Comparing Spacetime using Geometric Scalars

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21st International Conference on General Relativity and Gravitation
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July 11, 2016

Outline

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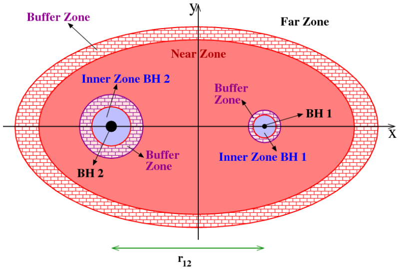
   - Motivation for comparison
   - Technique for the comparison: Geometric Scalars
2. Spacetime Comparison
   - Geometric scalars
   - Scalar analysis
   - Test of Geometric scalars
3. Where to go next...
   - Summary

What to compare

Analytic Spacetime

Using Approximations

- Post Minkowskian
- Post Newtonian
- Black hole perturbations

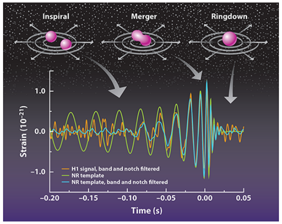


Credit: Johnson-Mcdaniel et al, 2009

Numerical Spacetime

Solving EFEqs on supercomputers

- 3+1 decomposition
- BSSN and other
- Numerical techniques



Credit:Pretorius, APS/Carin Cain

Remark

Numerical Spacetimes are accurate. Analytical spacetimes are approximately similar to Numerical spacetimes.

Why compare

Analytic Spacetime

- Approximations
- Computationally cheaper
- Accuracy limits
- Useful to study a variety of astrophysical phenomena

Numerical Spacetime

- Fully Relativistic
- Computationally expensive
- Accurate
- Limited by finite-resolution, inexact initial conditions

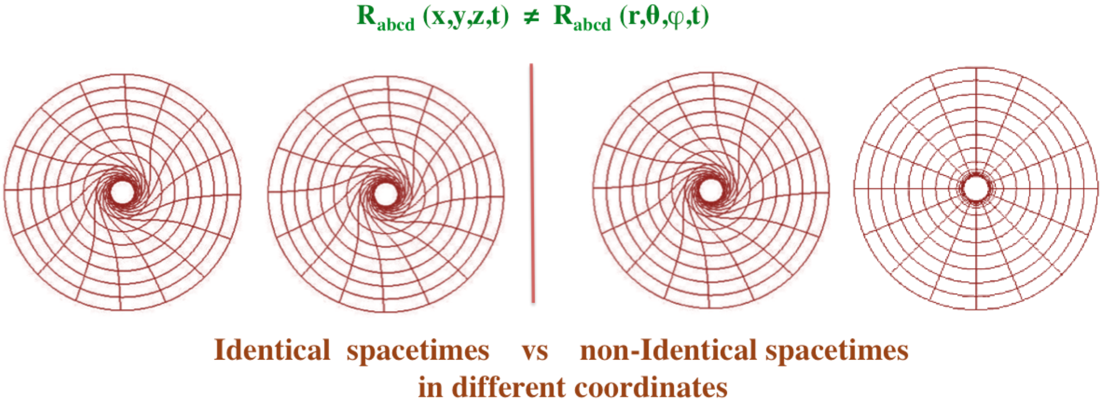
Remark

Numerical spacetimes are expensive and our goal is to understand how to improve the accuracy of analytic spacetimes that can be cheaper but as accurate everywhere as numerical spacetimes. This can be done if we understand how analytic spacetimes are different from numerical spacetimes.

How to compare

- To compare spacetimes we need to focus on how spacetimes differ at different phases of binary evolution
- This can be done using geometric quantities like curvature tensor
- We cannot use R_{abcd} to do the comparison because it is not gauge invariant

The Riemann tensor describes the curvature of spacetime. Components of the Riemann tensor differ in different coordinates even for identical spacetimes



Geometric Scalars

- Constructing geometrical scalars that are coordinate invariant
- Contracting components of the Riemann tensor R_{abcd}, with orthonormal vectors U^a, X^b, Y^c, Z^d which satisfy parallel propagation along geodesics
- We can construct 20 geometric scalars using R_{abcd} and combination of (U^a, X^b, Y^c, Z^d) which are coordinate invariant and follow geodesics
- Computing scalars say, S^1 and S^2, for two spacetimes
- Taking the difference of these two scalars and computing the relative error $\frac{||S^1 - S^2||}{||S^1||}$
- $\frac{||S^1 - S^2||}{||S^1||}$ will tell us about the difference in spacetimes provided spacetimes are identical even including the gauge

Scalar Analysis for Schwarzschild spacetime

Schwarzschild and approximate Schwarzschild case

- Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2\, d\theta^2 + r^2\, \sin^2\theta\, d\phi^2$$

- For approximate Schwarzschild using Taylor expansion

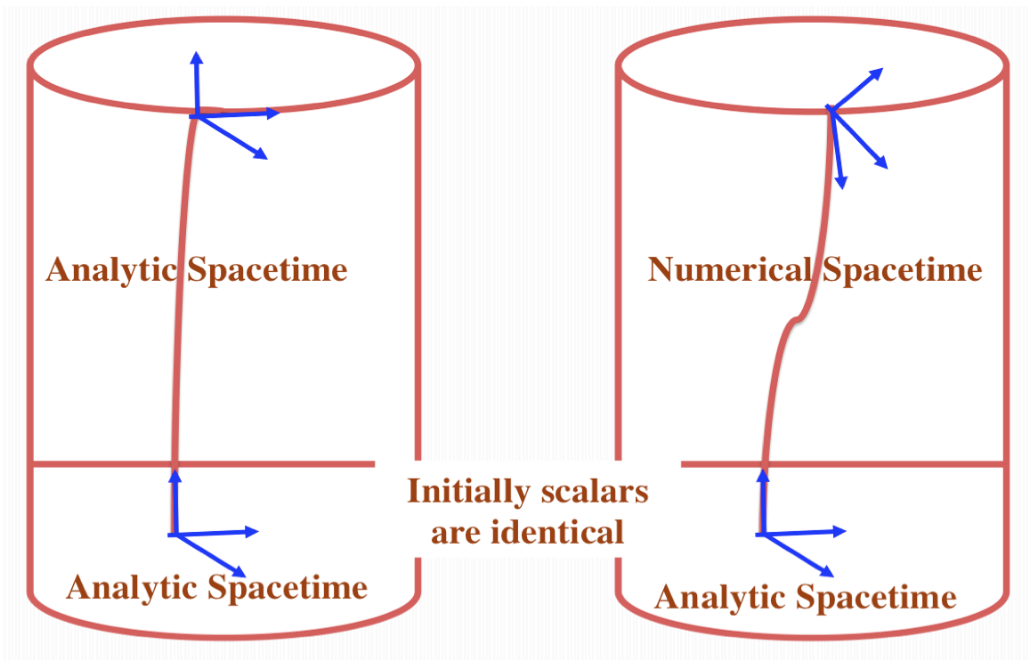
$$\left(1 - \frac{2GM}{r}\right)^{-1} = 1 + \sum_{N=1}^{k} \left[\frac{2GM}{r}\right]^N$$

- Two spacetime agree for more number of terms, N in Taylor expansion and at large r

Scalar Analysis for Schwarzschild spacetime

- There is a relationship between potentials and scalars for two spacetimes, $\frac{\delta V_{rr}}{V_{rr}} \propto \frac{\delta S}{S}$.
- For Schwarzschild and approximate Schwarzschild spacetime effective potentials, V and their first derivatives V_r are identically zero for near circular orbits, but $V_{rr} \neq 0$
- We find out for $0.01 \leq \frac{\delta V_{rr}}{V_{rr}} \leq 0.0001$ the upper limit for r gives $0.01 \leq \frac{\delta S}{S} \leq 0.0001$ within 10% limit.
- $\frac{\delta V_{rr}}{V_{rr}} = c\frac{\delta S}{S}$ with $0.9 \leq c \leq 1.2$
- This confirms that for spacetimes for which potential is unknown, the scalars can be used to probe geodesic nature
- Confirms that geometric scalars can probe the difference between the two spacetimes

Geometric Scalars



Geometric scalars, contracting R_{abcd} with (U^a, X^b, Y^c, Z^d), to probe the difference between spacetimes

Kerr spacetime for varying a

- Kerr metric

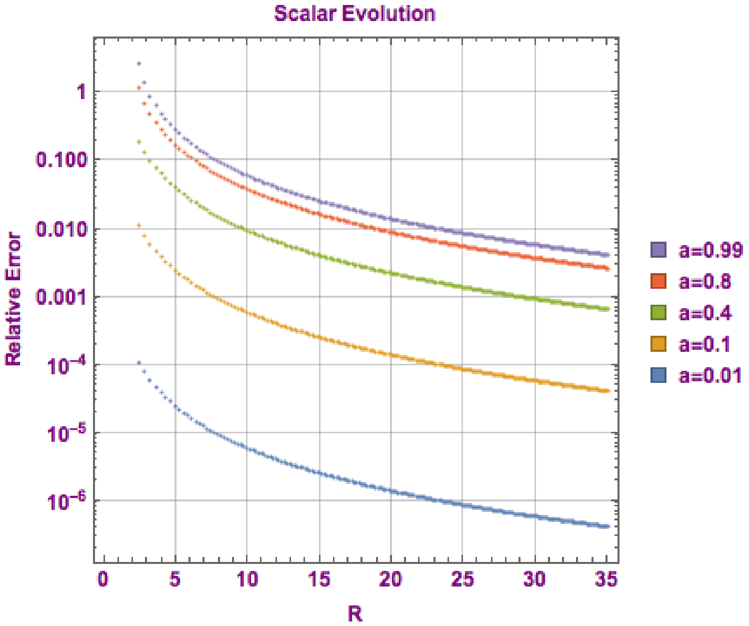
$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\Sigma}\Big((r^2+a^2)d\phi - a dt\Big)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

$\Delta = r^2 - 2Mr + a^2 + Q^2$
$\Sigma = r^2 + a^2\cos^2\theta$
$a = J/M$, the angular momentum per unit mass of the black hole

- Schwarzschild a special case for Kerr spacetime for $a = 0$
- At large r Kerr and Schwarzschild spacetimes are approximately the same

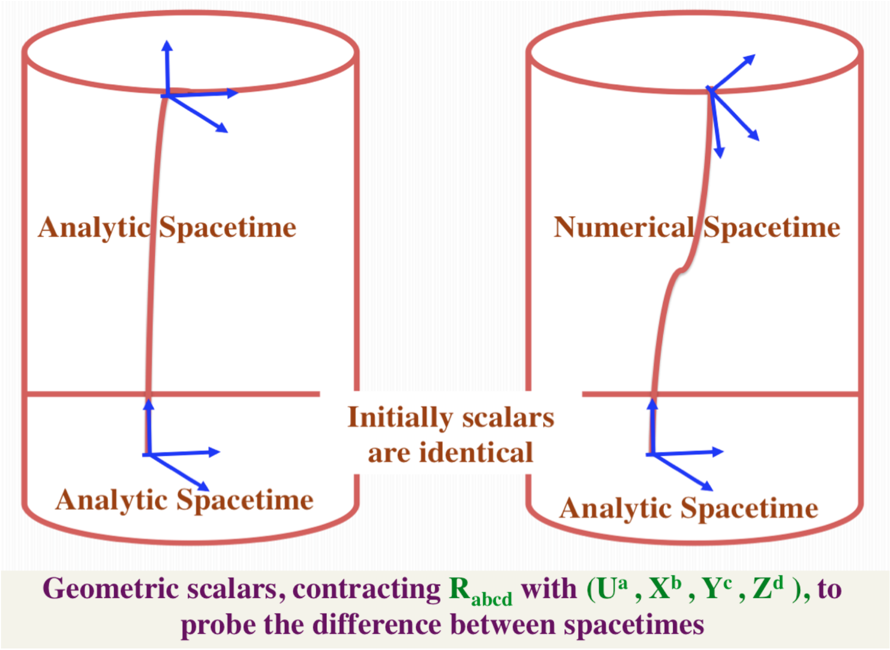
Kerr spacetime as test

- ✓ Test of geometric scalars for the Kerr spacetime
- ✓ Relative error of scalars decreases as R decreases
- ✓ Relative error of scalars also changes with a
- ✓ Confirms that geometric scalars can probe the difference of two spacetimes



Where to go next...

- Apply this method for analytic spacetimes
- Scalars can probe the difference which makes it possible to improve analytic spacetimes



Summary

- We propose a new method for the comparison of spacetimes
- We test it for the Schwarzschild and approximate Schawarzchild cases to find a relationship
- This method will provide us with a measure for accuracy of analytical spacetimes and possibly hints to improve it
- This method is tested and satisfies the initial test
- In future we want to use this analysis to analyze the improvements in analytic spacetimes