Analytic models for compact binaries
spin and **dynamic tides**

Jan Steinhoff

in collaboration with Tanja Hinderer, Andrea Taracchini, and Alessandra Buonanno

Steinhoff etal, in preparation   Hinderer etal, PRL **116** (2016) 181101

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Results for post-Newtonian approximation with spin

conservative part of the motion of the binary; see talk by M. Levi on Monday

post-Newtonian (PN) approximation: expansion around $\frac{1}{c} \to 0$ (Newton)

order	c^0 N	c^{-1}	c^{-2} 1PN	c^{-3}	c^{-4} 2PN	c^{-5}	c^{-6} 3PN	c^{-7}	c^{-8} 4PN
non spin	✓		✓		✓		✓		✓
spin-orbit				✓		✓		✓	
S_1^2					✓		✓		✓
$S_1 S_2$					✓		✓		✓
Spin^3								✓ (✓)	
Spin^4									✓ (✓)
⋮									⋱

✓ known (✓) partial ✓ derived last year

Work by many people ("just" for the spin sector): Barker, Blanchet, Bohé, Buonanno, O'Connell, Damour, D'Eath, Faye, Hartle, Hartung, Hergt, Jaranowski, Marsat, Levi, Ohashi, Owen, Perrodin, Poisson, Porter, Porto, Rothstein, Schäfer, Steinhoff, Tagoshi, Thorne, Tulczyjew, Vaidya

Zones, separation of scales, and effective theory

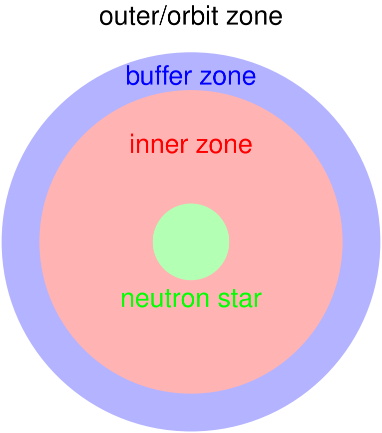
EFT program in classical gravity: Goldberger, Rothstein, PRD **73** (2006) 104029; . . .

various zones $\rightarrow$ separation of scales

scales continue down the star:
$\rightarrow$ fluid, nucleons, quarks, ?

The physics at "smaller" scales admits
an Effective Field Theory (EFT) description!

Here: Effective theory for dynamical tides
$\rightarrow$ dynamical, time-dependent response
(of the inner zone to perturbations from the outer zone)
$\rightarrow$ harmonic oscillator effective theory for multipoles



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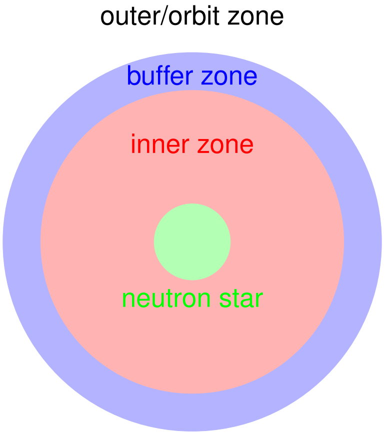

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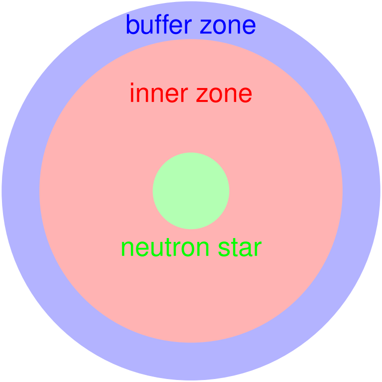

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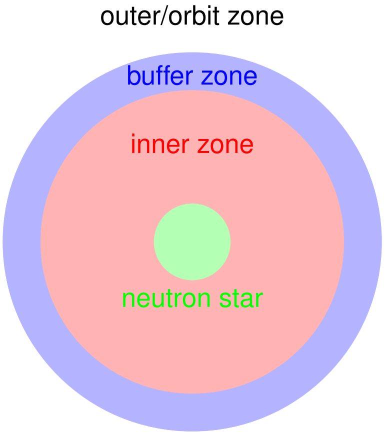

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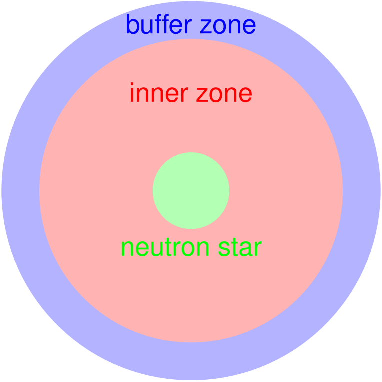

Dynamical tides in general relativity

Their description through an effective action [JS, Hinderer, Taracchini, Buonanno, in preparation]

Relativistic effective Lagrangian for dynamical tides: $Q_{\mu\nu} u^\nu = 0$

$$L_Q = \frac{z}{4\lambda\omega_f^2}\left[\frac{1}{z^2}\frac{DQ_{\mu\nu}}{d\sigma}\frac{DQ^{\mu\nu}}{d\sigma} - \omega_f^2 Q_{\mu\nu}Q^{\mu\nu}\right] - \frac{z}{2}E_{\mu\nu}Q^{\mu\nu} + \frac{z}{4}K\,E_{\mu\nu}E^{\mu\nu} + \ldots$$

$$u^\mu = \frac{Dx^\mu}{d\sigma}, \qquad z = \sqrt{-u^\mu u_\mu} \quad \text{(is the redshift for } \sigma = t\text{)}$$

- Newtonian case: [Flanagan, Hinderer, PRD **77** (2008) 021502]
- λ is the tidal deformability (Love number)
- identify ω_f with real part of quasi-normal-mode frequency
- K linked to (almost) completeness of modes: $K \approx 0$

ω_f and K are not fixed by a matching, but by physical intuition!

a prescription for the dynamical response is in Chakrabarti, Delsate, JS, arXiv:1304.2228

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What are the genuine relativistic effects?

- redshift effect
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Both effectively shift
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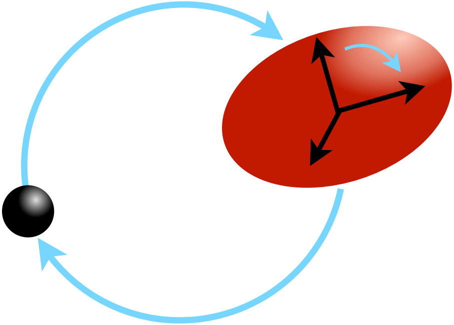

frame of the neutron star is dragged
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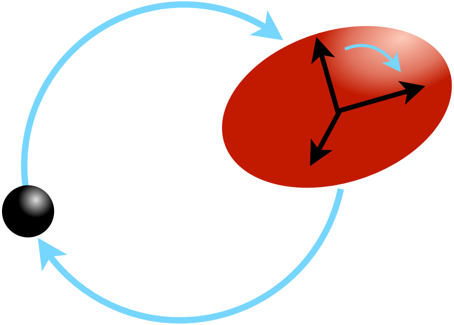

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Computing the post-Newtonian (PN) corrections

Frame dragging interaction

tidal spin: $S_Q^{ij} = 4Q^{k[i}P^{j]k}$
generates infinitesimal rotations
$\rightarrow$ frame dragging

substitute $S^{ij} \rightarrow S_Q^{ij}$ in known potentials! $\rightarrow$ lazy

The tidal driving force

$$\text{tidal:} \quad -\frac{1}{2}E_{\mu\nu}Q^{\mu\nu} \qquad \text{vs.} \qquad \text{spin induced:} \quad \frac{C_{\mathrm{ES}^2}}{2m}E_{\mu\nu}S^{\mu}S^{\nu}$$

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super lazy!!! agrees with Vines, Flanagan, PD **88** (2013) 024046

Harder: implementation into effective-one-body, analyze various models, ...

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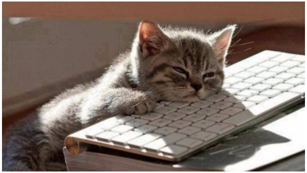

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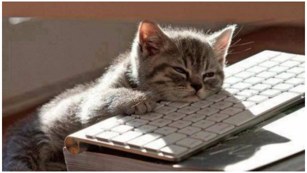

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Conclusions

All you need is λ! ?

Almost, need more coefficients
linked to dynamical tides!

λ, ω_f, K, ...

Dynamical tides become important
close to resonance with ω_f

Increase tidal effect by $\sim$ 30%!

Dynamical tides are important for accurate waveform models

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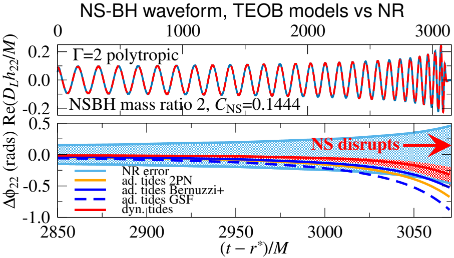

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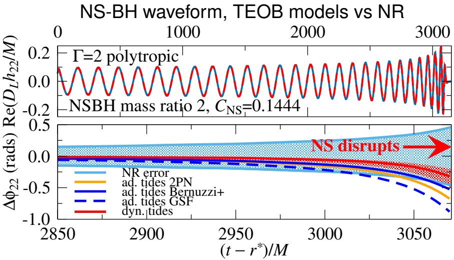

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Effective-one-body Hamiltonian

for 1PN dynamical tides, see also Hinderer etal, PRL **116** (2016) 181101

- effective test-particle Hamiltonian (point-mass potentials A, D)

$$H_{\text{eff}} = \sqrt{(A + \mathcal{E}_{ij} Q^{ij}) \left[\mu^2 \left(1 + \frac{2}{\mu} z_c H_o + \mathcal{C}_{ij} Q^{ij} \right) + \frac{p_\phi^2}{r^2} + \frac{p_r^2}{D} + \mathcal{O}(p_r^4) \right]} + f_{\text{DT}}$$

- oscillator Hamiltonian: $H_o = \lambda \omega_f^2 P_{ij} P_{ij} + \frac{Q^{ij} Q^{ij}}{4\lambda}$
- 1PN tidal force $X_A = m_A/M, \quad M = m_1 + m_2, \quad \nu = X_1 X_2, \quad \mu = M\nu, \quad u = M/r$

$$\mathcal{E}_{ij} = -\frac{3Gm_2}{\mu r^3} n^i n^j \left\{ 1 - [2X_2 - (1 - c_1)\nu] u \right\}$$

$$\mathcal{C}_{ij} = \frac{3Gm_2}{\mu^3 r^3} \left\{ \frac{L^i L^j}{r^2} + [1 + (c_2 - 2c_1)\nu] \, n^i p_j p_r + \left[(1 - c_1) p^2 + (5c_1 - c_2) p_r^2 \right] \nu n^i n^j \right\}$$

- gauge parameters c_1, c_2. blue term: no gauge parameters!
- redshift factor (normalized to 1 for $m_1 \ll m_2$)

$$z_c = 1 + \frac{3}{2} X_1 u + \frac{\nu}{2} (1 + 2c_1) \left[\frac{p^2}{\mu^2} - u \right]$$

- frame dragging terms $\sim$ spin-orbit + corotating frame, "$S_Q = Q \times P$"

$$f_{\text{DT}} = -\vec{S}_Q \cdot \vec{L} \frac{1}{\mu^2 r^2} \left\{ 1 + [3X_1 - 5 - (1 + c_2)\nu] \frac{u}{2} - (1 - c_2\nu) \frac{p^2}{2\mu^2} - c_2 \nu \frac{p_r^2}{\mu^2} \right\}$$

Test-particle and effective-one-body Hamiltonians

Test-particle Hamiltonian 101:

- get mass-shell constraint: $0 = \mu^2 + p^\mu p_\mu +$ tidal terms, $p_\mu = \dfrac{\partial L}{\partial u^\mu}$
- solve for the energy $H \equiv -p_0$

Absorb interaction into the metric:

- notice $E \propto p^2$
- factorize p^2 terms: $0 = \mu^2 + 2\mu H_{\text{oszi}} + \underbrace{\left[g^{\mu\nu} - \frac{1}{2\mu^2} R^{\alpha\mu\beta\nu} Q_{\mu\nu} \right]}_{g_{\text{eff}}^{\mu\nu}} p_\mu p_\nu$
- also works for higher multipoles

When used for EOB: no pole at the light ring in H
pole can be always by removed Akcay, etal, PRD 86 (2012) 104041
but also no gauge-invariant centrifugal radius