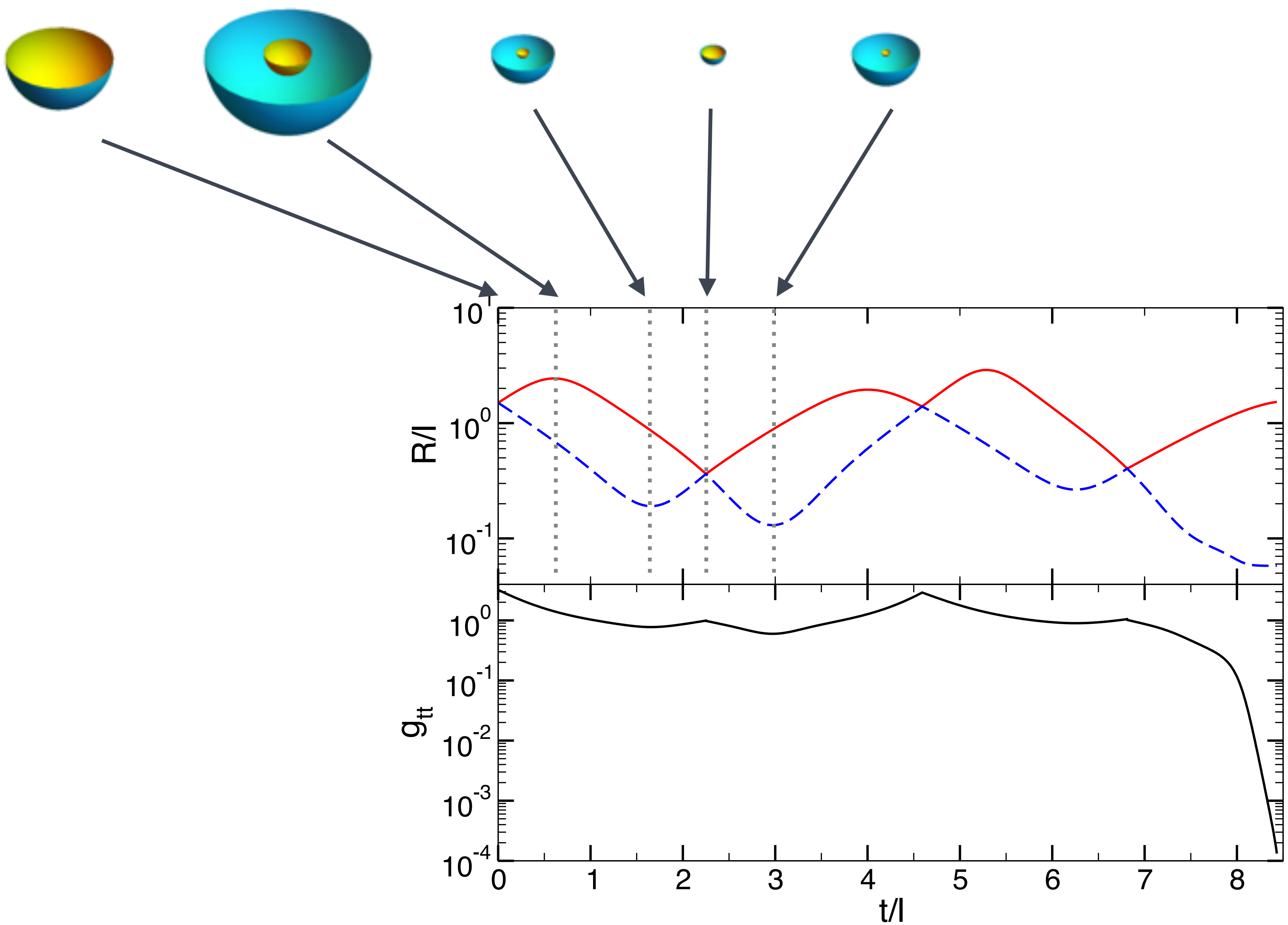

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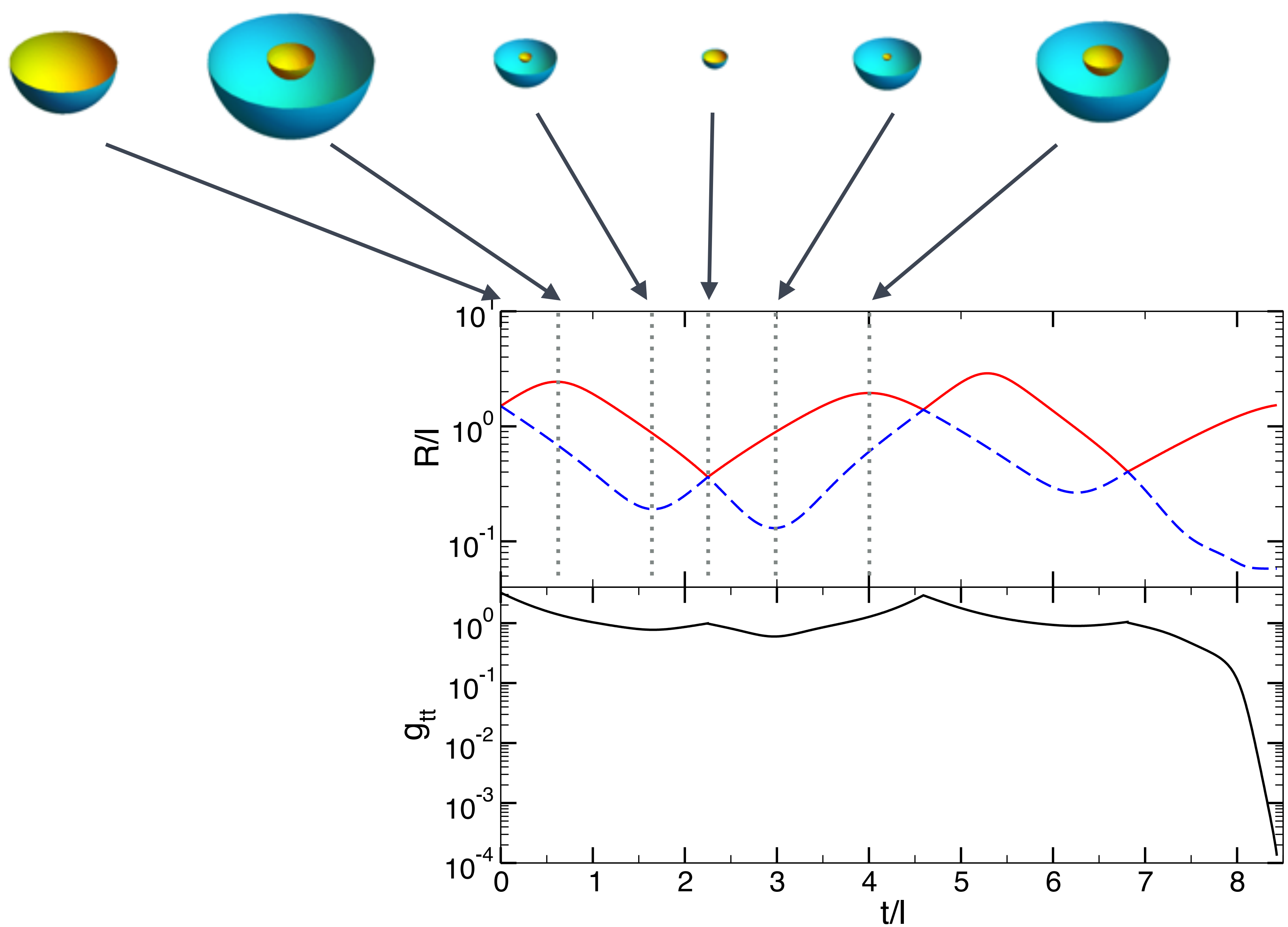

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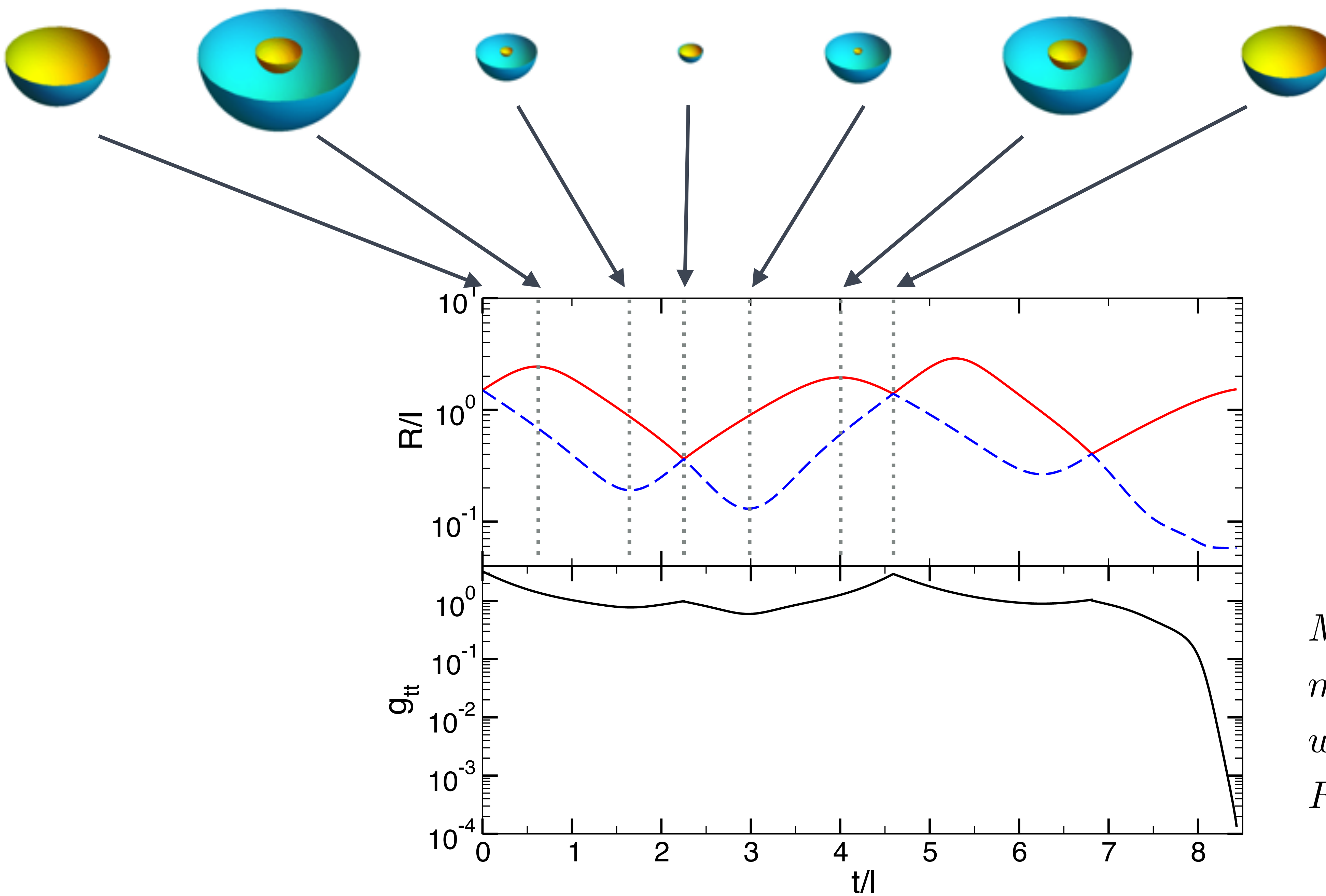

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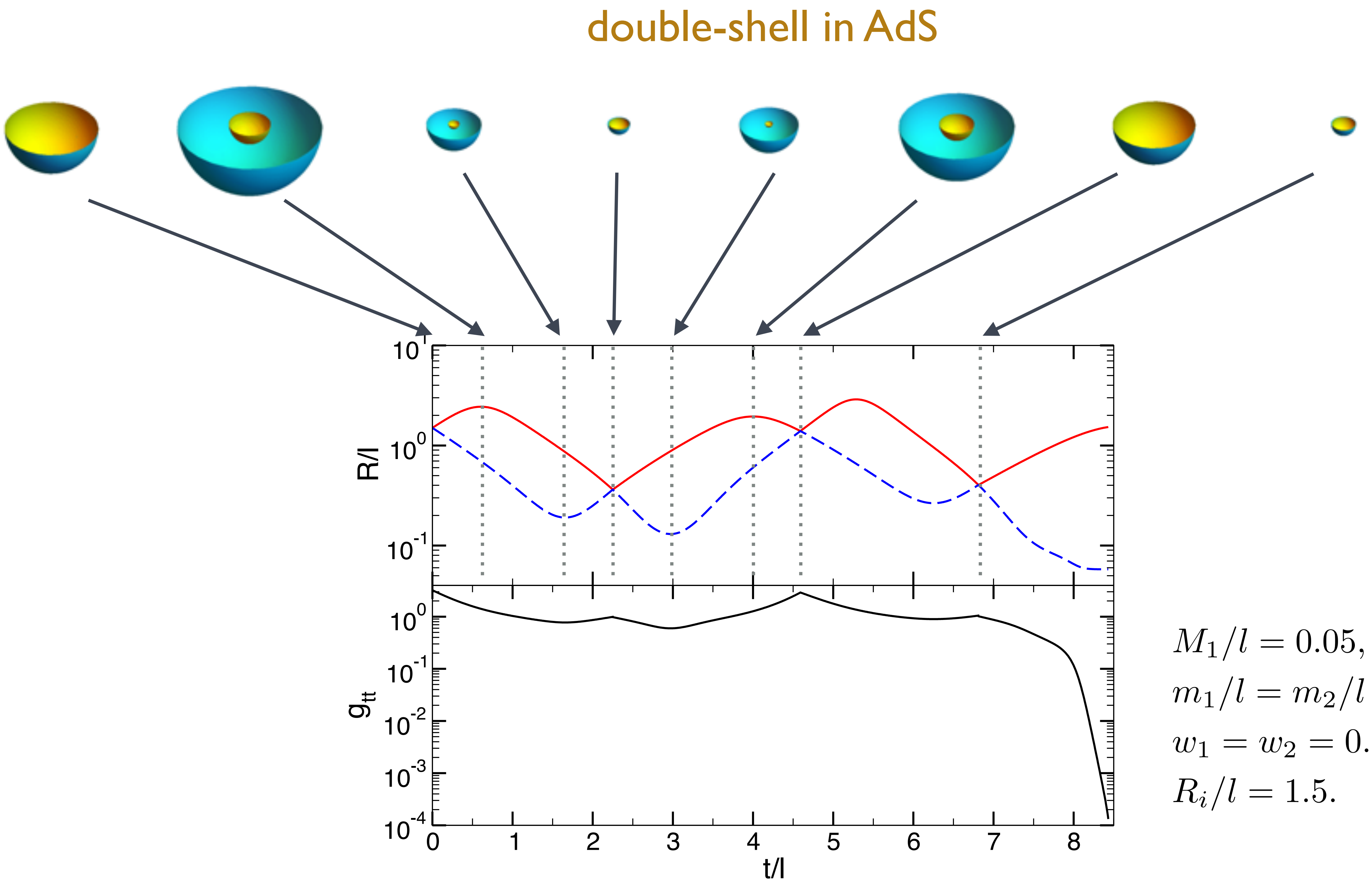

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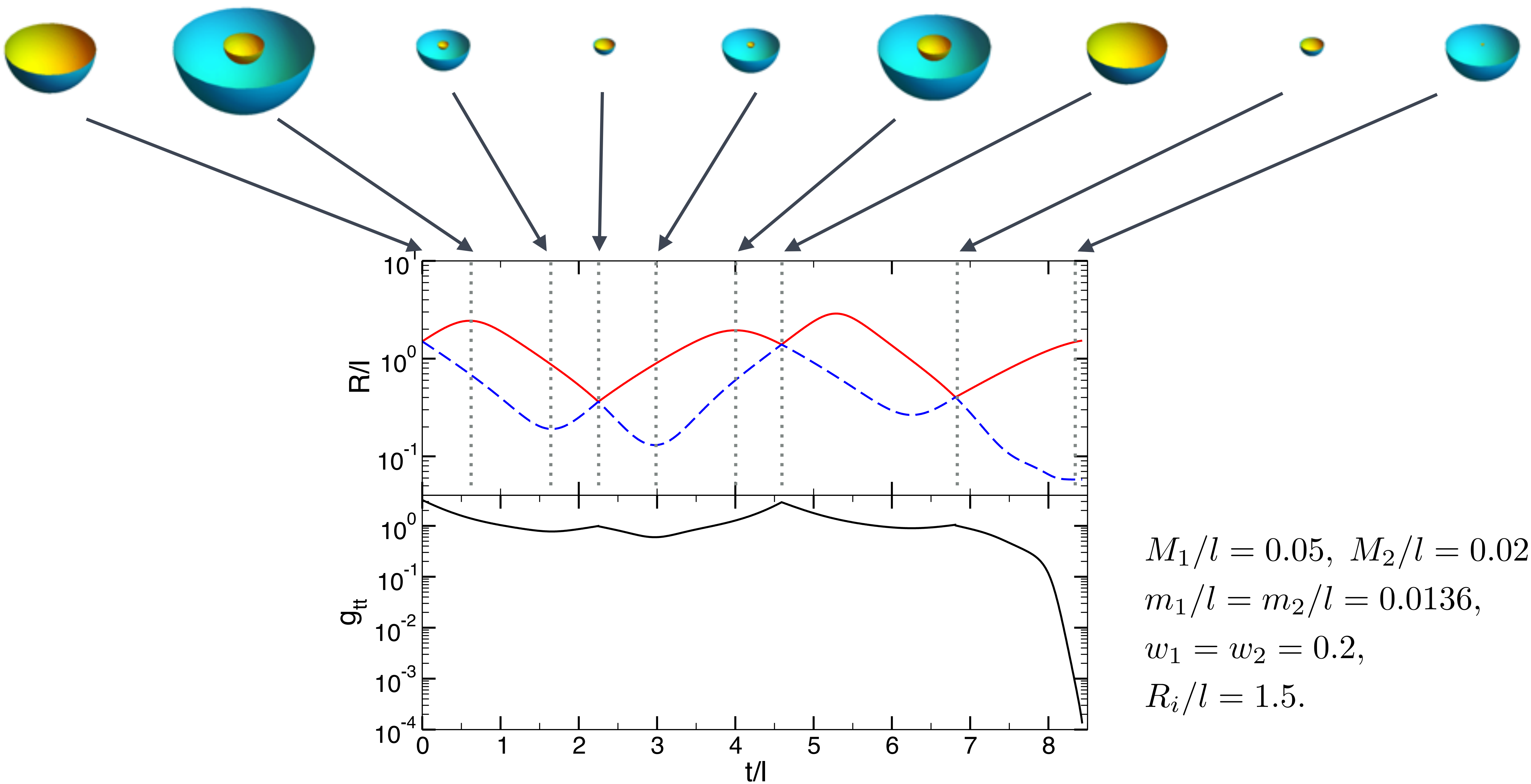

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Results: Critical phenomena

- **Number of crossings before collapse depends sensitively on initial conditions.**
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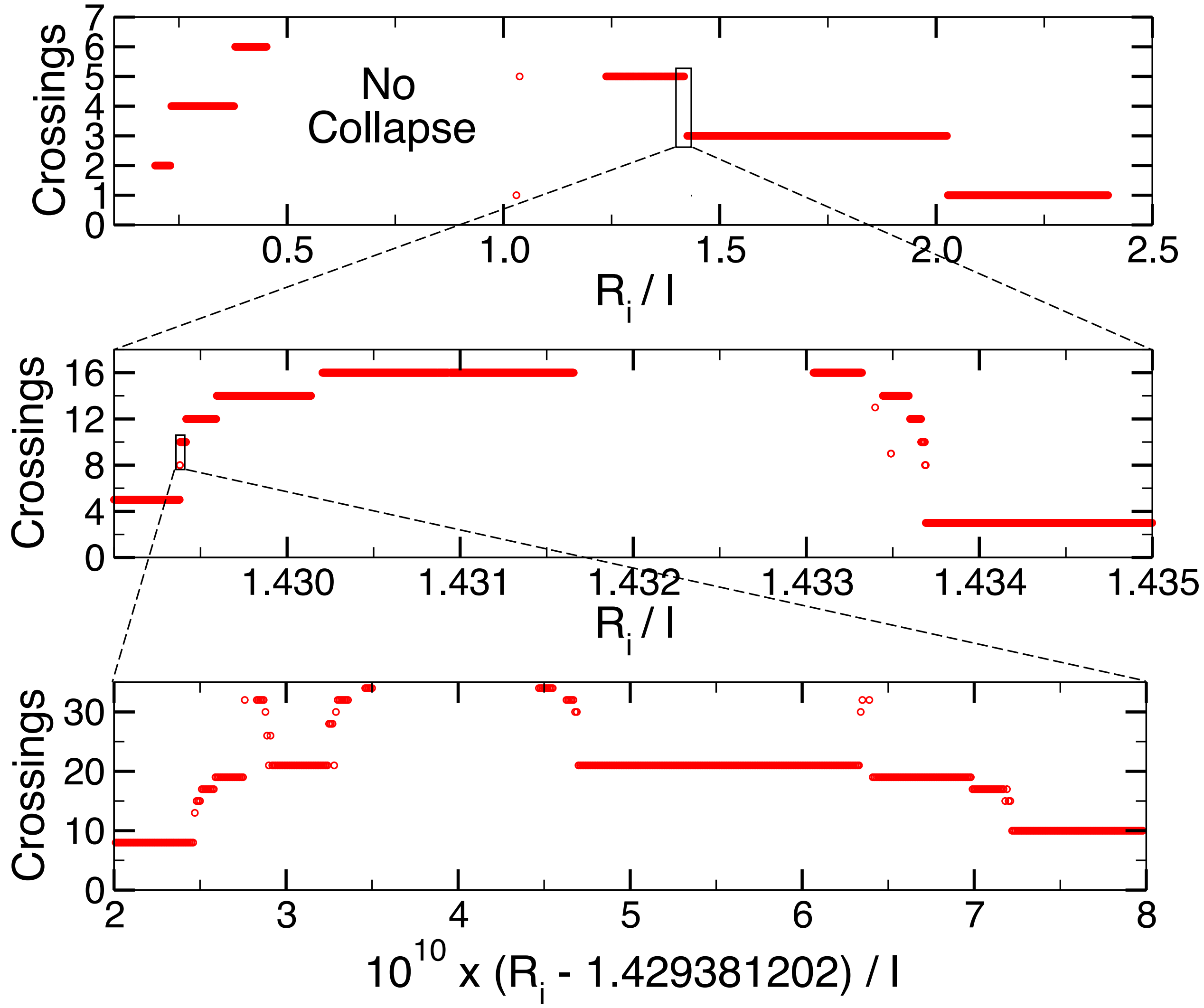

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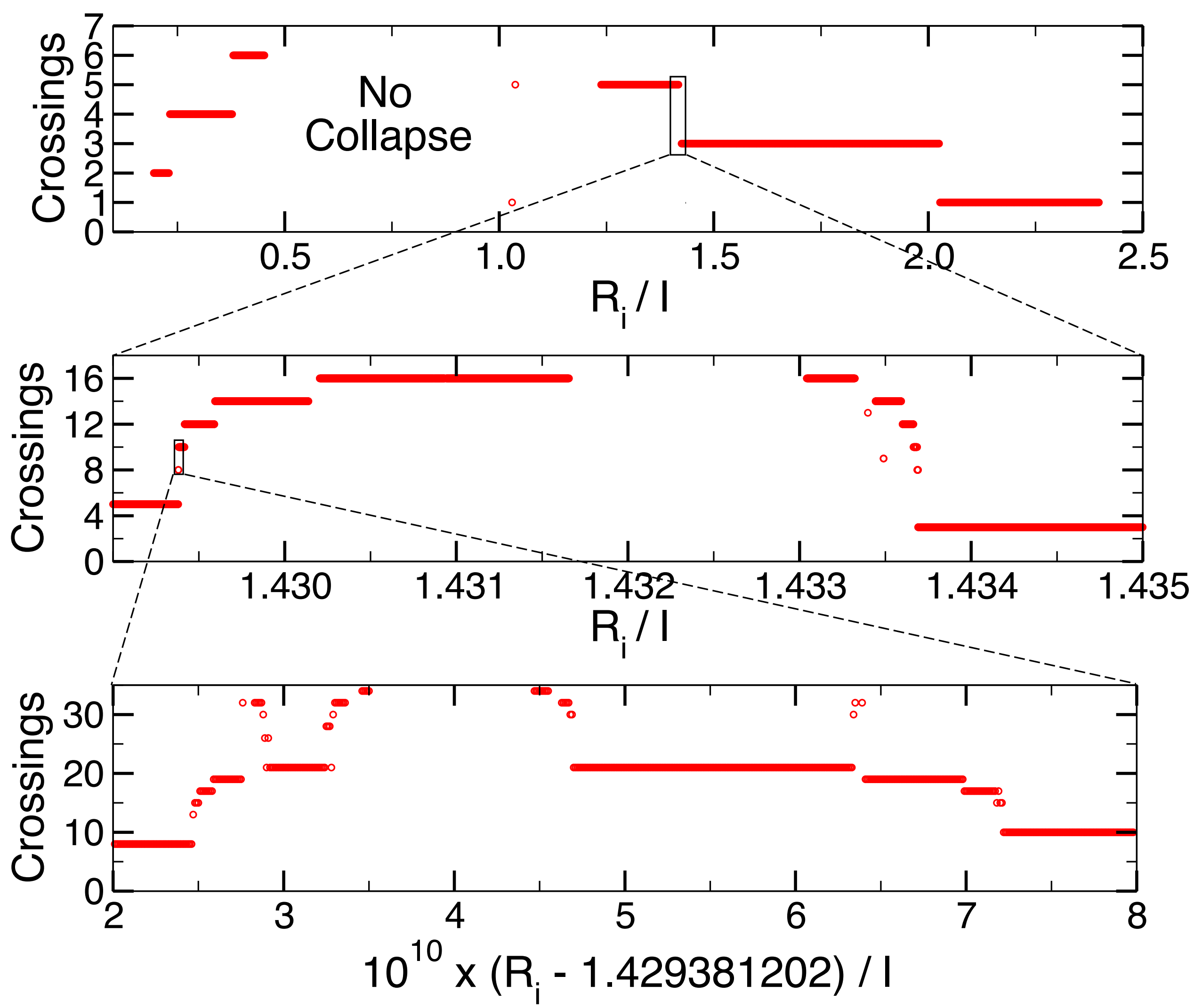

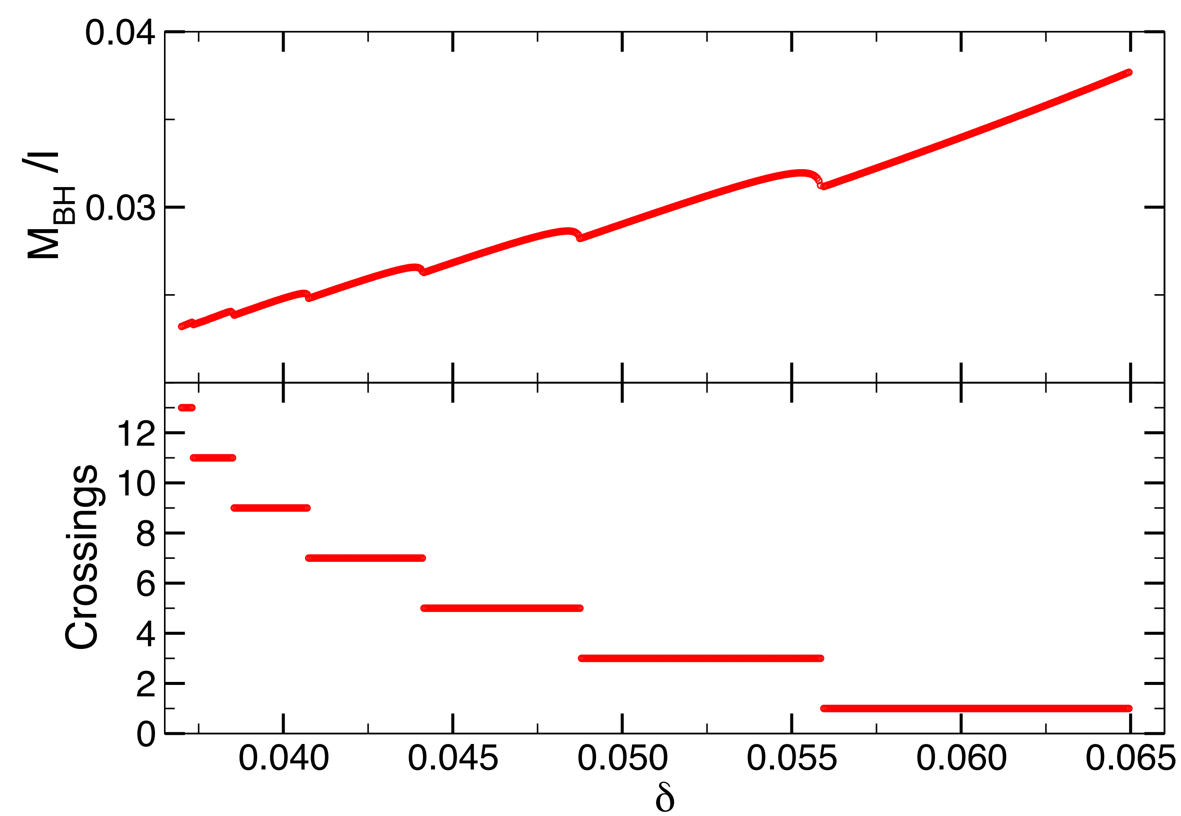

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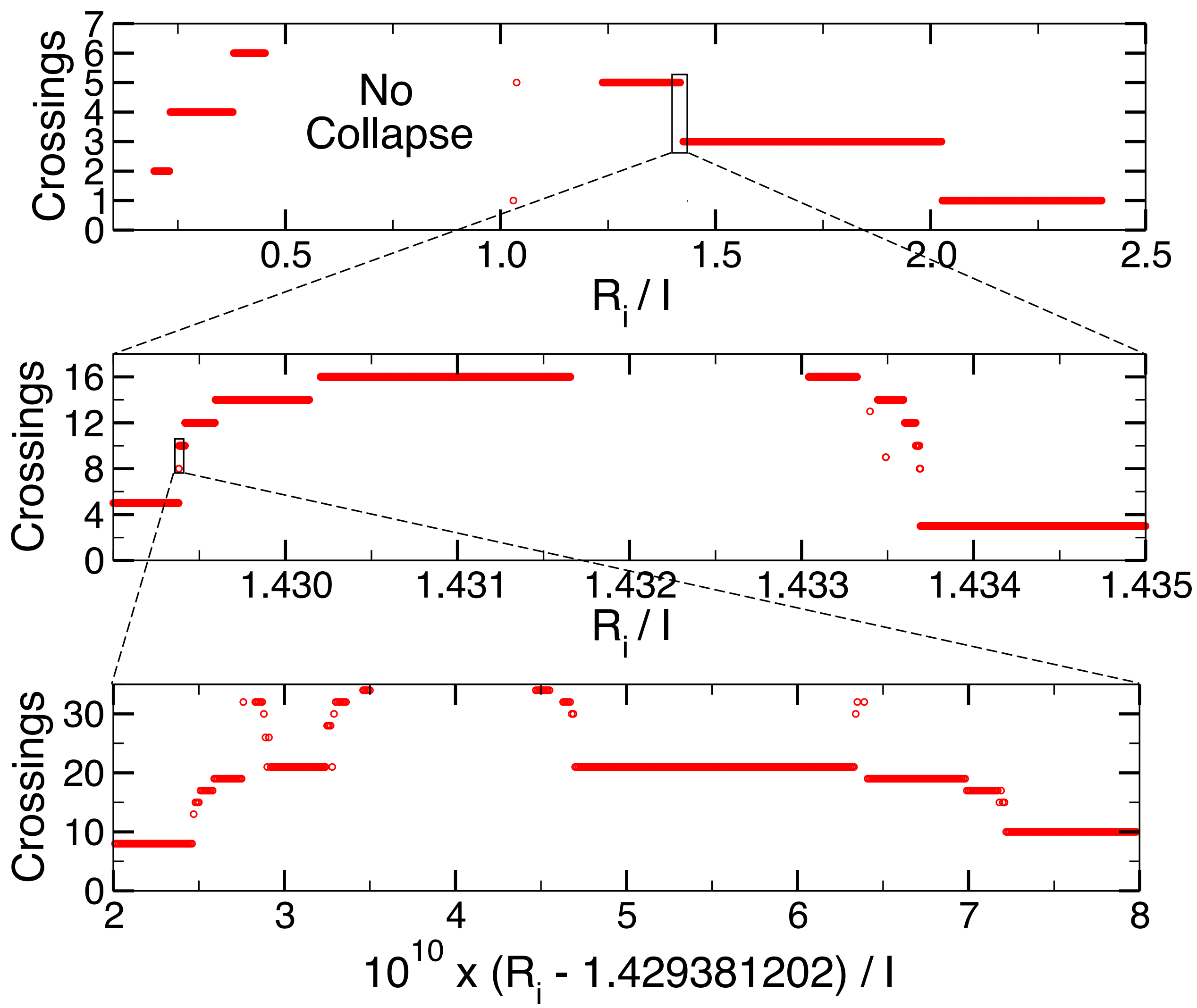

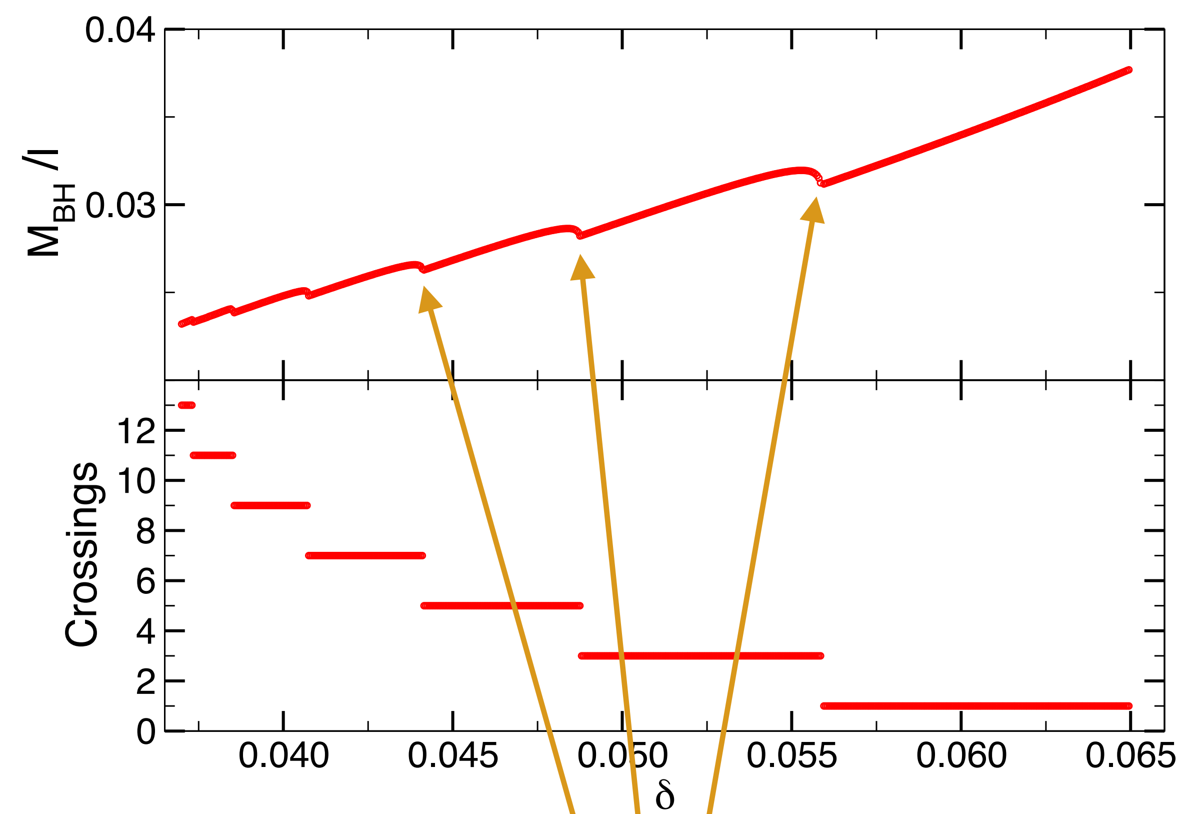

$$M_{BH} - M_0 \propto |\delta - \delta_*|^\gamma$$

Results: Comparison between 'cavity' and 'AdS'

- The critical exponent γ is sensitive to the matter content, i.e. the choice of w.

[Koike, Mishima (1995)]
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- A similar trend has been observed recently with massless scalar fields.

[Santos-Oliván, Sopuerta (2015)]
[Cai, Yang (2016)]

Results: Chaotic behavior

- The double-shell system in Newtonian gravity (without CC) is chaotic. How about in GR, in AdS?

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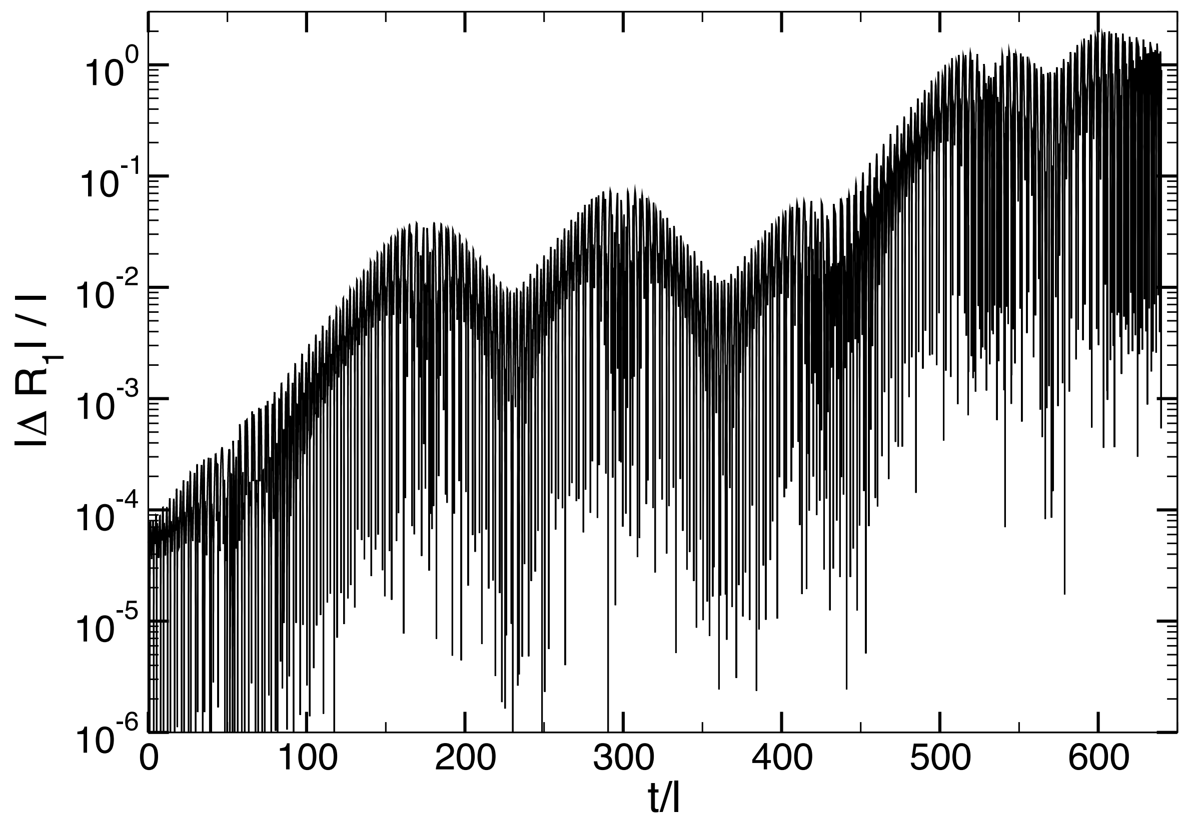

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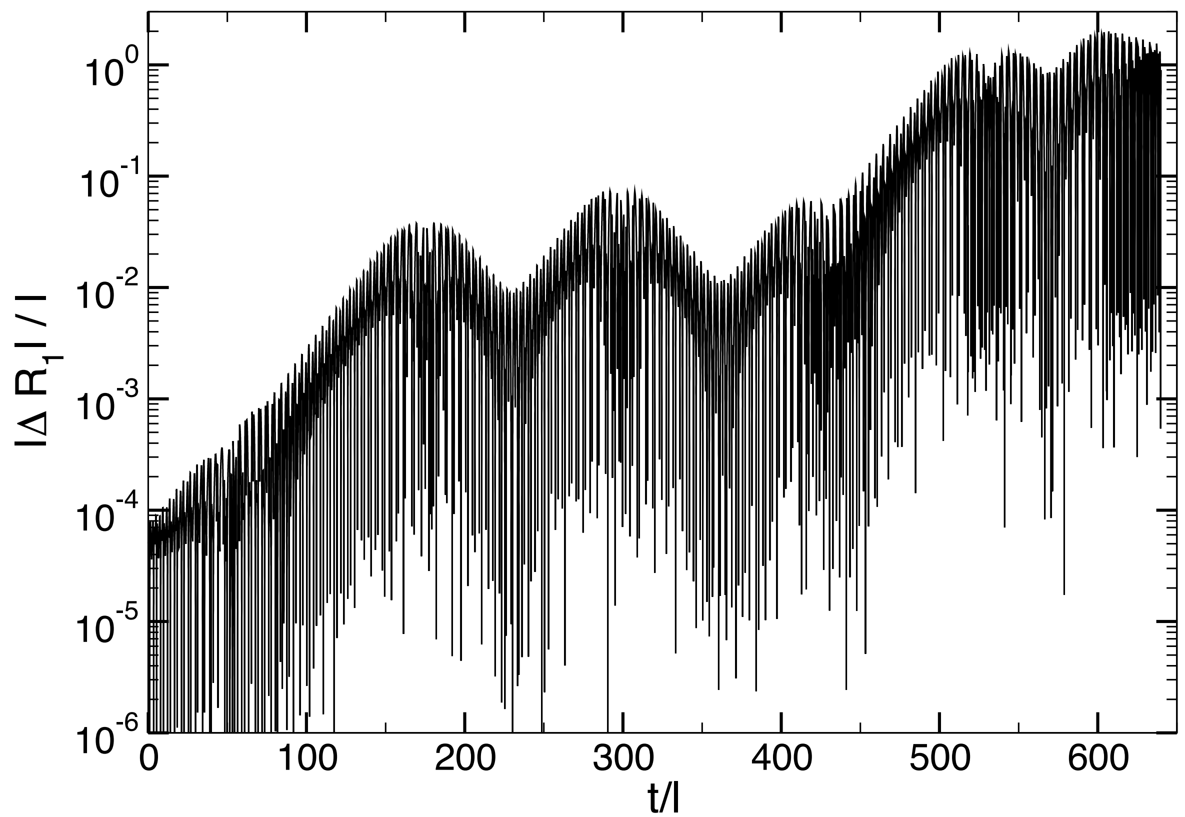

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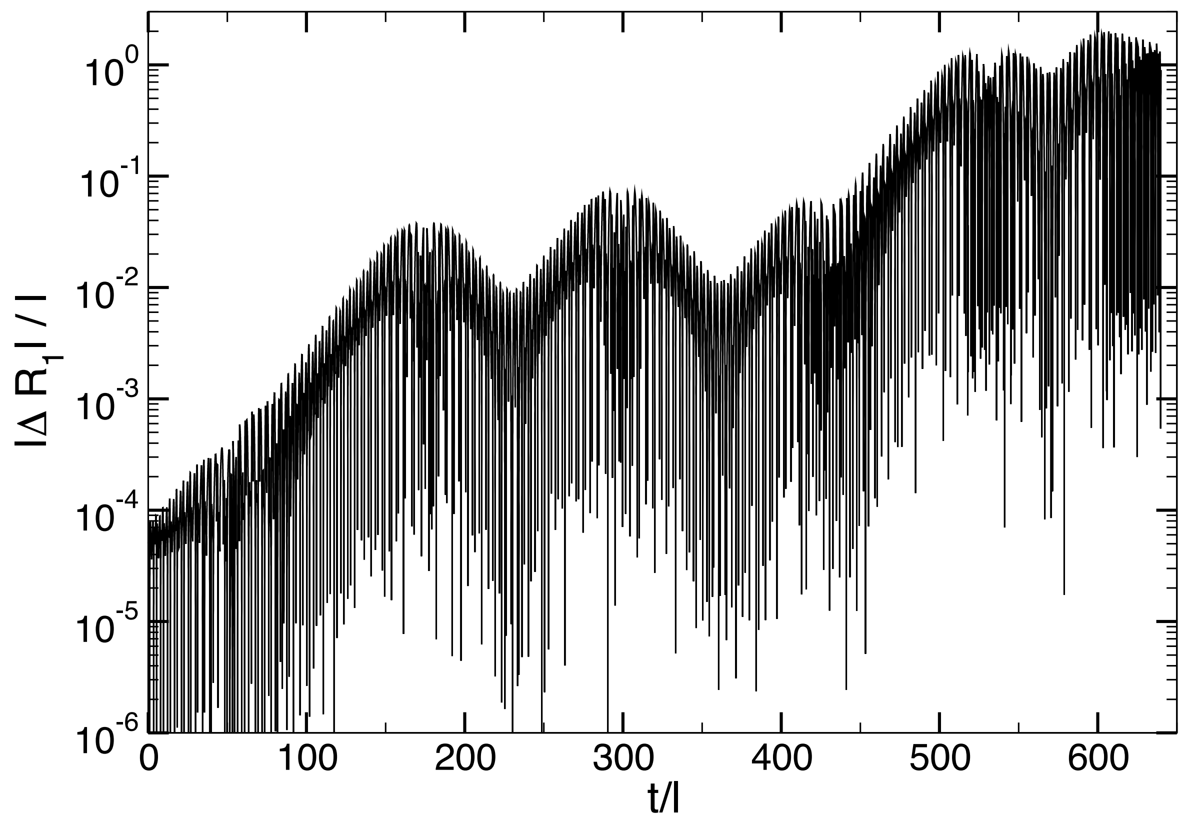

- Observe exponential growth in the separation between two initially nearby orbits. Eventually growth saturates because system is confined.

- Similar behavior was observed for a massless scalar field in AdS.

[Oliveira, Pando Zayas, Terrero-Escalante (2012)]
[Farahi, Pando Zayas (2014)]

Conclusion:

- The simplicity + richness afforded by this setup provides an **ideal testbed** for explorations of *gravitational collapse in confining geometries* and its *holographic dual interpretations*.

  The study of the dynamics of these systems only requires solving two decoupled ODEs.

- Depending on initial conditions one finds:
  — prompt collapse to a black hole
  — delayed collapse
  — perpetual oscillatory motion.

- We observe traces of **critical behavior** and **chaotic nature** in this setting, reminiscent of *confined scalar fields undergoing 'turbulent' dynamics*.

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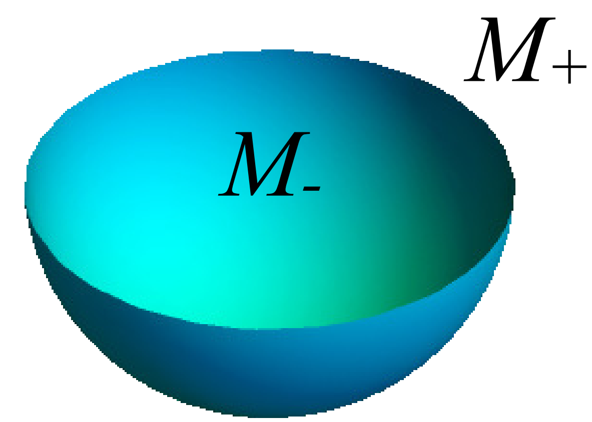
Thank you.

Extra: Junction conditions

- Start with a single spherical shell.
- Geometries inside and outside are both Schwarzschild-AdS:



$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2\,, \qquad f(r) = \left(1 - \frac{2M}{r} + \frac{r^2}{l^2}\right)$$

- Apply junction conditions for the timelike hypersurface, $t = T(\tau), \quad r = R(\tau)\,.$

induced metric →
$$\mathfrak{g}_{ij}^{(+)} = \mathfrak{g}_{ij}^{(-)} \equiv \mathfrak{g}_{ij}\,,$$

extrinsic curvature →
$$(k_{ij}^{(+)} - k_{ij}^{(-)}) - \mathfrak{g}_{ij}(k^{(+)} - k^{(-)}) = -8\pi G \mathcal{S}_{ij}$$
← shell's stress-energy tensor

- The 2nd junction condition dictates the form of the shell stress-energy tensor. It's a perfect fluid:

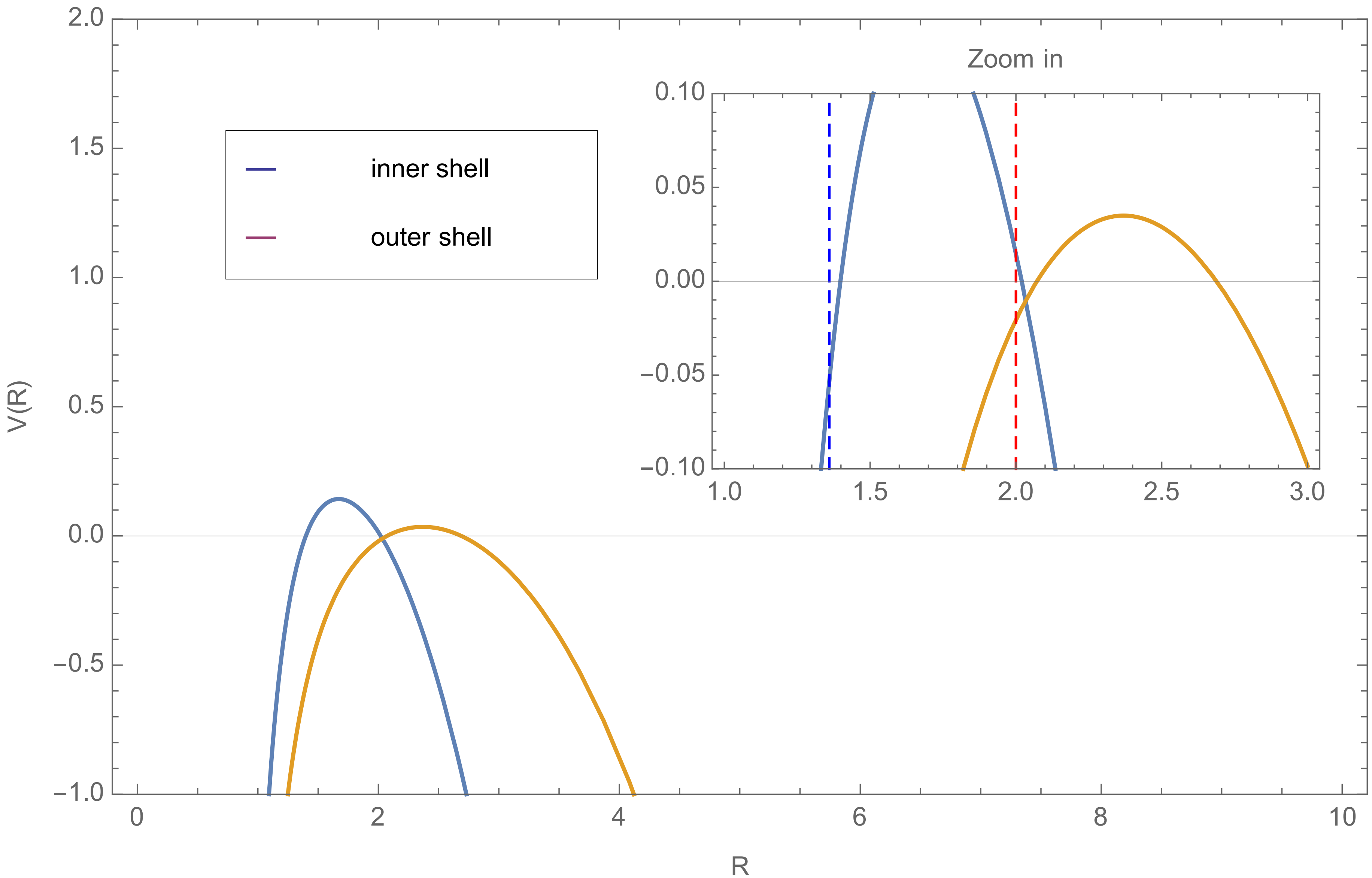
$$\mathcal{S}_{ij} = (\rho + P)u_i u_j + P\mathfrak{g}_{ij} \qquad (u = \partial_\tau)$$

energy density

pressure

Extra: Non collapsing configurations

- Self-sustained non collapsing configurations of multiple shells can exist, even without a confining mechanism.



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$m_1 = m_2 = 2.9$

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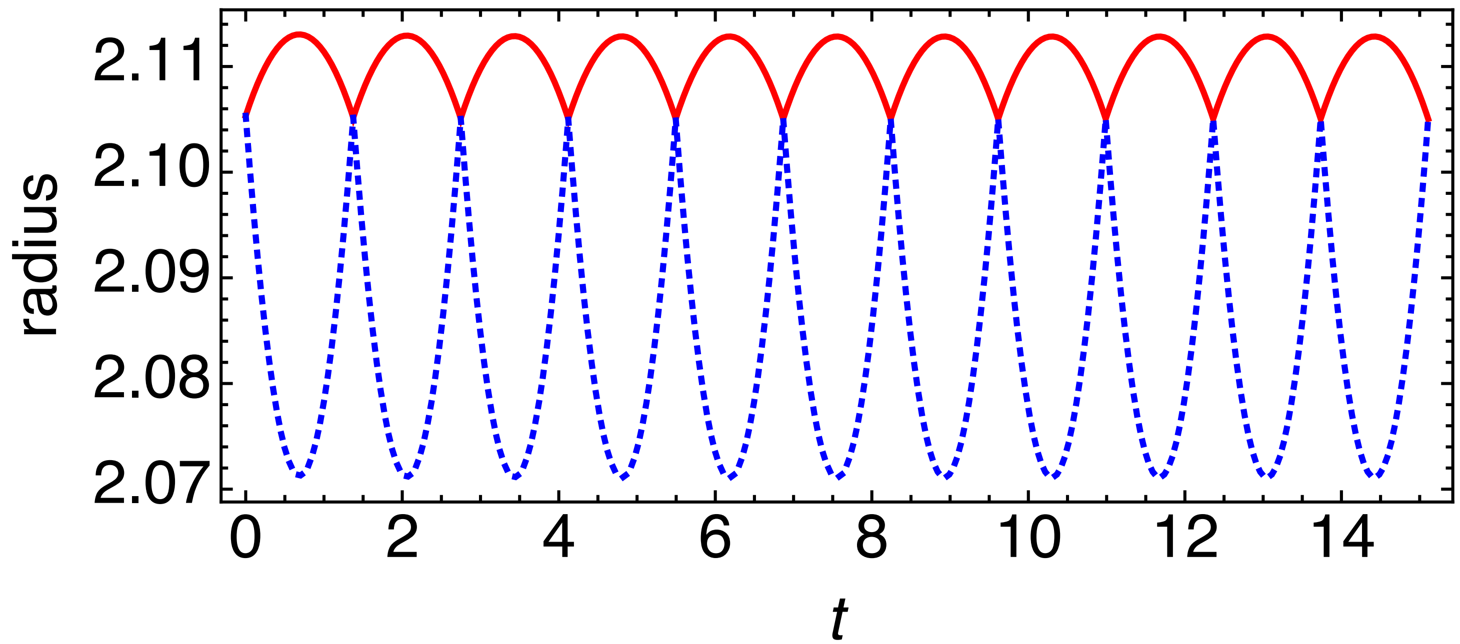

$M_1 = 1$

$M_2 = 0.663$

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Extra: Ida-Nakao formula

- Post-collision 'ADM' mass:

$$M_2' = -(\nu_1^2 + \nu_2^2)\frac{r_0}{4} + (f_2 - 1 - f_1)\frac{r_0}{4} \quad -(\nu_1^2 - 1)(\nu_2^2 - f_1)\frac{r_0}{4f_2} + \frac{r_0}{2}$$
$$-\epsilon_1\epsilon_2\frac{r_0}{4f_2}\left[\nu_1^4 - 2(1+f_2)\nu_1^2 + (1-f_2)^2\right]^{1/2}\left[\nu_2^4 - 2(f_1+f_2)\nu_2^2 + (f_1-f_2)^2\right]^{1/2}$$

expanding shell

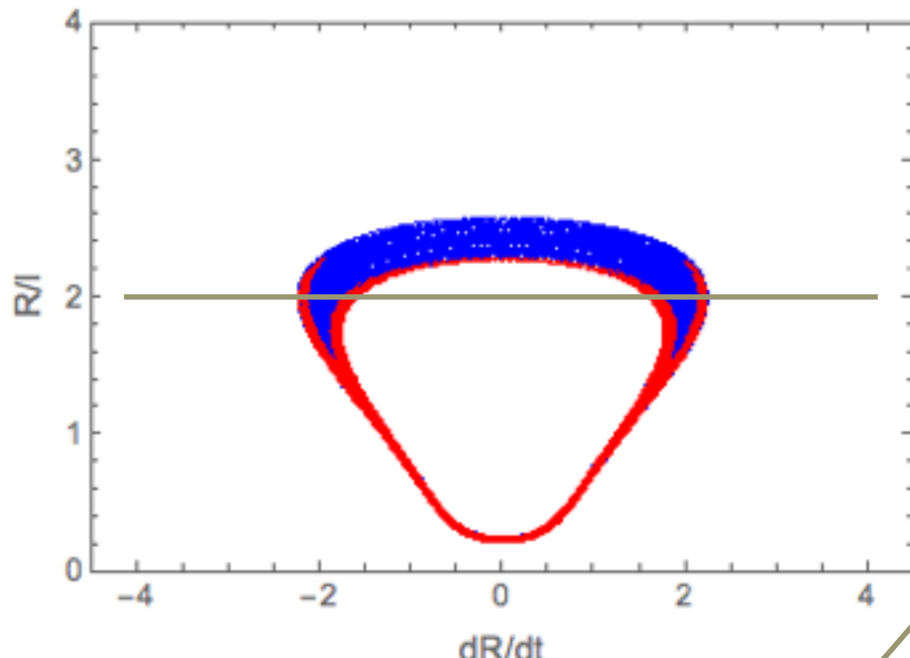
where $$f_i = 1 - \frac{2M_i}{r_0}\,, \qquad \nu_i = \frac{m_i}{r_0^{1+2w}}\,, \qquad \epsilon_i = \pm 1\,.$$

contracting shell

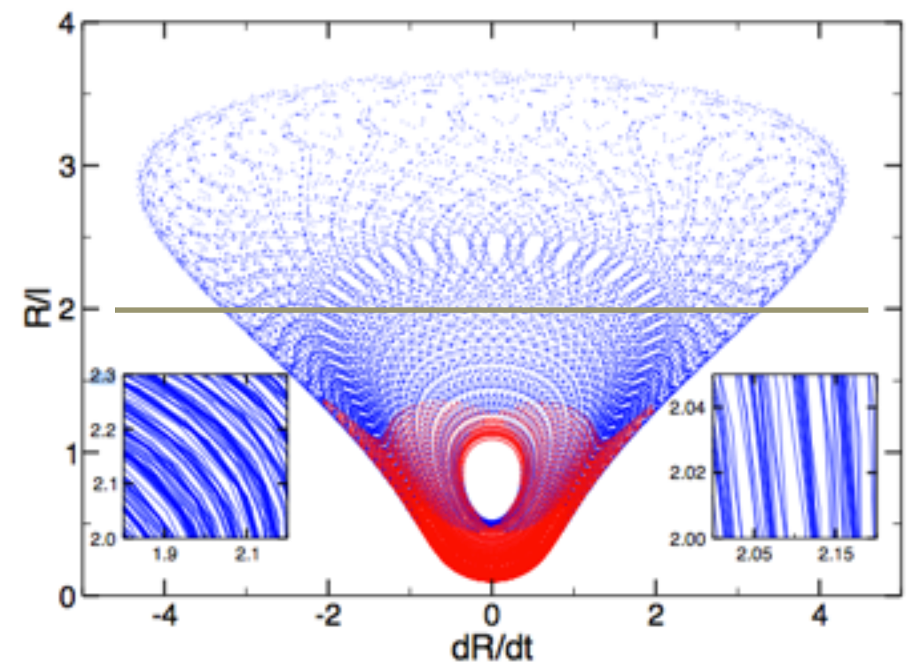
collision radius

Extra: Chaotic phase space

- Scan non-collapsing configurations: system is quasi-periodic in the low energy regime.



$\delta = 0.0005$



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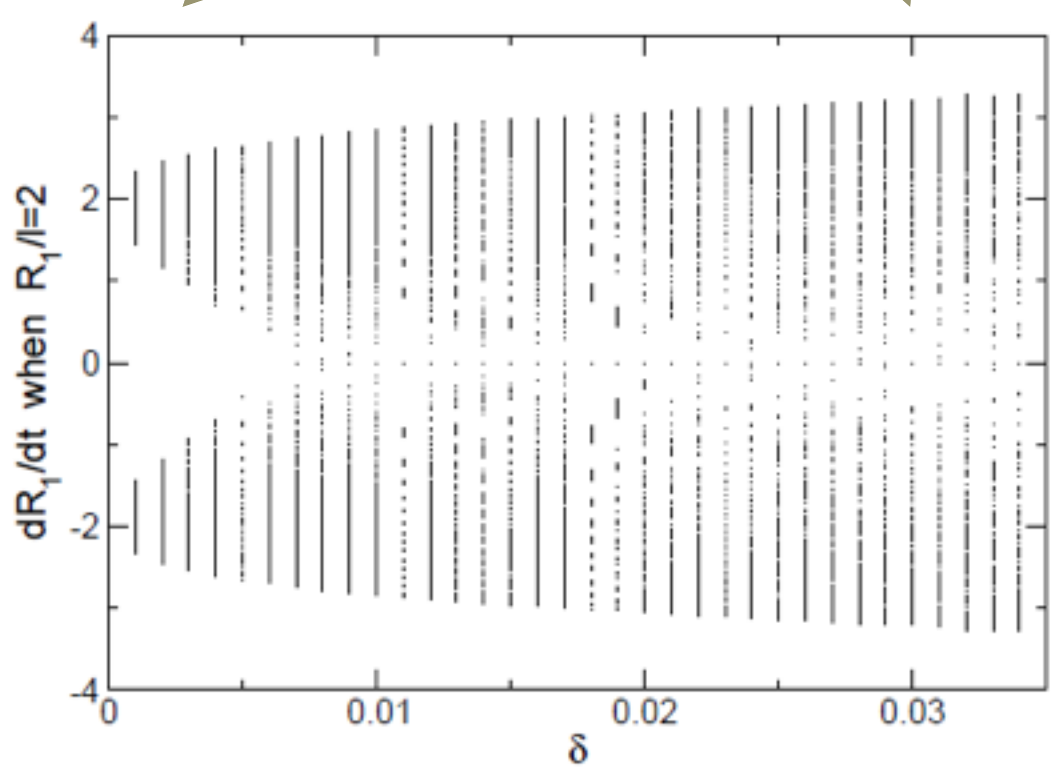

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