Constraining the Ultra-Large Scale Structure of the Universe Using Numerial Relativity

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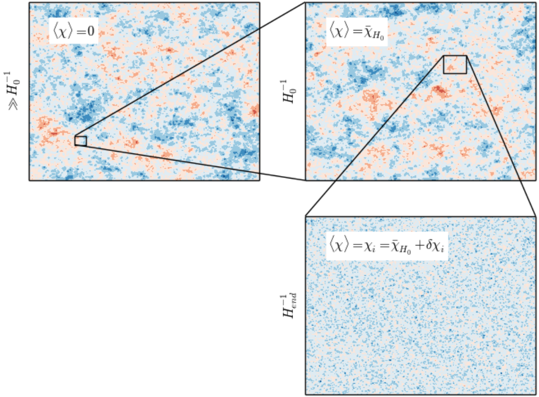
w/ **Hiranya Peiris**, **Matthew Johnson**, and Anthony Aguirre
based on arXiv:1604.04001 and *in progress*

Ultra-Large Scale Structure



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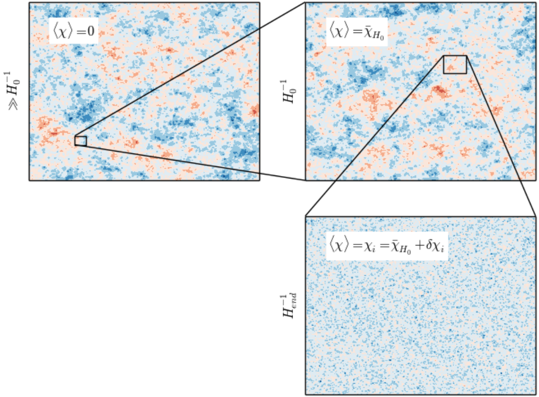
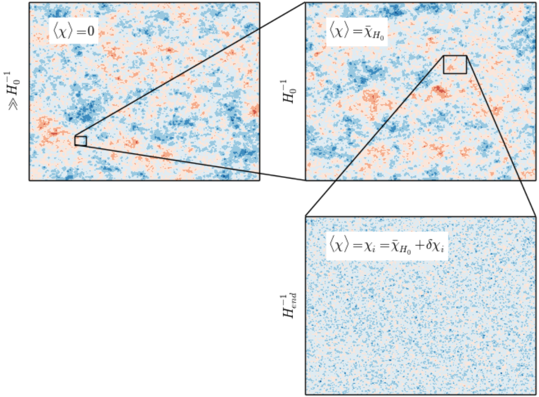


Local Remnants of Ultra-Large Scale Structure?

Ultra-Large Scale Structure



Local Remnants of Ultra-Large Scale Structure?

- Structure present at start of inflation
- Conversion of structure during or after inflation

Modelling Initial Conditions

Monte Carlo Sampling: Planar Symmetry

$$ds^2 = -d\tau^2 + a_{\|}^2(x,\tau)dx^2 + a_{\perp}^2(x,\tau)\left(dy^2 + dz^2\right)$$

Inflaton on $a_{\|}(\tau = 0) = 1 = a_{\perp}(\tau = 0)$

$$\phi(x) = \bar{\phi} + \delta\hat{\phi}$$

$$\bar{\phi} \text{ gives } \mathcal{N} \text{ e-folds} \qquad 3H_{\mathrm{I}}^2 \equiv V(\bar{\phi})$$

Field Fluctuations

$$\delta\hat{\phi}(x_i) = A_{\phi}\sum_{n=1}\hat{G}e^{ik_n x_i}\sqrt{P(k_n)} \qquad \hat{G} = \sqrt{-2\ln\hat{\beta}}e^{2\pi i\hat{\alpha}}$$

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$$P(k) = \Theta(k_{\rm max} - k) \qquad H_{\rm I}^{-1} k_{\rm max} = 2\pi\sqrt{3}$$

Model Choices

$$V(\phi) = V_0 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_P}}\right)^2$$

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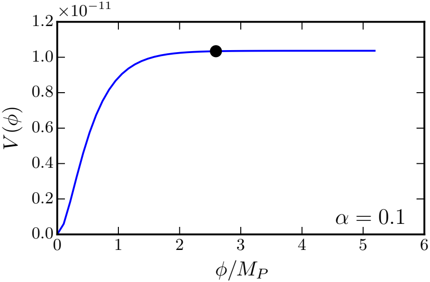

- $\alpha \ll 1 \rightarrow$ small-field model

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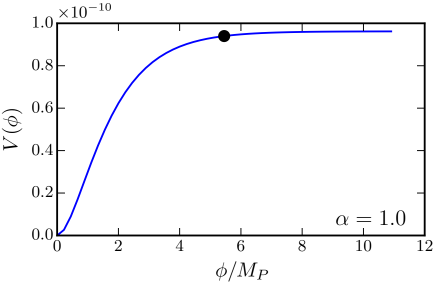

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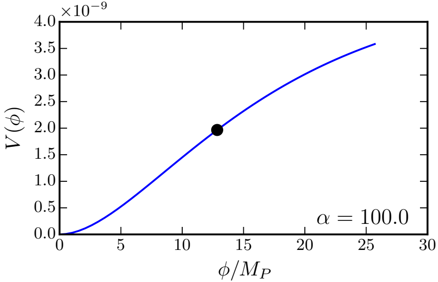

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- $\alpha = 1 \to$ Starobinski
- $\alpha \gg 1 \to m^2\phi^2/2$

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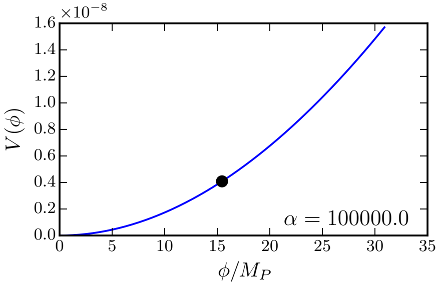

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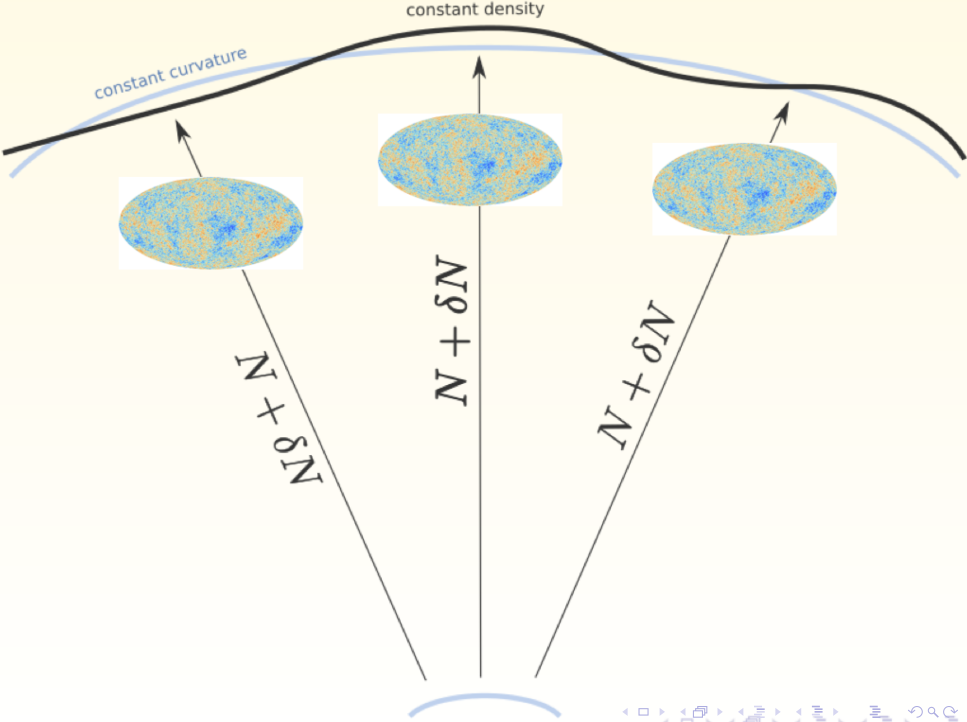
constant density
constant curvature
$N + \delta N$
$N + \delta N$
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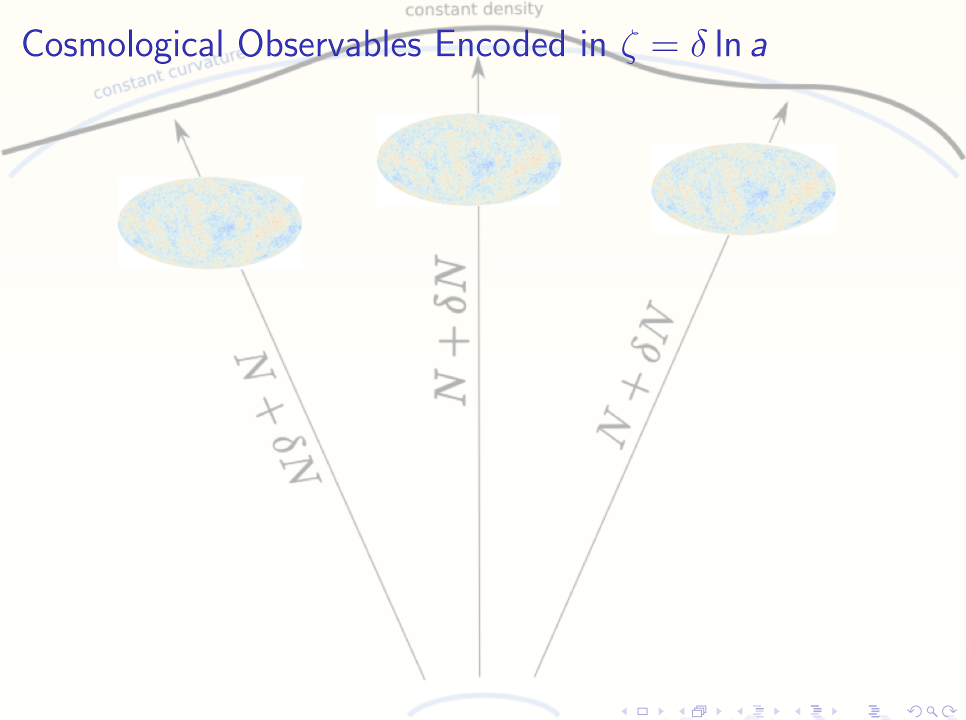
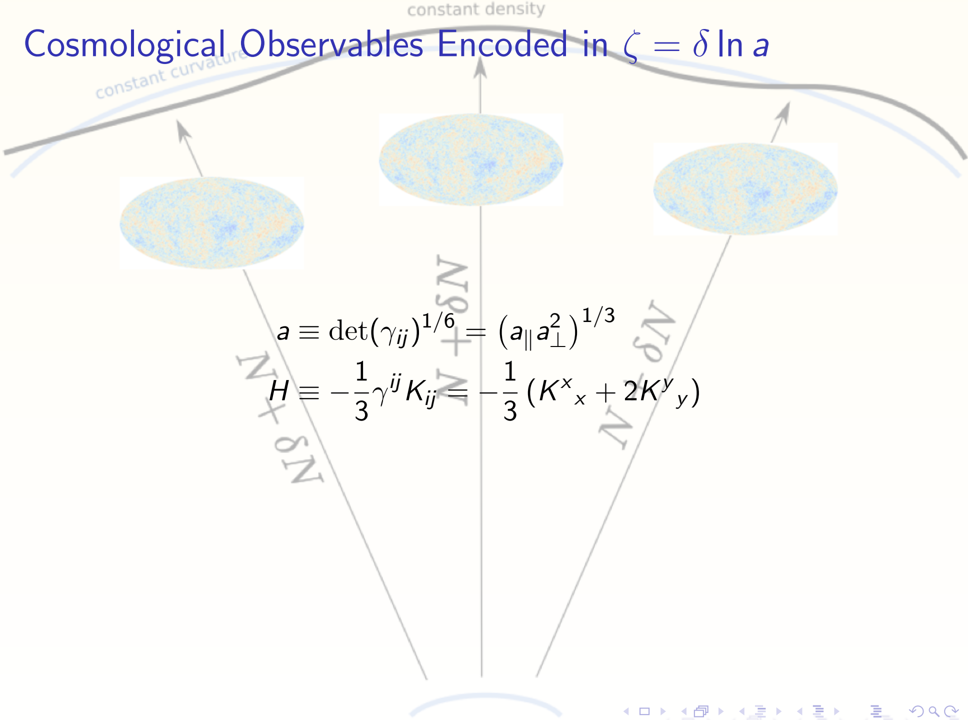
Cosmological Observables Encoded in $\zeta = \delta \ln a$



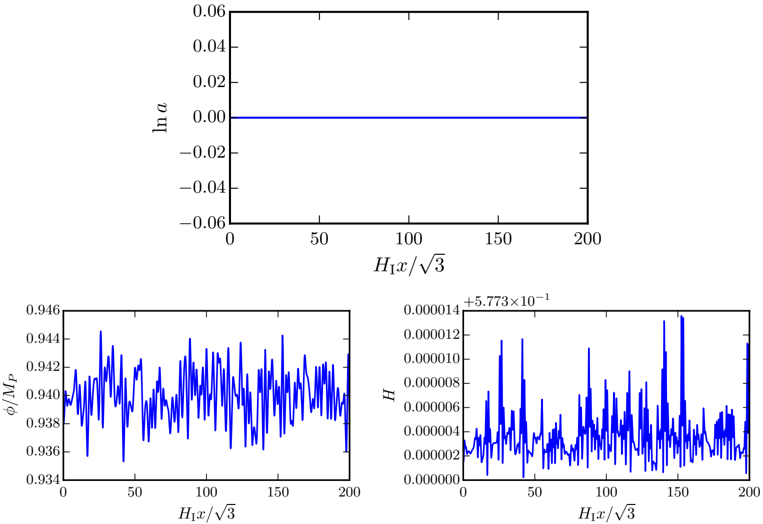
Cosmological Observables Encoded in $\zeta = \delta \ln a$



$$a \equiv \det(\gamma_{ij})^{1/6} = \left(a_{\parallel} a_{\perp}^2\right)^{1/3}$$

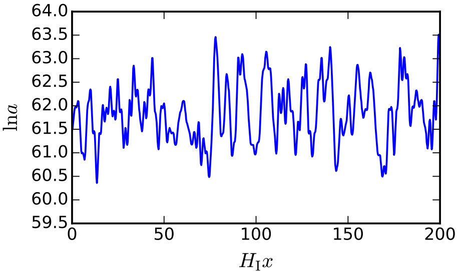
$$H \equiv -\frac{1}{3}\gamma^{ij} K_{ij} = -\frac{1}{3}\left(K^x{}_x + 2K^y{}_y\right)$$

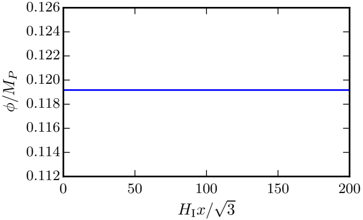
Required Evolution

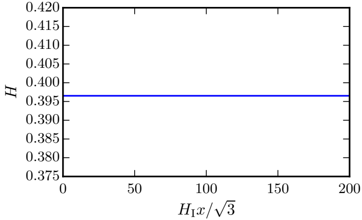


Initial Conditions $(\tau = 0)$

Required Evolution







End of Inflation ($\epsilon_H = -d \ln H / d \ln a = 1$)

Solving Einstein's Equations for a Self-Gravitating Inflaton

Machine precision accuracy

- Gauss-Legendre time-integrator ($\mathcal{O}(dt^{10})$, symplectic)
- Fourier pseudospectral discretisation (exponential convergence)

Fast to allow sampling

- Adaptive time-stepping
- Adaptive grid spacing

$\mathcal{O}(1s - 10s)$ **to evolve through 60 e-folds of inflation**
Machine precision convergence and constraint preservations

Observational Constraints

$$\mathrm{Pr}(A_\phi, H_I L_{\rm obs} | C_2^{\rm obs}, \dots) \propto \mathcal{L}(A_\phi, H_I L_{\rm obs}) \mathrm{Pr}(A_\phi, H_I L_{\rm obs} | \dots)$$

$$\mathcal{L} = \mathrm{Pr}(C_2^{\rm obs} | A_\phi, H_I L_{\rm obs}, \dots)$$

- A_ϕ : Fluctuation Amplitude $P(k) \propto A_\phi^2$
- $H_I L_{\rm obs}$: Uncertain post-inflation expansion history
- $\dots$: $V(\phi)$, spectrum shape, IC hypersurface, $C_2^{\rm high-\ell}$, etc.

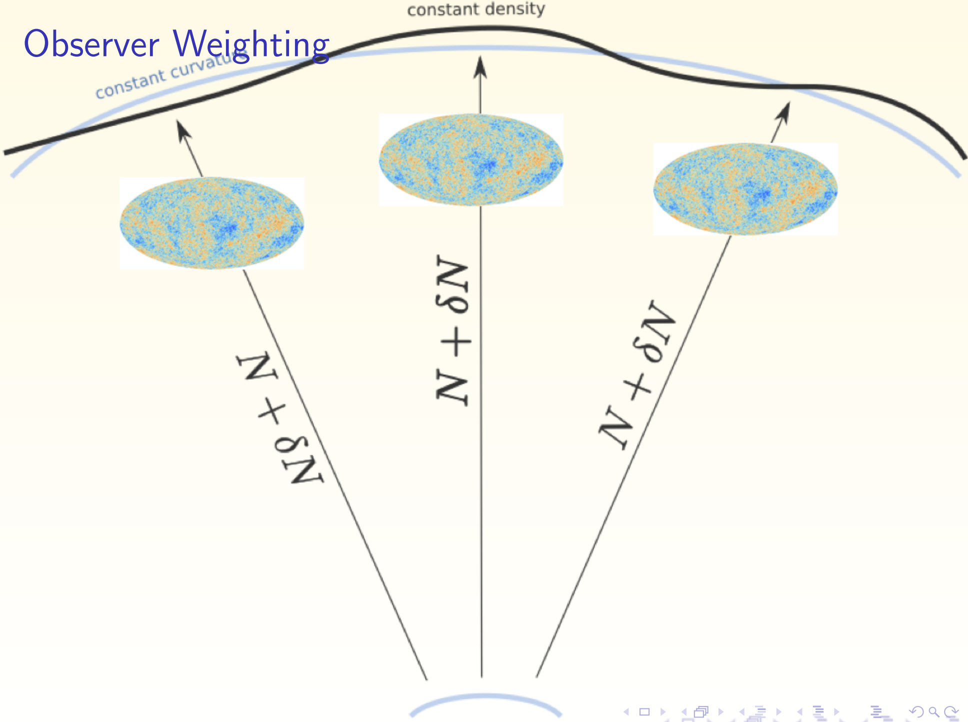
Planck measured C_ℓ

$$C_2^{\rm obs} = 253.6 \mu K^2 \qquad C_2^{\rm high-\ell} = 1124.1 \mu K^2$$

Numerical GR Input

$$\mathrm{Pr}\left(\hat{C}_2 | A_\phi, H_I L_{\rm obs}, \dots\right)$$

Observer Weighting



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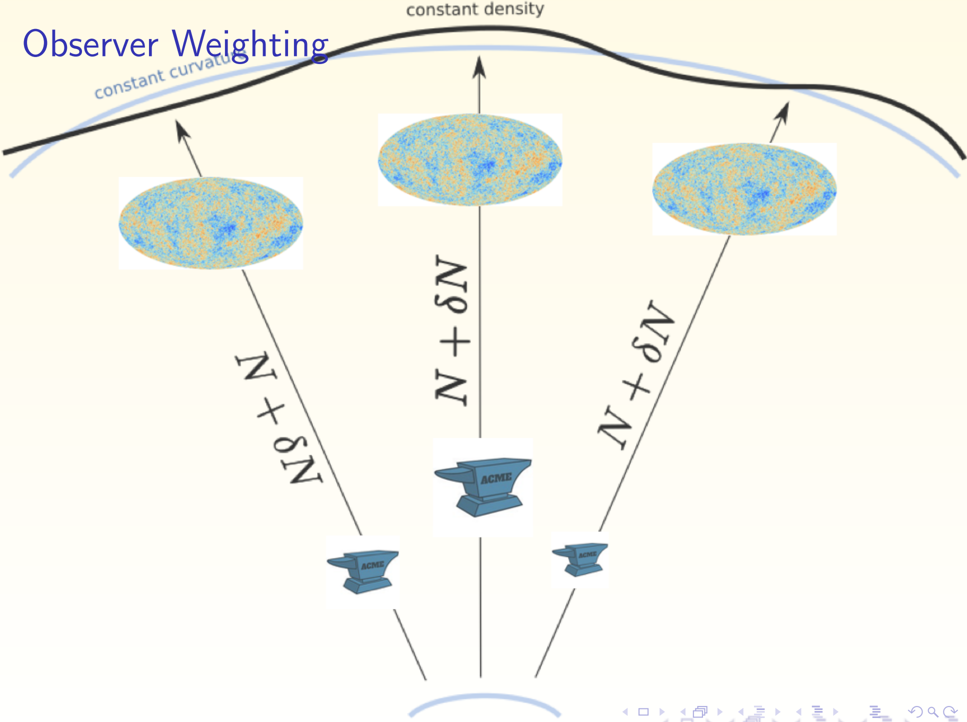

Evaluation of CMB Quadrupole

$$\hat{C}_2 = \frac{1}{5}\left[\left(a_{20}^{(\mathrm{UL})} + a_{20}^{(\mathrm{Q})}\right)^2 + \sum_{m=-2,m\neq 0}^{m=2}\left(a_{2m}^{(\mathrm{Q})}\right)^2\right]$$

Evaluation of CMB Quadrupole

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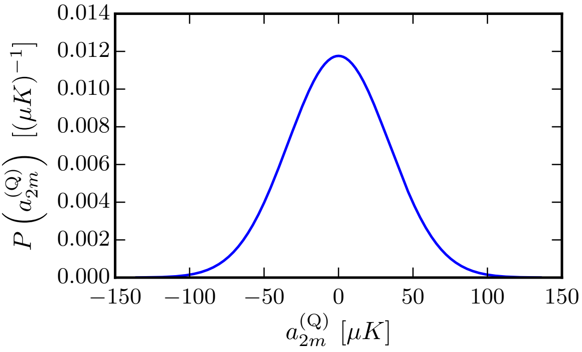
$$a_{2m}^{(\mathrm{Q})} : \text{ Gaussian with } \langle (a_{2m}^{(\mathrm{Q})})^2 \rangle = 1124.1 \mu K^2$$



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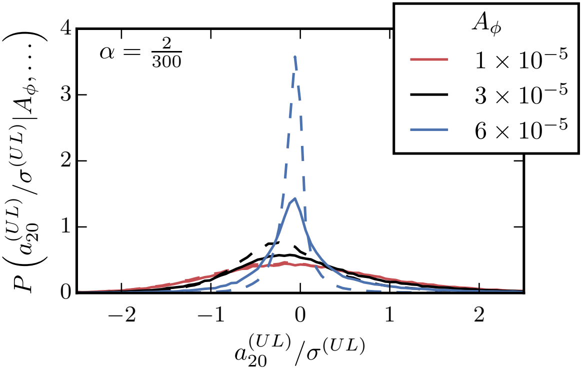
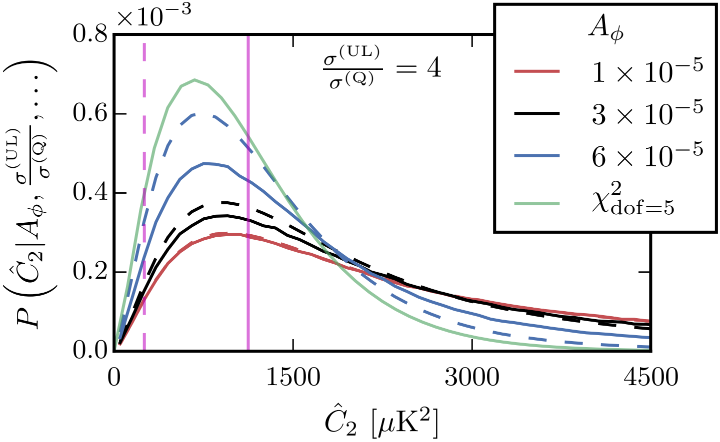
$a_{20}^{(\mathrm{UL})}$: NR simulations



Dependence of C_2 on Model Parameters



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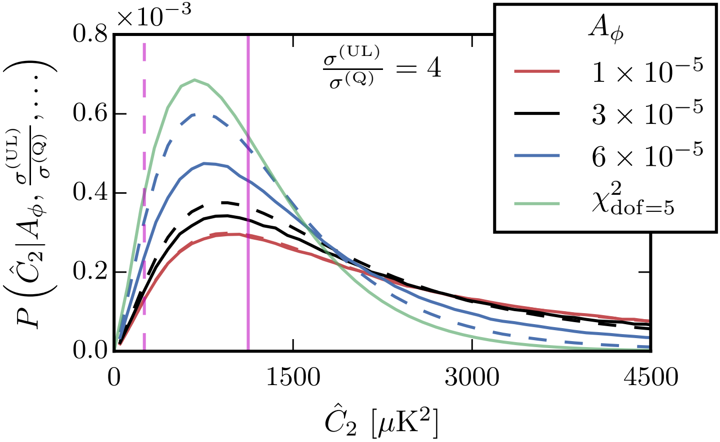
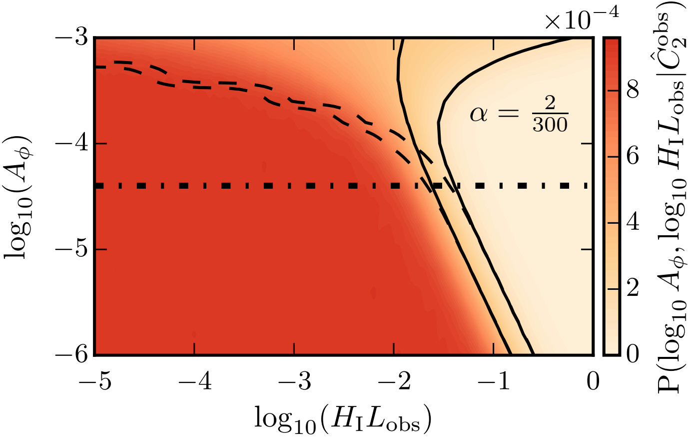


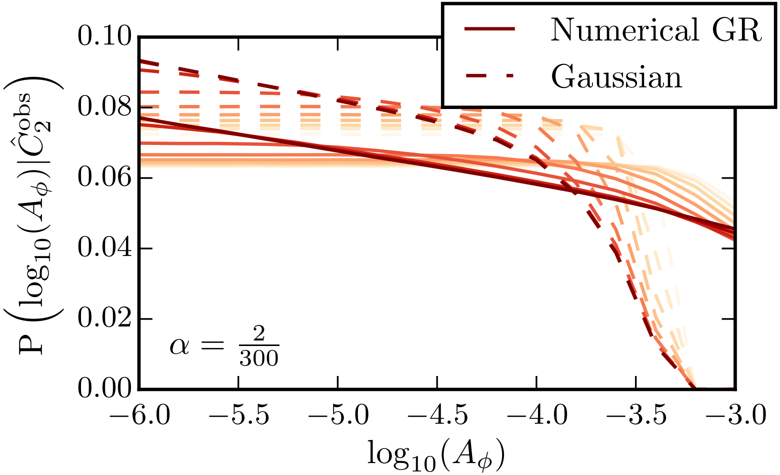
$$a_{20}^{(\mathrm{UL})} \approx F\left(H_I L_{\mathrm{obs}}\right)^2 \partial_{x_p x_p} \zeta \approx F\left(H_I L_{\mathrm{obs}}\right)^2 \frac{1}{a_\parallel} \frac{\partial}{\partial x} \left(\frac{1}{a_\parallel} \partial_x \zeta\right)$$

Final Posterior



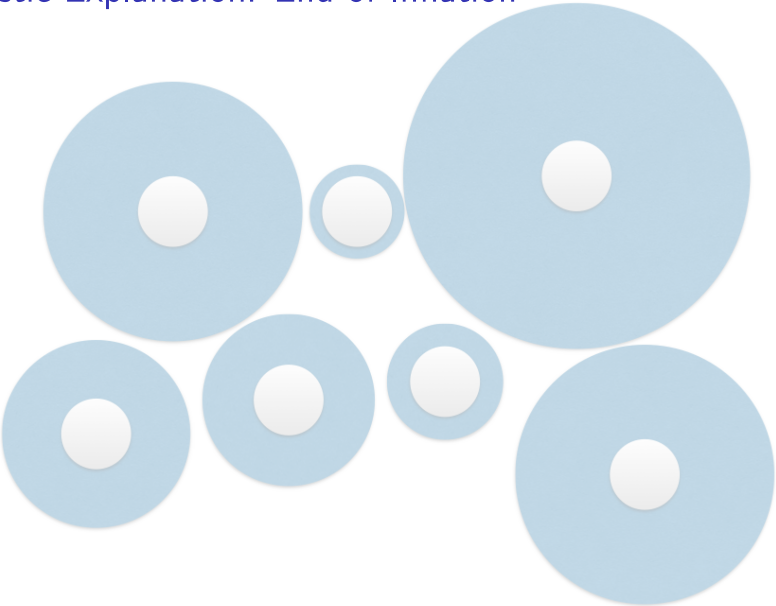
Significant deviations from Gaussian approximation

Marginalised Constraints on Model Parameters: Amplitude



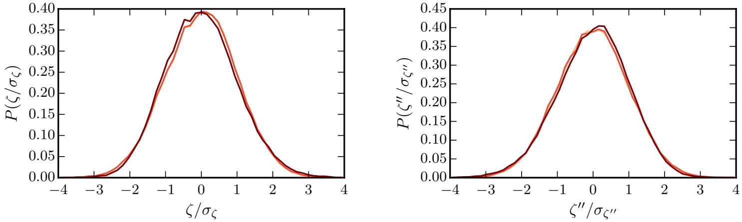
Heuristic Explanation: Initial Conditions

Heuristic Explanation: End-of-Inflation



Analytic Approximation

ζ and comoving derivatives nearly Gaussian



Treat as Gaussian random field

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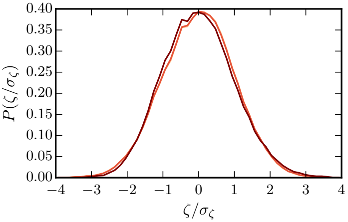
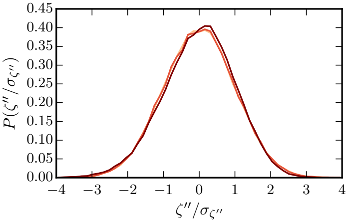


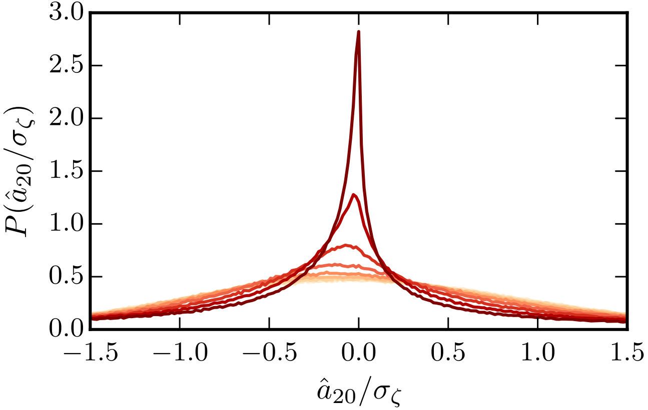


Treat as Gaussian random field

Large-Scale Approximation for a_{20}

$$a_{20}(x_0) \approx \mathcal{A} e^{-2\zeta(x_0)} \left(\zeta''(x_0) - \zeta'(x_0)^2\right)$$

Anaytic a_{20} Distributions



Vary σ_ζ at fixed $\sigma_{\zeta^{(p)}}/\sigma_\zeta$

Conclusions

- Numerical relativity is a useful framework for making cosmological predictions
  - Sometimes it is a *necessary* tool (deviations from Gaussianity)
- Robust qualitative conclusions over a variety of inflationary models
- Inflation is effective at hiding large amplitude initial fluctuations
- Gaussianity of ζ in comoving coordinates suggests analytic approach in 3D