The vector fields method for Vlasov fields

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Joint work with
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Stability problem

- Problem at hand : Stability of the Minkowski spacetime as a solution to the Einstein-Vlasov system
- In the right gauge : System of quasilinear wave equations coupled with a geometric transport equation
- Stability of Minkowski spacetime : Christodoulou-Klainerman (91) ; Lindblad-Rodnianski (05) : perturbative proof of the Minkowski stability ;
- Stability (spherical symmetry) : Rein-Rendall (92) for massive part., Dafermos (02) for massless part. ;
- Stability (no symmetry) : M. Taylor (2015), for massless part., through the control of the Jacobi equations ;
- Aim here : develop the tools to obtain a pertubative proof of the full stability of Minkowski for massive particles.

Weighted decay estimates for the massive transport equation

- Massive transport equation : $f : \mathbb{R}_{x^0} \times \mathbb{R}^n_x \times \mathbb{R}_v \to \mathbb{R}$ matter density :
$$Tf = v^0 \partial_{x^0} f + v^i \partial_{x^i} f \text{ with } v_i v^i + v_0 v^0 = -1$$
- Conservation of energy : $\int_x \int_v f dx dv = \text{constant}$
- Commutators with T : lifts of the Killing fields to the tangent bundles :
$$\partial_{x^\alpha}, x_\alpha \partial_{x^\beta} - x_\beta \partial_{x^\alpha} + v_\alpha \partial_{v^\beta} - v_\beta \partial_{v^\alpha}$$
- Use hyperboloidal foliation : H_ρ, $\rho = (x^0)^2 - x_i x^i$, for $\rho \geq 1$;
- Establish weighted $L^\infty - W^{1,n}$- Sobolev estimates for the velocity averages :
$$\int_v |f| \frac{dv}{v^0} \lesssim t^{-n} E_n[f](\rho)$$
- Altogether, decay of the velocity average :
$$\int_v |f| \frac{dv}{v^0} \lesssim t^{-n} E_n[f](\rho = 1)$$

The Vlasov-Nordström model

- Arise as a linearization of the Einstein-Vlasov system near the flat metric for a conformally flat metric in the right gauge + rescaling and changes of variable ;
- Studied (global existence, including some decay estimates), by Calogero-Rein (02 - 05) ;
- Linearized Einstein equations :

$$\Box\phi = -\int_v f \frac{dv}{v^0}$$

- Geodesic spray for the conformally flat metric :

$$T_\phi f = Tf - \left(T\phi v^i + \nabla^i \phi\right) \partial_{v^i} f - 4(n+1) T\phi f = 0$$

- Commute with the lift of the Killing fields of the Minkowski spacetime.

Theorem : Global existence - asymptotic behavior for small data (FJS - 2015)

Let $N > 3n+4$; Assume that the initial data on H_1 are small enough

$$E_{N+n}[f](1) + \mathcal{E}_N[\phi](1) \leq \epsilon;$$

- Then, for all $\rho > 1$, $E_N[f](\rho) \lesssim \epsilon \rho^{C\epsilon^{1/4}}$ and $\mathcal{E}_N[\phi](\rho) \lesssim \epsilon$;
- Furthermore,

$$\int_{H_\rho} \frac{t}{\rho} \left(\int_v \widehat{Z}^N f \frac{dv}{v^0} \right)^2 d\mu_{H_\rho} \lesssim \epsilon^2 \rho^{C\epsilon^{1/4} - n}$$

A glimpse on the 3 dimensional case

- Ongoing with D. Fajman and J. Smulevici.
- Work with the standard decay for the wave equation coming from the vector field method :

$$|\partial\phi| \lesssim (1+|t-r|)^{-\frac{1}{2}}(t+r)^{-1}$$

- Improve decay for the massive field in the outgoing directions

$$\int_v (v^0)^k |f| \frac{dv}{v^0} \lesssim t^{-n} \left(\frac{u}{v}\right)^k$$

- Improve the commutators with the transport equations :

$$\hat{Z} \to \hat{Z} + \Phi_Z Z$$

- Include other relevant commuting quantities ;
- Expected (ongoing) :

$$|\partial\phi| \sim (1+|t-r|)^{-\frac{1}{2}}(t+r)^{-1} \text{ and } \int_v |\hat{Z}^\alpha f| \frac{dv}{v^0} \sim t^{-3+C\epsilon}$$

where ϵ is the energy of the initial data.

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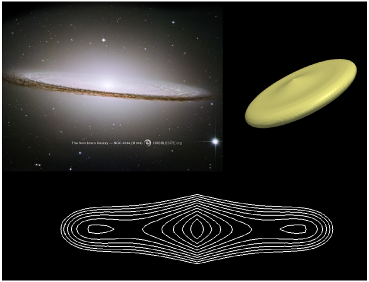
Workshop :	Geometric Transport Equations in General Relativity
Dates :	Feb. 20-24, 2017
Place :	Erwin Schrödinger Institute for Math. and Phys. Vienna, Austria
Organizers :	H. Andréasson, D. Fajman, J. Joudioux
Website :	http ://homepage.univie.ac.at/jeremie.joudioux/esi/



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