Curvature Perturbations, Gravitational Collapse and Primordial Black Hole Formation

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GR21 (NYC) - 13 July 2015

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Background model

- The most general **spherical symmetric** form of the metric, to describe a deviation from the uniform background, can be written as,

$$ds^2 = -a^2\,dt^2 + b^2\,dr^2 + R^2 d\Omega^2$$

- This defines *a*, *b*, and *R* being functions of the comoving coordinate *r* the often called **cosmic time** *t* and involves a choice of the **time slicing** to keep the metric diagonal (gauge choice). The radius *R* is the **circumferential radial coordinate**.

- The **unperturbed solution,** describing an expanding homogeneous universe**,** is given by the FRW metric: $K = \pm 1,\ 0$ is the **curvature parameter**, $\tilde{a}(t)$ is the **scale factor**, and $R = \tilde{a}(t)\,r$ is the **circumferential radial coordinate**.

$$ds^2 = -dt^2 + \tilde{a}(t)[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2]$$

- In case of BH formation it is more convenient to use a **null slicing** *(r,u)*:

$$f\,du = a\,dt - b\,dr \qquad\qquad ds^2 = -f^2\,du^2 - 2fb\,dr\,du + R^2 d\Omega$$

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \qquad T_{\mu\nu} = (e+p)u_\mu u_\nu - p g_{\mu\nu}$$

COSMIC TIME

$$D_t \equiv \frac{1}{a}\left(\frac{\partial}{\partial t}\right) \qquad D_r \equiv \frac{1}{b}\left(\frac{\partial}{\partial r}\right)$$

$$U \equiv D_t R \qquad \Gamma \equiv D_r R$$

$$D_t U = -\left[\frac{\Gamma}{(e+p)} D_r p + \frac{M}{R^2} + 4\pi R p\right]$$

$$D_t \rho = -\frac{\rho}{\Gamma R^2} D_r (R^2 U)$$

$$D_t e = \frac{e+p}{\rho} D_t \rho$$

$$D_t M = -4\pi R^2 p U$$

$$D_r a = -\frac{a}{e+p} D_r p$$

$$D_r M = 4\pi R^2 \Gamma e$$

$$\Gamma^2 = 1 + U^2 - \frac{2M}{R}$$

NULL TIME

$$D_t \equiv \frac{1}{f}\left(\frac{\partial}{\partial u}\right) \qquad D_k \equiv D_r + D_t$$

$$D_t U = -\frac{1}{1-c_s^2}\left[\frac{\Gamma}{(e+p)} D_k p + \frac{M}{R^2} + 4\pi R p + \right.$$
$$\left. + c_s^2 \left(D_k U + \frac{2U\Gamma}{R}\right)\right]$$

$$D_t \rho = \frac{\rho}{\Gamma}\left[D_t U - D_k U - \frac{2U\Gamma}{R}\right]$$

$$D_t e = \left(\frac{e+p}{\rho}\right) D_t \rho$$

$$D_t M = -4\pi R^2 p U$$

$$D_k \left[\frac{(\Gamma + U)}{f}\right] = -4\pi R (e+p) f$$

$$D_k M = 4\pi R^2 [e\Gamma - pU],$$

$$\Gamma = D_k R - U = 1 + U^2 - \frac{2M}{R}$$

Equation of State

energy density: $e = \rho(1 + \epsilon)$ pressure: $p = (\gamma - 1)\rho\epsilon$

rest mass density

adiabatic index - particle degree of freedom

specific internal energy (velocity dispersion)

- Barotropic fluid (no rest mass density): $p = we$ with $w \in [0, 1]$
  - radiation dominated era: $w = 1/3$ RADIATION $(\gamma = 4/3)$
  - matter dominated era: $w = 0$ DUST $(\gamma = 1)$
- Polytropic fluid: $p = K(s)\rho^{\gamma}$ $(\gamma = 5/3,\ 4/3,\ 2)$
  - If the fluid is adiabatic (no entropy change): $K(s) = K$ (constant)

Quasi homogeneous solution

- Defining the scale of the cosmological perturbations as R_0 and the **cosmological horizon** scale as $R_H := 1/H_b$. In the linear regime of supra horizon growing modes we can construct a **small parameter** $\epsilon(t) << 1$ as:

$$\epsilon(t) := \left(\frac{R_H}{R_0}\right)^2 = \epsilon(t_0)\left(\frac{t}{t_0}\right)^{\frac{2(1+3w)}{3(1+w)}} \qquad H_b^2 = \frac{8\pi}{3}e_b =: \left(\frac{1}{R_H}\right)^2$$

First order perturbations in ϵ are given by:

$$a = 1 + \epsilon\tilde{a} \qquad e = e_b(1+\epsilon\tilde{e})$$

$$b = \frac{\partial_r R}{\sqrt{1-K(r)r^2}}(1+\epsilon\tilde{b}) \qquad U = H_b R(1+\epsilon\tilde{U})$$

$$R = R_b(1+\epsilon\tilde{R}) \qquad M = \frac{4}{3}\pi e_b R(1+\epsilon\tilde{M})$$

In the **linear regime**, when $\epsilon << 1$, $K(r)r^2 = 1-\Gamma^2$ is a **time independent curvature profile** because pressure gradients are negligible, and can be used as the only independent source of perturbations.

- Solution of Einstein equations to the first order in ϵ (**Polnarev** & **Musco** 2007):

$$\tilde{e} = \Phi \frac{r_0^2}{3r^2} \partial_r \left[K(r) r^3\right]$$

$$\tilde{U} = \frac{1}{2} \left[\Phi - 1\right] K(r) r_0^2$$

$$\tilde{M} = \Phi K(r) r_0^2$$

$$\tilde{a} = -\Phi \frac{\omega}{1+\omega} \frac{r_0^2}{3r^2} \partial_r \left[K(r) r^3\right]$$

$$\tilde{b} = \Psi_1 r \partial_r \left[\frac{r_0^2}{3r^2} \partial_r \left(K(r) r^3\right)\right]$$

$$\tilde{R} = -\Psi_1 \frac{r_0^2}{3r^2} \partial_r \left[K(r) r^3\right] + \Psi_2 \frac{K(r)}{2} r_0^2$$

$$\Phi = \frac{3(1+w)}{5+3w}$$

$$\Psi_1 = \frac{3w}{(1+3w)(5+3w)}$$

$$\Psi_2 = \frac{2}{(1+3w)(5+3w)}$$

Constraints: $\lim_{r\to\infty} r^3 K(r) = 0$ and $K(r_0) + \frac{r_0}{3} \partial_r K(r_0) = 0$

- The **perturbation amplitude** δ can be measured by the mass excess inside the over dense region.

$$\delta \equiv \left(\frac{4}{3}\pi r_0^3\right)^{-1} \int_0^{r_0} 4\pi \frac{e - e_b}{e_b} r^2 dr = \epsilon(t)\Phi K(r_0) r_0^2$$

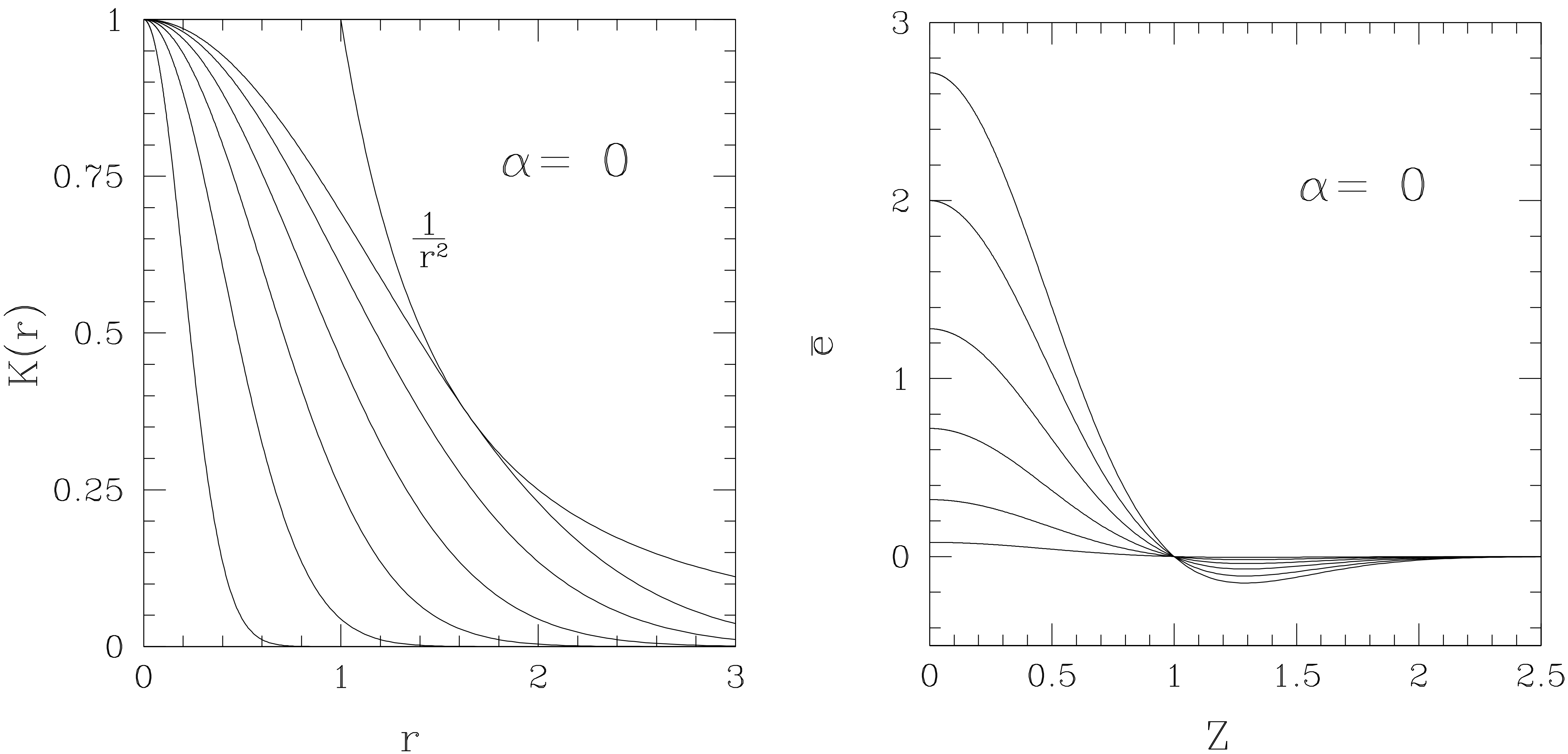
- The simplest curvature profile is given by a **Gaussian profile** of *K(r)* which gives a **Mexican Hat profile** for the energy density *e*:

$$K(r) = \exp\left(-\frac{r^2}{2\Delta^2}\right) \qquad \Rightarrow \qquad r_0^2 = 3\Delta^2$$

$$\tilde{e} = \frac{3(1+\omega)}{5+3\omega}\Delta^2 \left[1 - \left(\frac{R_b}{R_0}\right)^2\right] \exp\left(-\frac{3}{2}\left(\frac{R_b}{R_0}\right)^2\right)$$

$$\delta = \epsilon(t)\frac{9(1+\omega)}{5+3\omega}\Delta^2 \exp\left(-\frac{3}{2}\right)$$

- **Gaussian Curvature - Mexican hat energy density perturbation**: the amplitude *Δ* of the Gaussian profile of *K(r)* gives a measure of the central peak of the Mexican hat energy density profile $\bar{e}(r)$ that integrated on the 3D spherical volume gives the perturbation amplitude δ.



- The curvature profile can be expressed also in different gauges:

$$b = s(t)e^{\zeta(\hat{r})}(1+\epsilon\tilde{b}) \qquad\qquad r = e^{\zeta(\hat{r})}\hat{r}$$

$$K(r)r^2 = -\hat{r}\zeta'(\hat{r})\left[2+\hat{r}\zeta'(\hat{r})\right]$$

$$\boxed{\tilde{e} = \frac{\delta e}{e_b} = -\frac{2(1+\omega)}{5+3\omega}e^{2(\zeta(\hat{r}_0)-\zeta(\hat{r}))}\left[\frac{d^2\zeta}{d\hat{r}^2}+\frac{2}{\hat{r}}\frac{d\zeta}{d\hat{r}}+\frac{1}{2}\left(\frac{d\zeta}{d\hat{r}}\right)^2\right]}$$

- In the non liner regime of PBH formation the connection between the two gauges does not conserve the curvature profile shape! The zeta-gauge allows to study both type I and type II PBHs

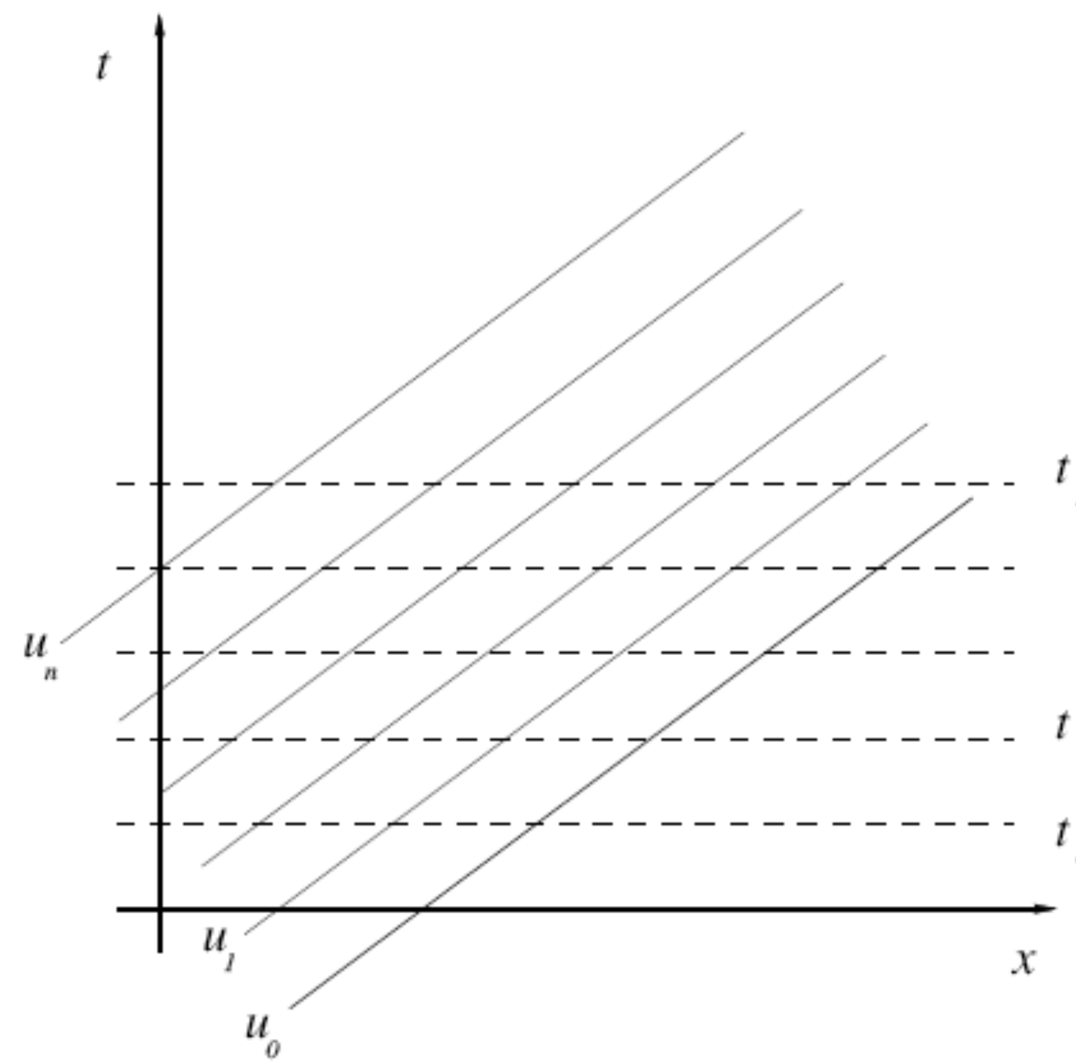
Numerical Results: the method

- Simulations are performed using a **Lagrangian spherically symmetric GR hydro code with an adaptive grid** (AMR).

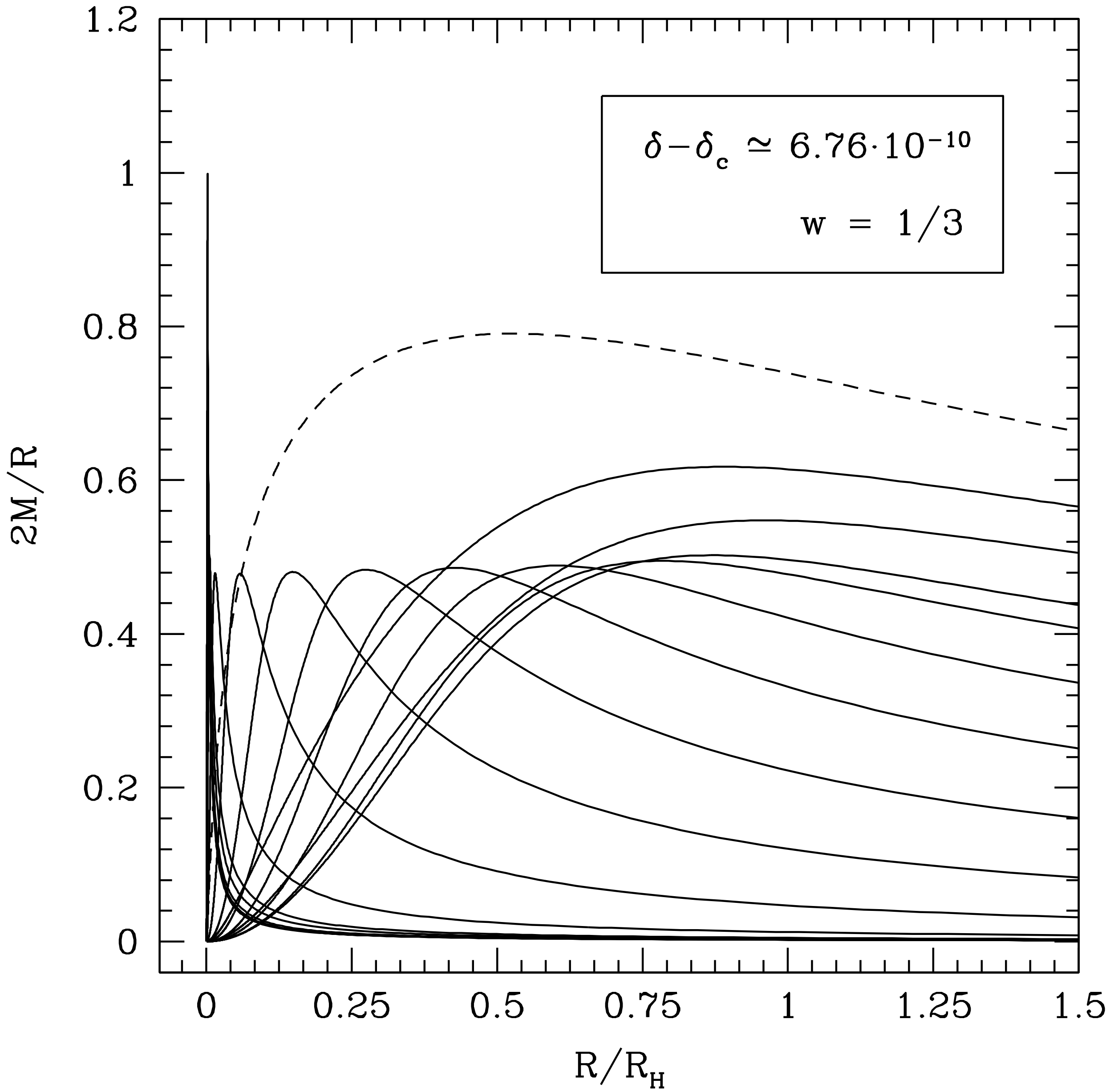
- We set initial conditions using a **cosmic time coordinate** t.

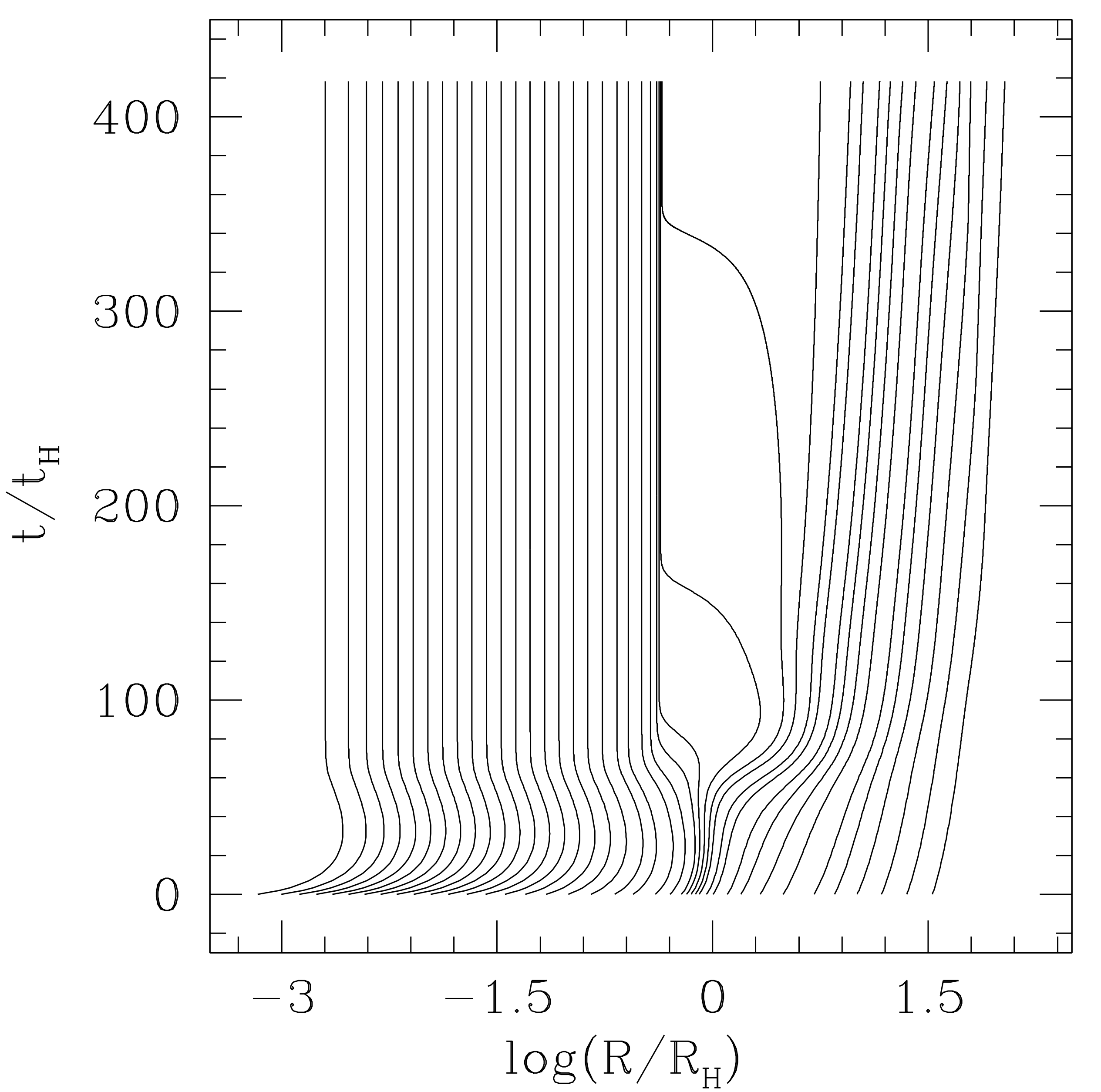
- We transfer those onto a **null foliation** of the space time, then evolved using an **observer time coordinate** u.

- The **formation of a PBH is seen by a distant external observer** (the singularity is hidden by the asymptotic formation of the apparent horizon).

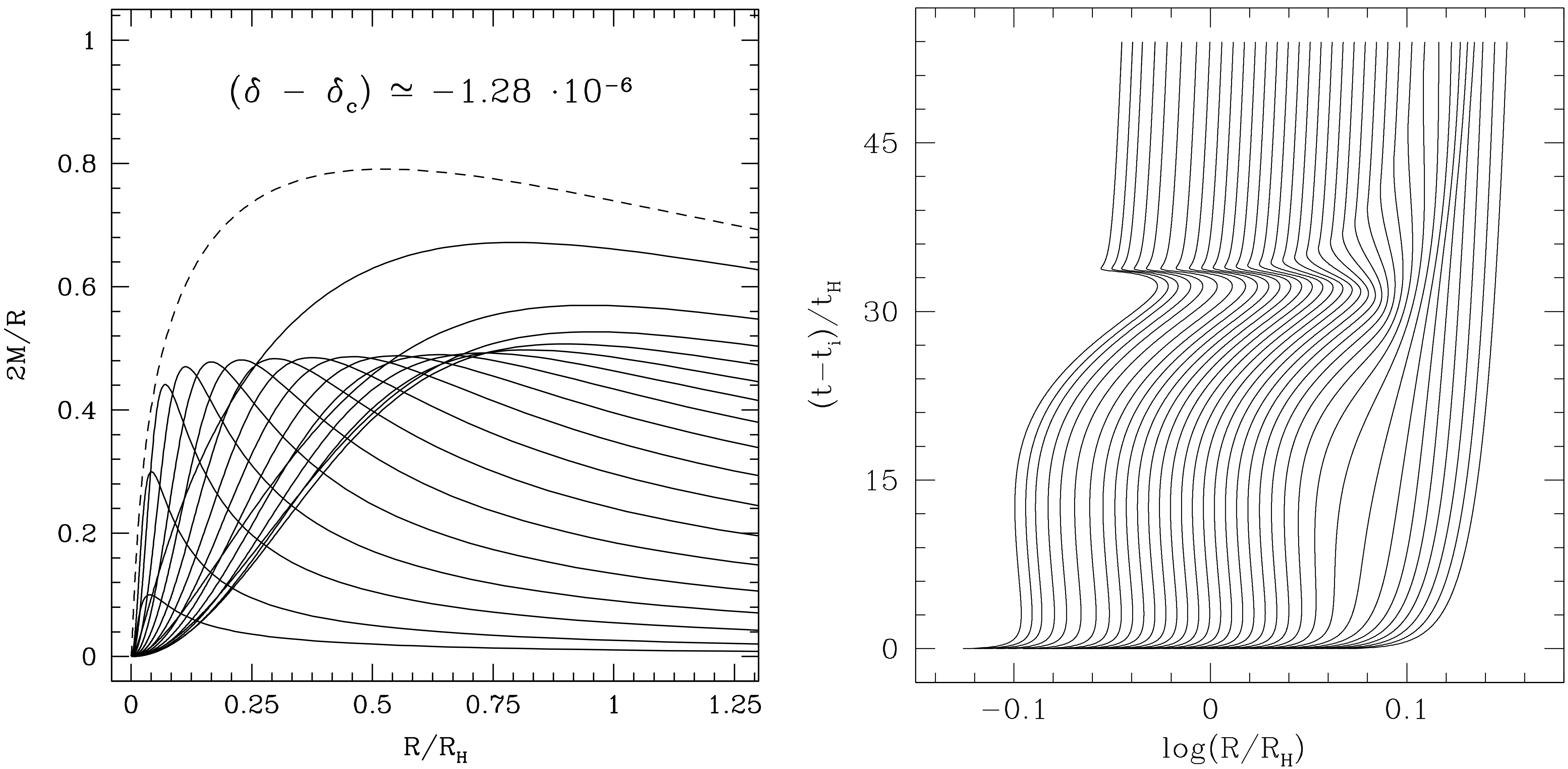


Numerical Results: BH formation



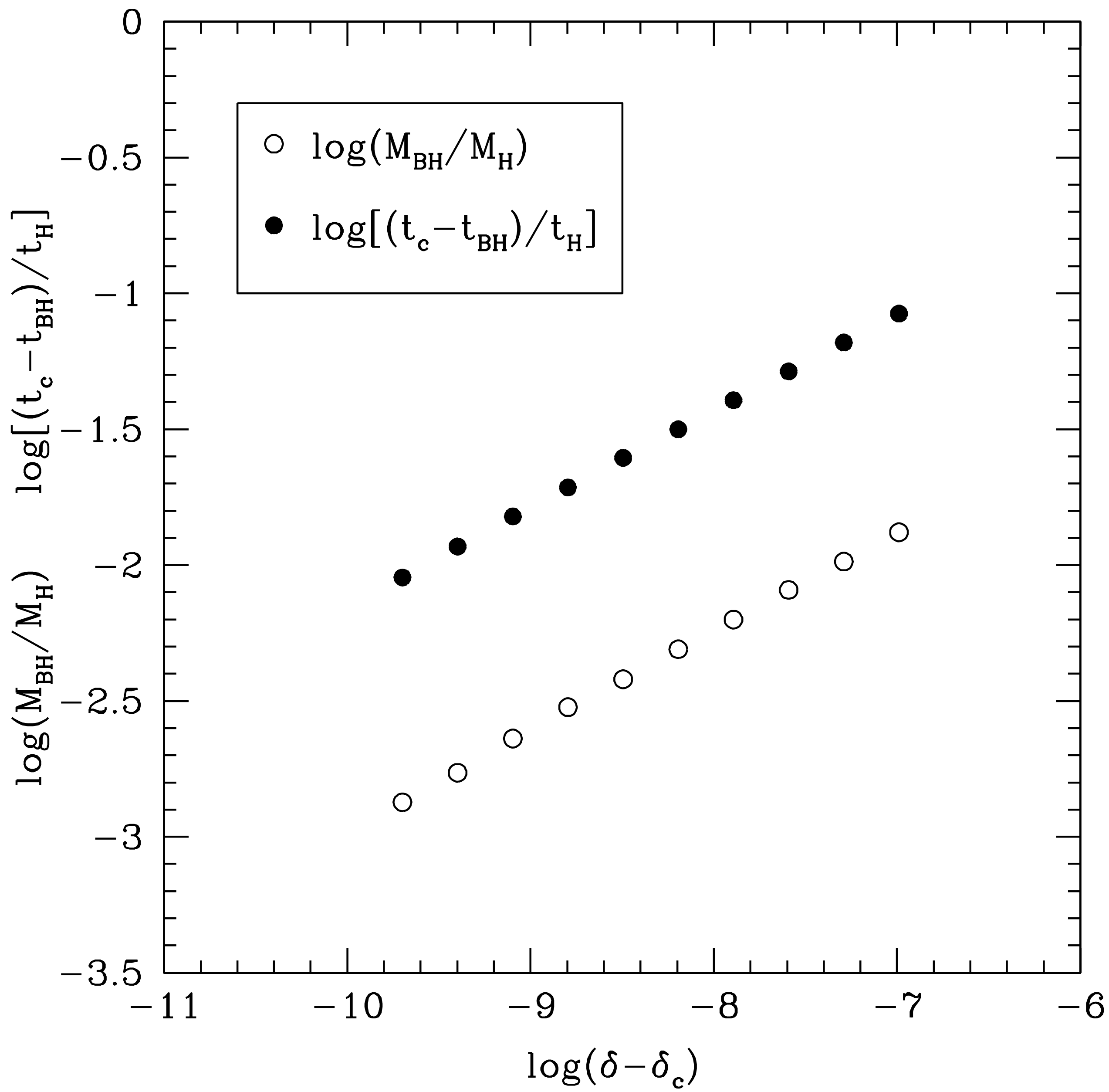


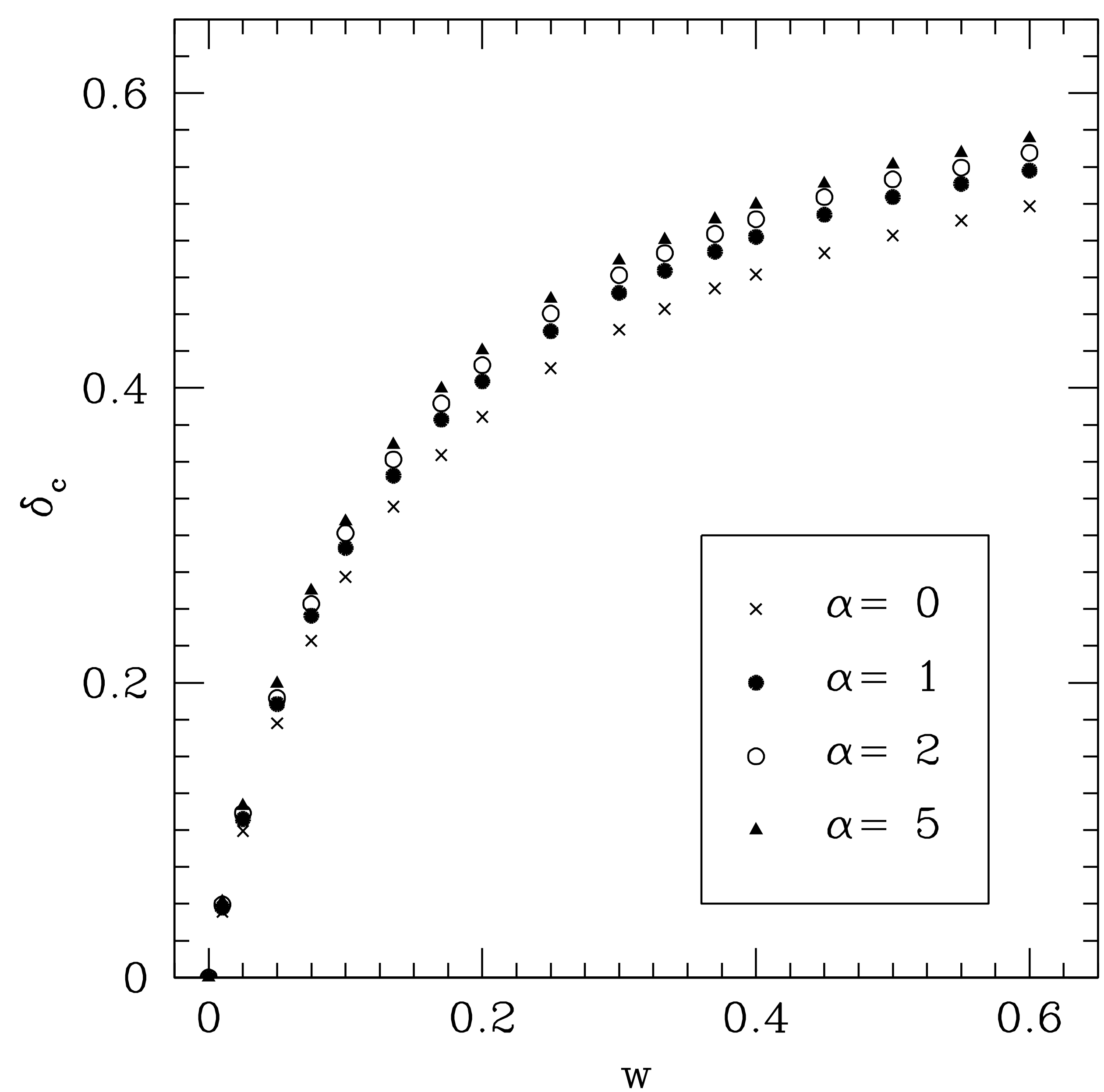
Numerical Results: no-BH formation



Numerical Results: scaling law and threshold

$$M_{BH} = K(\delta - \delta_c)^\gamma M_H$$



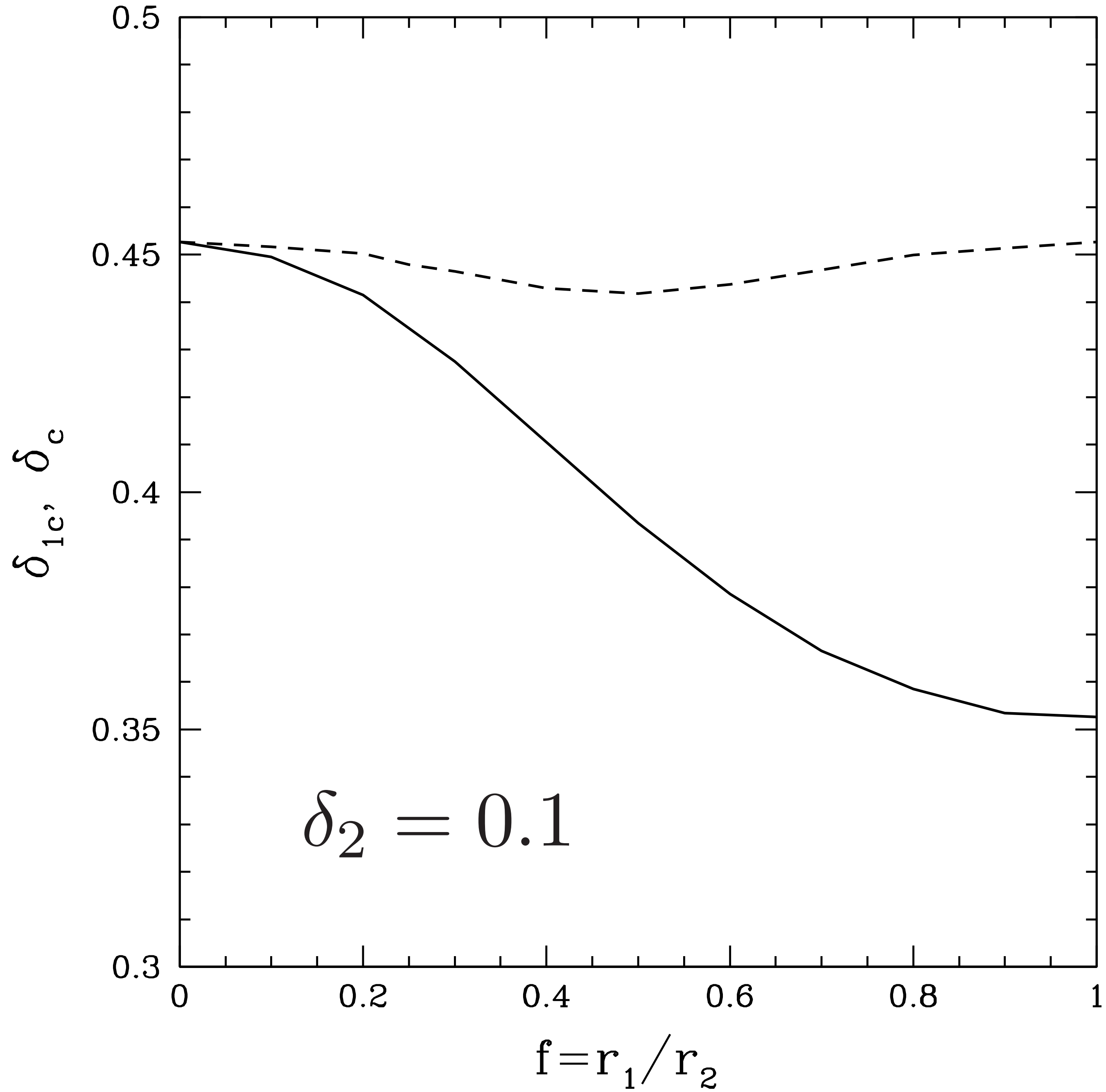


REFERENCE: **IM & J.C. Miller, CQG 30 (2013) 145009**

- Combination of two modes on different length scales to study the increase of PBH formation due to the long-mode. This could be particularly relevant in presence of positive non-Gaussianity.

$$K(r) = K_1(r) + K_2(r) = A \exp\left(-\frac{r^2}{2\Delta_1^2}\right) + B \exp\left(-\frac{r^2}{2\Delta_2^2}\right)$$

- Change of threshold for PBH formation when 2 modes are considered.



REFERENCE: **IM , C. Byrnes & S. Young (2016) - in preparation**

Conclusions

- PBH could have been formed in the early Universe from the tail of perturbation spectrum corresponding to non linear curvature perturbations.

- Determining the exact value for the threshold of PBH formation is crucial to calculate the probability of PBH formation.

- PBH could have been formed on top a second perturbation on a larger scale. This scenario is enhancing the probability of PBH formation and is favoured in vase of positive non Gaussianity.

- PBH seems to be more likely to be formed within cluster of structures and this is consistent with the possibility of PBH being the seeds of Super Massive Black Holes inside galaxies.

THANK YOU