

# Quasi-static solutions for compact objects in Chameleon models



***Ilia Musco***

*(CNRS, Observatoire de Paris/Meudon - LUTH)*

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Collaborators:

*Pier Stefano Corasaniti* (Meudon)

*Pedro Ferreira* (Oxford)

*David Mota* (Oslo)

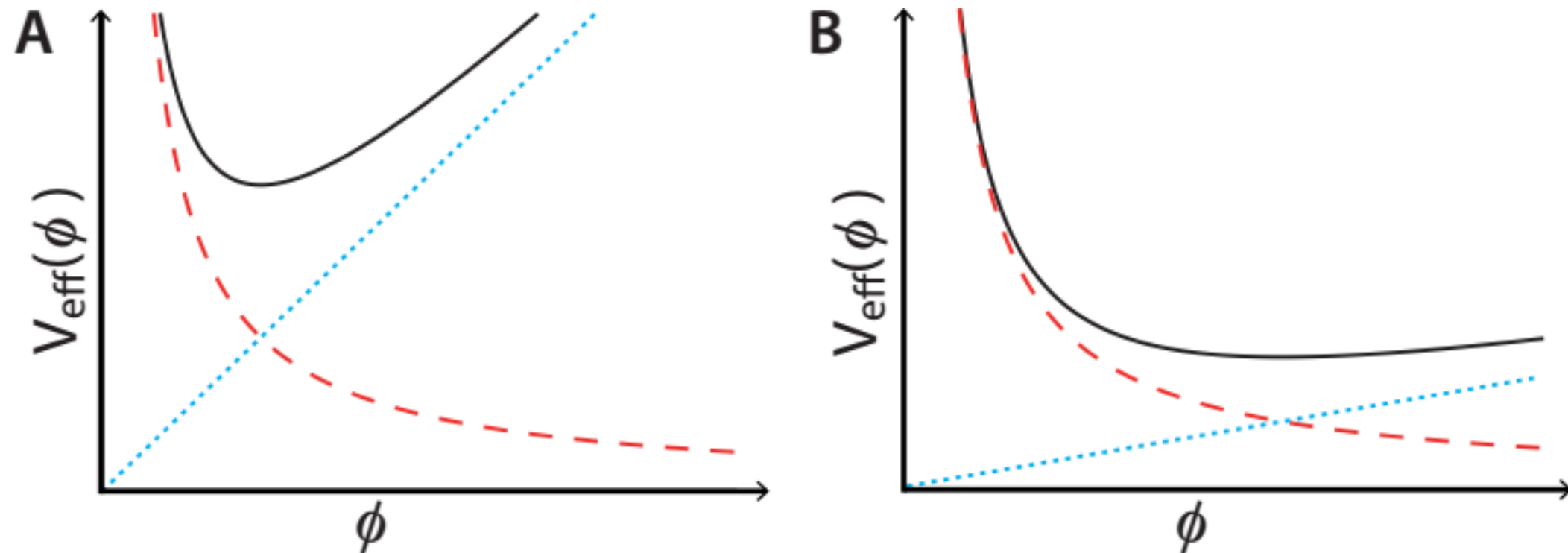
# Outline

- Introduction: What's/Why Chameleon?
- TOV-KG equations in quasi-static approximation
- Results (Incompressible model)
- Conclusions and future perspective

# ***RELATIVISTIC STARS IN SCALAR TENSOR THEORIES:***

## ***Quasi Static solution of Chameleon mechanism***

- In **spherical symmetry** the static structure of a star is obtained solving the **TOV equations** (Euler equation + mass-energy conservation equation).
- The presence of a coupled non minimally coupled scalar field introduces the **Klein-Gordon equation** into the system.
- Investigation on the viability of the **chameleon (screening) mechanism** proposed by *[Khoury & Weltman](#)* (2004).
- The scalar field  $\phi$  is characterised by a **coupling**  $\beta(\phi)$  and a **potential**  $V(\phi)$  that at large distances should describe a Dark Energy field.
- The coupling of the field with the matter is screening the field with a transition (**thin/thick shell**) to an expanding FRW Universe (  $R \rightarrow \infty$  ).



- To match a static solution with a non static background we need **quasi static (slow roll) regime** (  $\dot{\phi}, \ddot{\phi} \sim 0$  ).
- The **Hubble flow** (  $U = HR$  ) is not negligible.
- Quasi- static approximation keeps into account Universe expansion outside the star (no need of artificial matter outside as in *Babichev & Langlois* - 2010)
- Is chameleon consistent with Astrophysics?

# TOV-KG (QUASI-STATIC) EQUATIONS

$$\begin{aligned}
 \frac{dM}{dR} &= 4\pi R^2 e \\
 \frac{1}{a} \frac{da}{dR} &= -\frac{1}{e_m + p_m} \left[ \frac{dp_m}{dR} + \beta(e_m - 3p_m) \frac{d\phi}{dR} \right] \\
 \frac{1}{a} \frac{\partial U}{\partial t} &= \frac{\Gamma^2}{a} \frac{da}{dR} - \frac{M}{R^2} - 4\pi R p \\
 \frac{d^2 \phi}{dR^2} + \left[ \frac{2}{R} + \frac{1}{2\Gamma^2} \frac{d\Gamma^2}{dR^2} + \frac{1}{a} \frac{da}{dR} \right] \frac{d\phi}{dR} &= \frac{1}{\Gamma^2} \left[ \frac{dV(\phi)}{d\phi} + \beta(e_m - 3p_m) \right] \\
 \Gamma^2 &= 1 + U^2 - \frac{2M}{R} \\
 e &= e_m + e_\phi, \quad e_\phi = \frac{\Gamma^2}{2} \left( \frac{d\phi}{dR} \right)^2 + V(\phi) \\
 p &= p_m + p_\phi, \quad p_\phi = \frac{\Gamma^2}{2} \left( \frac{d\phi}{dR} \right)^2 - V(\phi)
 \end{aligned}$$

# INCOMPRESSIBLE STAR MODEL

	$e = e_0$	
<u>Inside the star:</u>	$p(0) = p_0$	$\phi_0$ shooting parameter
	$U = 0$	

<u>Star surface:</u>	$p(R_s) = 0$	junction condition
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	$e = e_b$	
	$p = 0$	
<u>Outside the star:</u>	$U = HR$	$\phi_\infty$ Dark Energy
	$H^2 = \frac{8\pi}{3}e_b$	

<u>Boundary Condition:</u>	$\frac{dV(\phi)}{d\phi} + \beta e_b = 0$	Equilibrium at infinity
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$$S_E = \int d^4x \sqrt{-g} \left[ \kappa R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + S_m(\Omega^2(\phi) g_{\mu\nu}, \psi_m)$$

$$V(\phi) = \frac{\mu^{n+4}}{\phi^n} \qquad \Omega(\phi) = \exp\left(\frac{\sqrt{8\pi G}}{c^2} \beta \phi\right) \qquad \beta(\phi) \equiv \frac{\Omega'(\phi)}{\Omega(\phi)}$$

$$\mu^{n+4} = \frac{\hbar}{n} \frac{\sqrt{8\pi G}}{c^2} \beta e_b \phi_\infty^{n+1} \implies V(\phi) = \frac{\sqrt{8\pi G}}{c^2} \frac{\beta}{n} e_b \phi_\infty \left(\frac{\phi_\infty}{\phi}\right)^n$$

$$\phi_\infty = \frac{1}{4\sqrt{8\pi}\beta} W\left(4n\frac{1-\Omega_m}{\Omega_m}\right) \qquad W_4 = \frac{1}{4} W\left(4n\frac{1-\Omega_m}{\Omega_m}\right) \simeq 2 \text{ (for } n=1, \Omega_m=0.3\text{)}$$

$$m_\phi^2 \equiv \left. \frac{d^2 V_{\text{eff}}}{d\phi^2} \right|_{\phi=\phi_\infty} \qquad \frac{8\pi\beta^2}{W_4} = \frac{m_\phi^2 c^4}{(n+1)Ge_b} = \frac{m_\phi^2}{(n+1)R^2}$$

# Normalised Equations (with background):

$$\frac{d\hat{p}_m}{d\hat{R}} = -\frac{(\hat{e}_m + \hat{p}_m) \left(4\pi\hat{p}\hat{R}^3 + \hat{M}\right)}{\Gamma^2 \hat{R}^2} - W_4(\hat{e}_m - 3\hat{p}_m)\hat{\lambda}$$

$$\frac{d\hat{M}}{d\hat{R}} = 4\pi\hat{R}^2\hat{e}$$

$$\hat{\lambda} = \frac{d\hat{\phi}}{d\hat{R}}$$

$$\frac{d\hat{\lambda}}{d\hat{R}} = -\left[\frac{2}{\hat{R}} - \frac{4\pi\hat{R}^3(\hat{e} - \hat{p}) - 2\hat{M}}{\hat{\Gamma}^2 \hat{R}^2}\right]\hat{\lambda} + \frac{\hat{\beta}^2}{W_4\Gamma^2} \left[\hat{e}_m - 3\hat{p}_m - \hat{\phi}^{-(n+1)}\right]$$

$$\hat{\beta} = \sqrt{8\pi}\beta \quad \Omega(\hat{\phi}) = \exp\left(W_4\hat{\phi}\right) \quad \hat{V}(\hat{\phi}) = \frac{W_4}{n}\hat{\phi}^{-n}$$

$$\hat{e}_\phi = \frac{W_4^2\Gamma^2}{2\hat{\beta}^2}\hat{\lambda}^2 + \frac{W_4}{n}\hat{\phi}^{-n} \quad \hat{e} = \hat{e}_m + \hat{e}_\phi$$

$$\hat{p}_\phi = \frac{W_4^2\Gamma^2}{2\hat{\beta}^2}\hat{\lambda}^2 - \frac{W_4}{n}\hat{\phi}^{-n} \quad \hat{p} = \hat{p}_m + \hat{p}_\phi$$



Series expansion:

$$\hat{\phi} = \hat{\phi}_0 + \hat{\phi}_1 \hat{R} + \frac{\hat{\phi}_2}{2} \hat{R}^2 + \frac{\hat{\phi}_3}{6} \hat{R}^3$$

$$\hat{\lambda} = \hat{\lambda}_0 + \hat{\lambda}_1 \hat{R} + \frac{\hat{\lambda}_2}{2} \hat{R}^2 + \frac{\hat{\lambda}_3}{6} \hat{R}^3$$

$$\hat{e}_m = \hat{e}_0 + \hat{e}_1 \hat{R} + \frac{\hat{e}_2}{2} \hat{R}^2 + \frac{\hat{e}_3}{6} \hat{R}^3$$

$$\hat{p}_m = \hat{p}_0 + \hat{p}_1 \hat{R} + \frac{\hat{p}_2}{2} \hat{R}^2 + \frac{\hat{p}_3}{6} \hat{R}^3$$

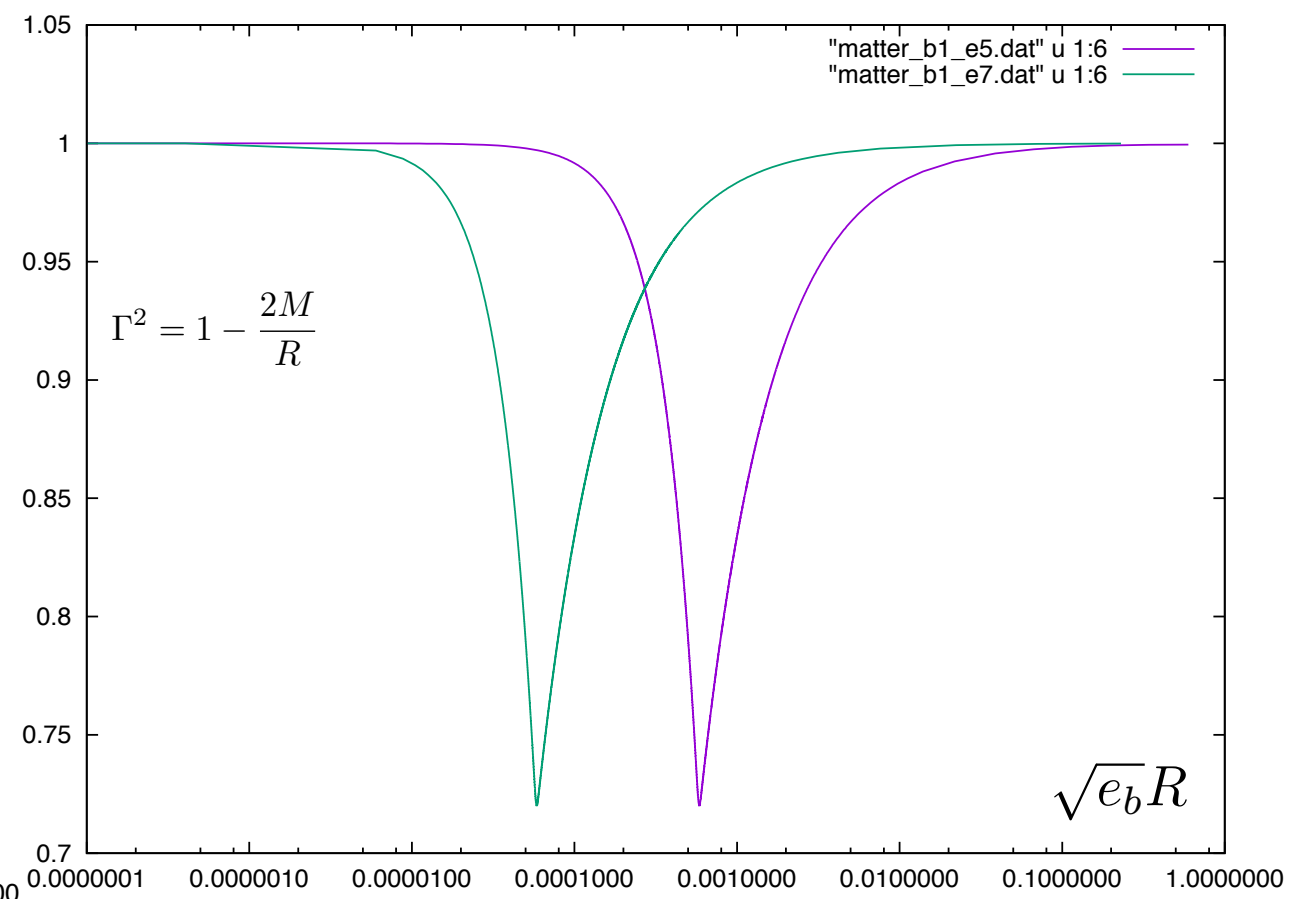
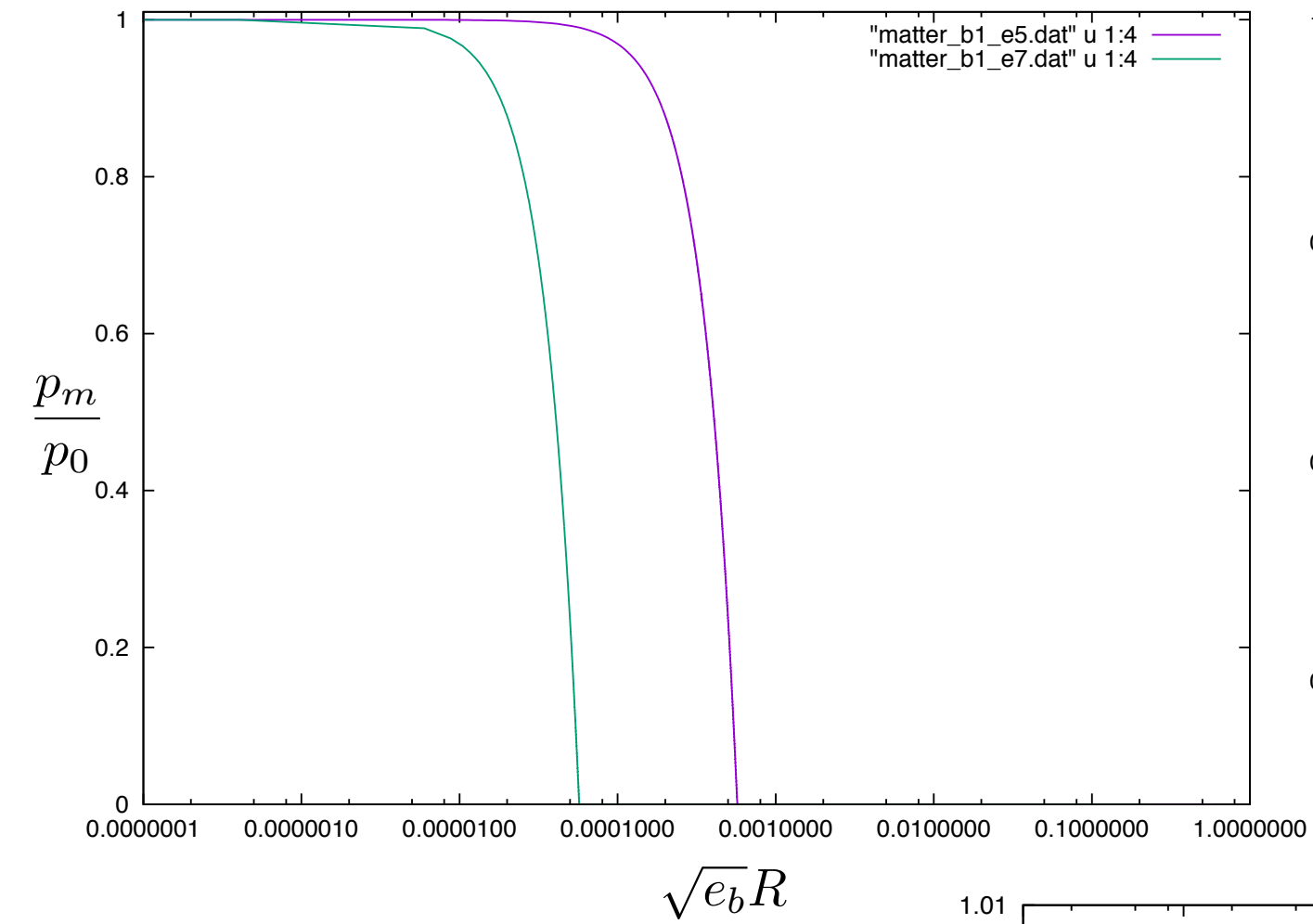
$$\hat{M} = \hat{M}_0 + \hat{M}_1 r + \frac{\hat{M}_2}{2} \hat{R}^2 + \frac{\hat{M}_3}{6} \hat{R}^3$$

$$\hat{\phi}_2 = \hat{\lambda}_1 = \frac{\hat{\beta}^2}{3W_4} \left[ \hat{e}_0 - 3\hat{p}_0 - \hat{\phi}_0^{-(n+1)} \right]$$

$$\hat{p}_2 = -\frac{4\pi}{3} (\hat{e}_0 + \hat{p}_0) \left[ \hat{e}_0 + 3\hat{p}_0 - 2\frac{W_4}{n} \hat{\phi}_0^{-n} \right] - W_4 (\hat{e}_0 - 3\hat{p}_0) \hat{\phi}_2$$

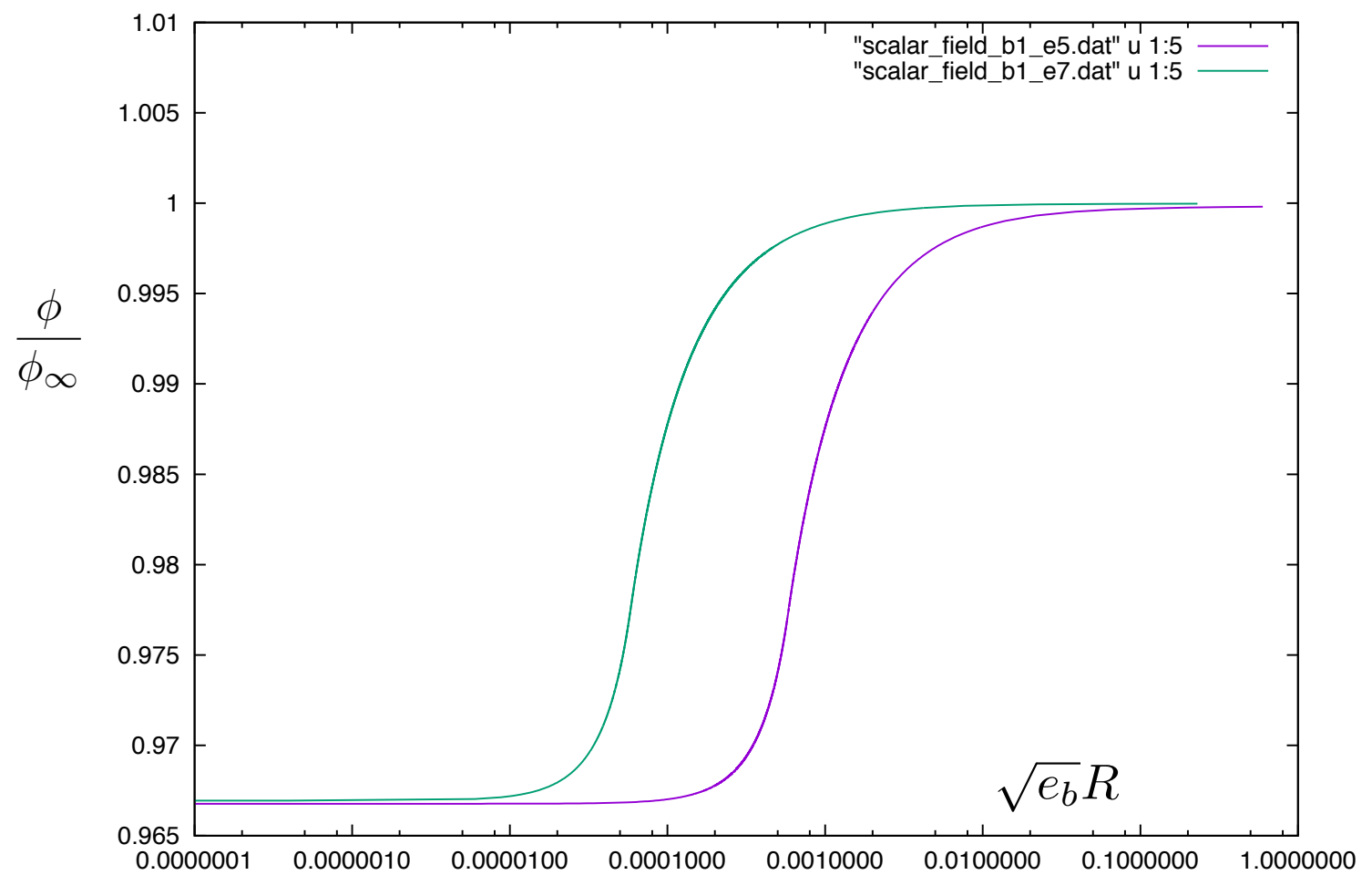
$$\hat{m}_3 = 8\pi \left[ \hat{e}_0 + \frac{W_4}{n} \hat{\phi}_0^{-n} \right]$$

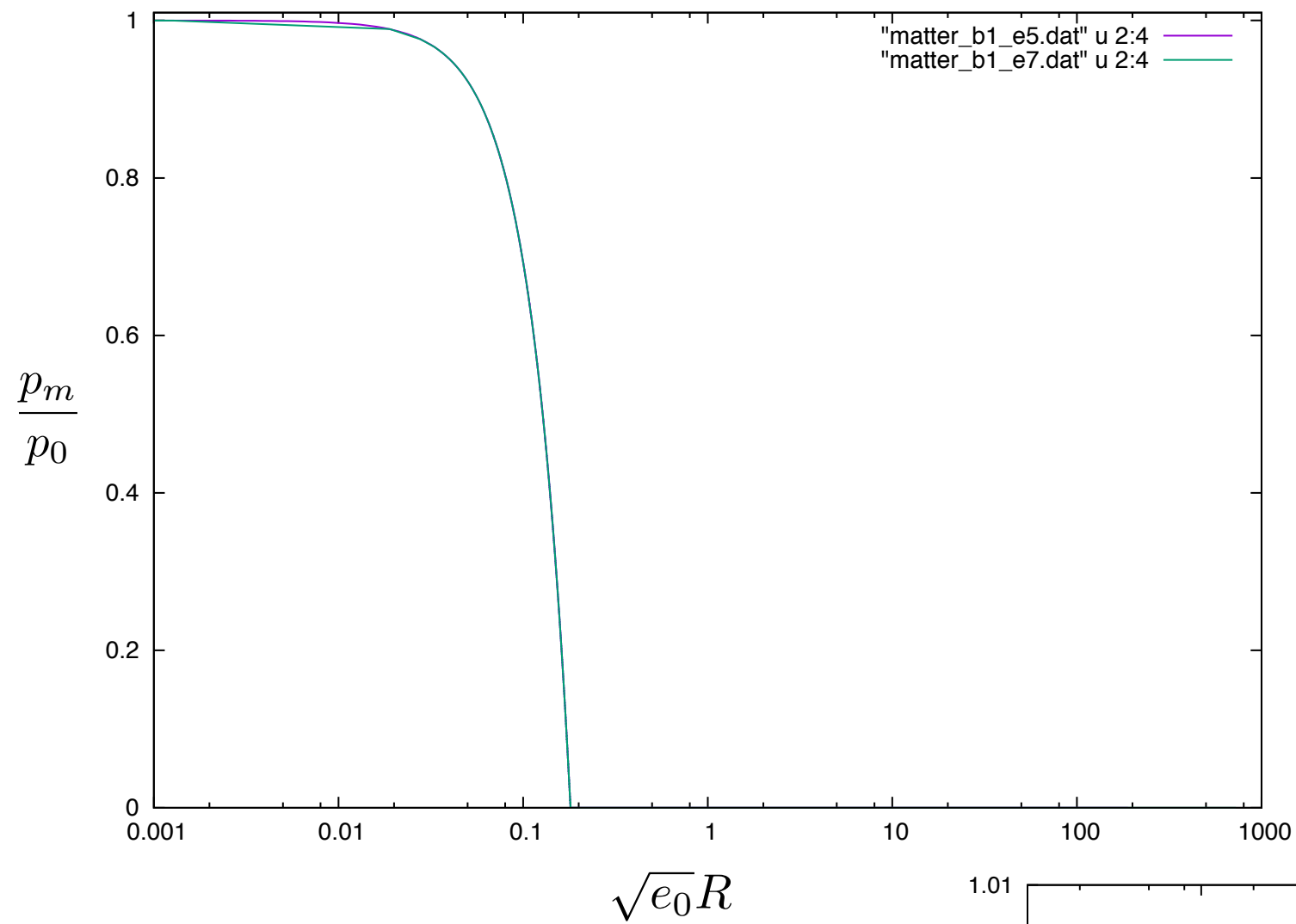
$$\hat{\lambda}_3 = \frac{8\pi}{5} \left[ 7\hat{e}_0 - 3\hat{p}_0 + 10\frac{W_4}{n} \hat{\phi}_0^{-n} \right] \hat{\phi}_2 + \frac{3\hat{\beta}^2}{5W_4} \left[ \hat{e}_2 - 3\hat{p}_2 + (n+1)\hat{\phi}_2 \hat{\phi}_0^{-(n+2)} \right]$$



$$\beta = 1$$

$$\frac{e_0}{e_b} = 10^5 - 10^7$$

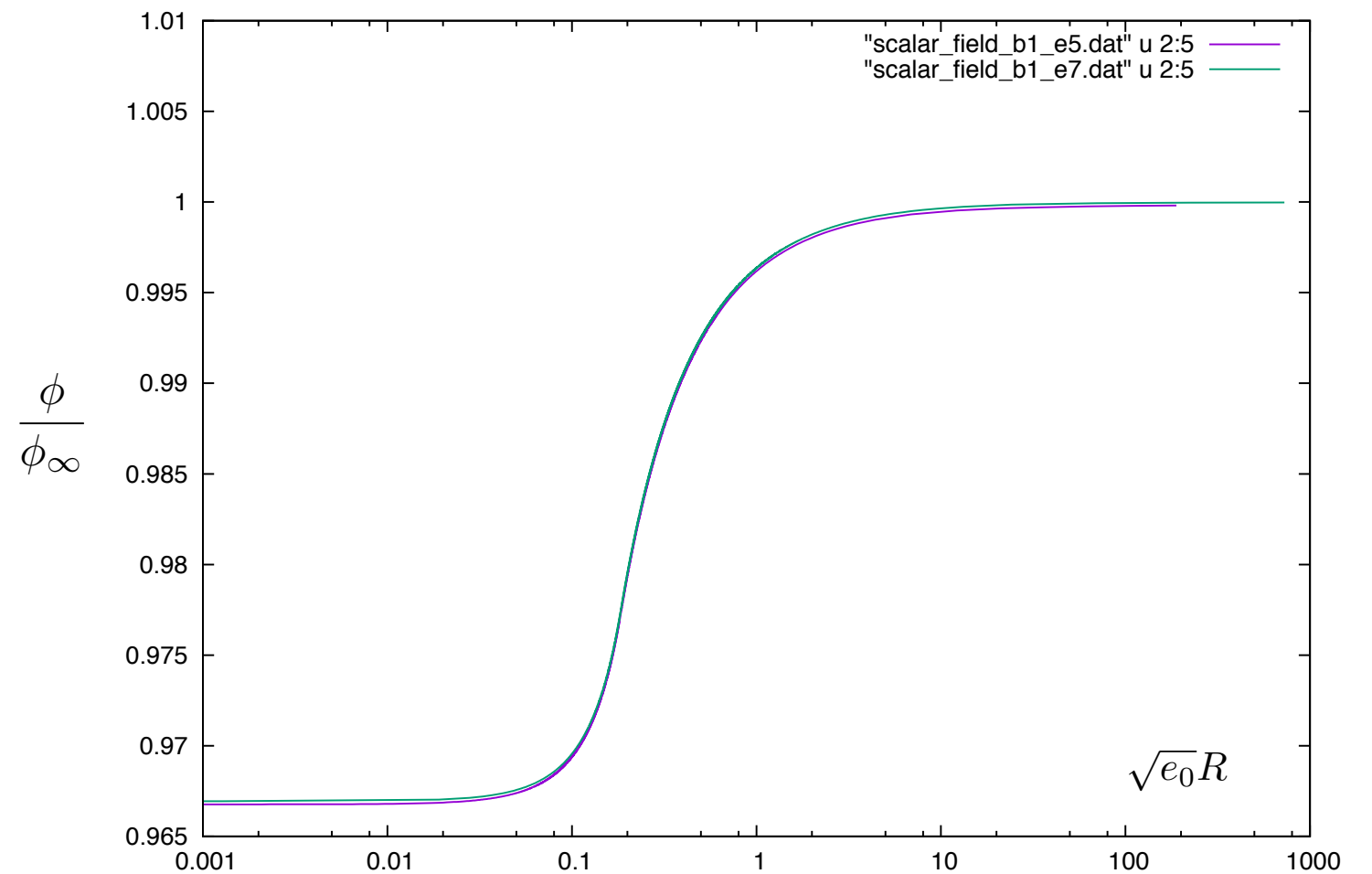


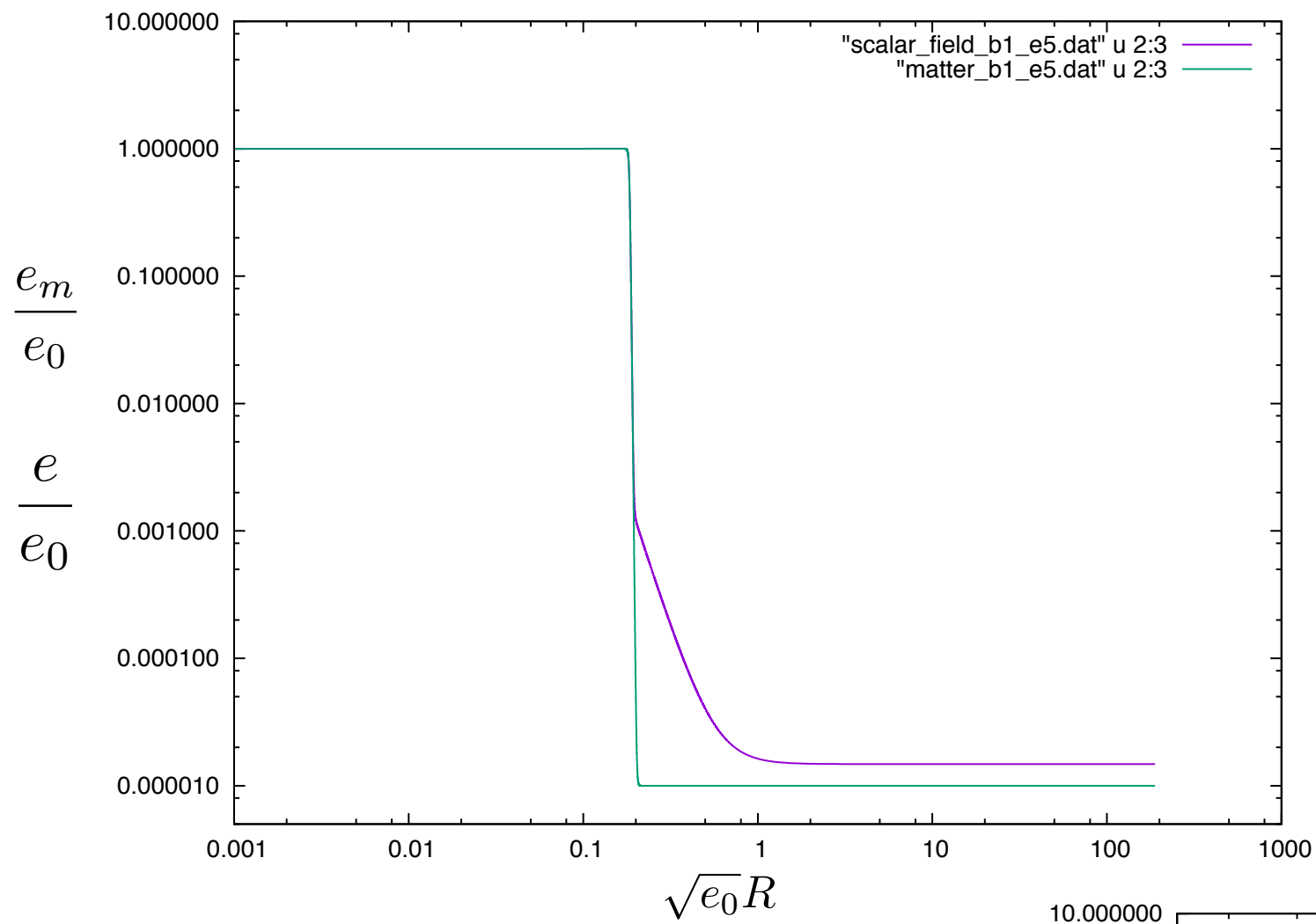


$$\beta = 1$$

$$\frac{e_0}{e_b} = 10^5 - 10^7$$

“Universal” solution:  
 central value  $\phi_0$  depends  
 on compactness  $M/R$   
 and coupling strength  $\beta$





$$\beta = 1$$

$$\frac{e_0}{e_b} = 10^5 - 10^7$$

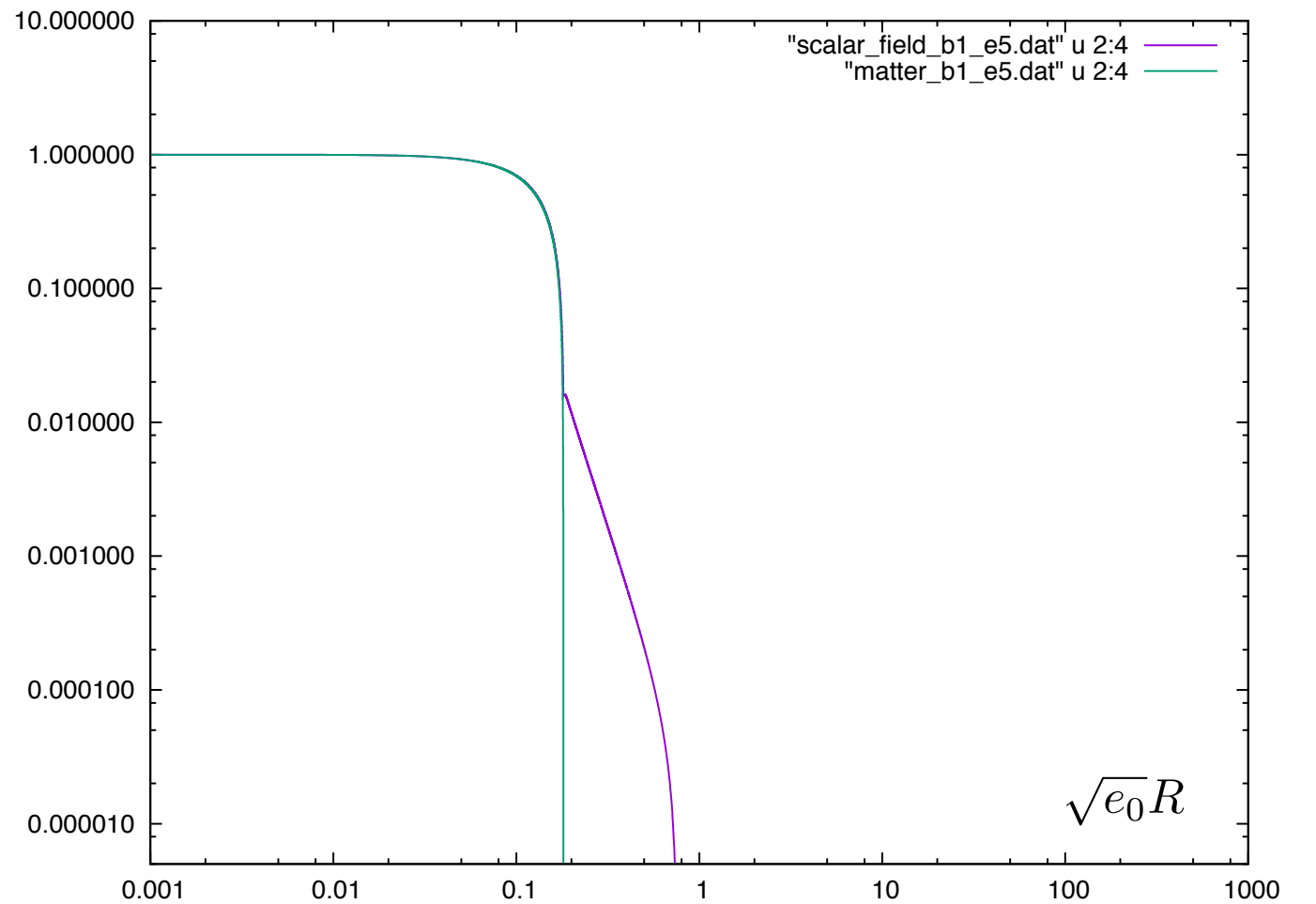
$$e = e_m + e_\phi \quad p = p_m + p_\phi$$

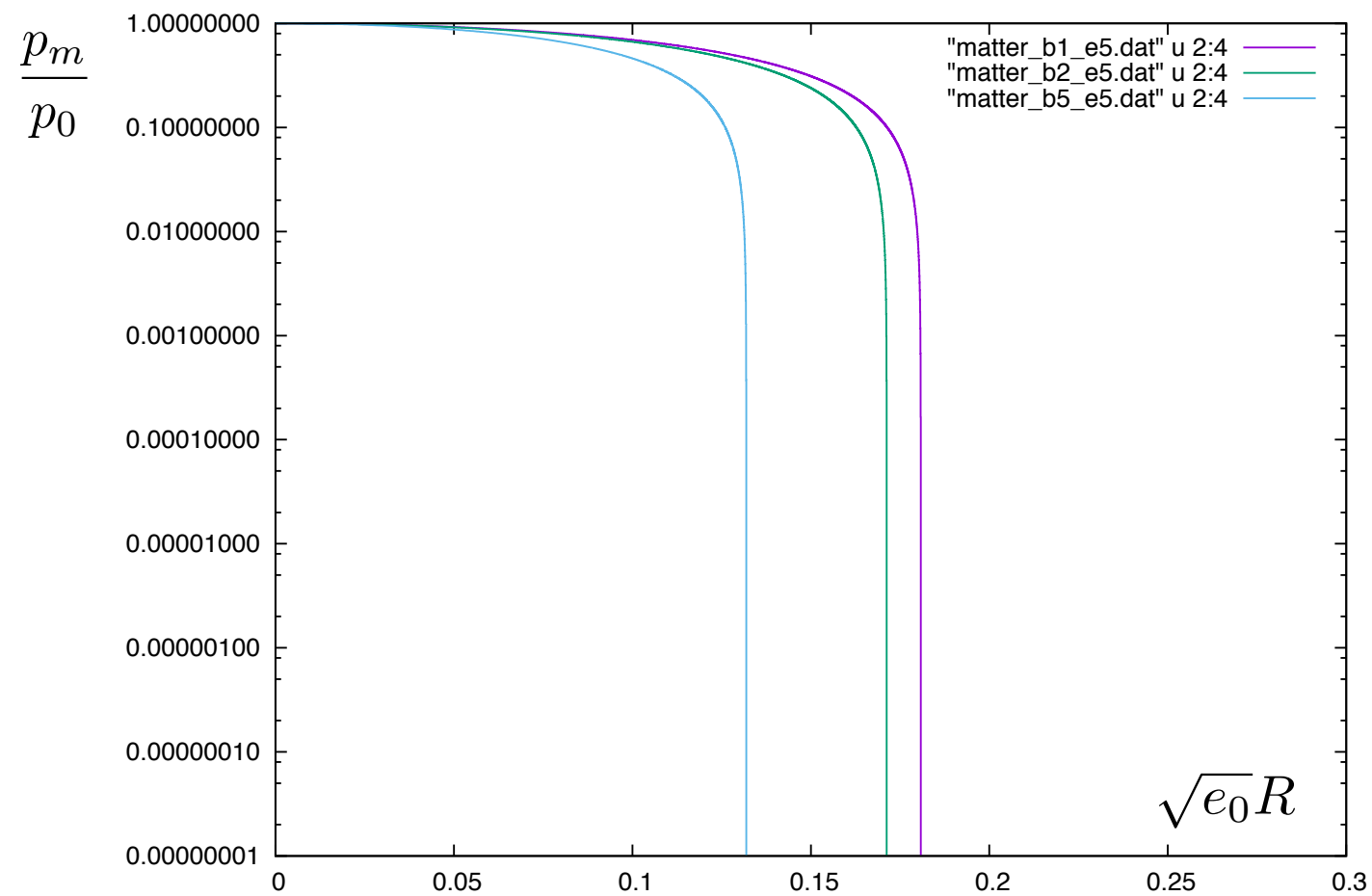
$$e_\phi = \frac{\Gamma^2}{2} \left( \frac{d\phi}{dR} \right)^2 + V(\phi)$$

$$p_\phi = \frac{\Gamma^2}{2} \left( \frac{d\phi}{dR} \right)^2 - V(\phi)$$

$$\frac{p_m}{p_0}$$

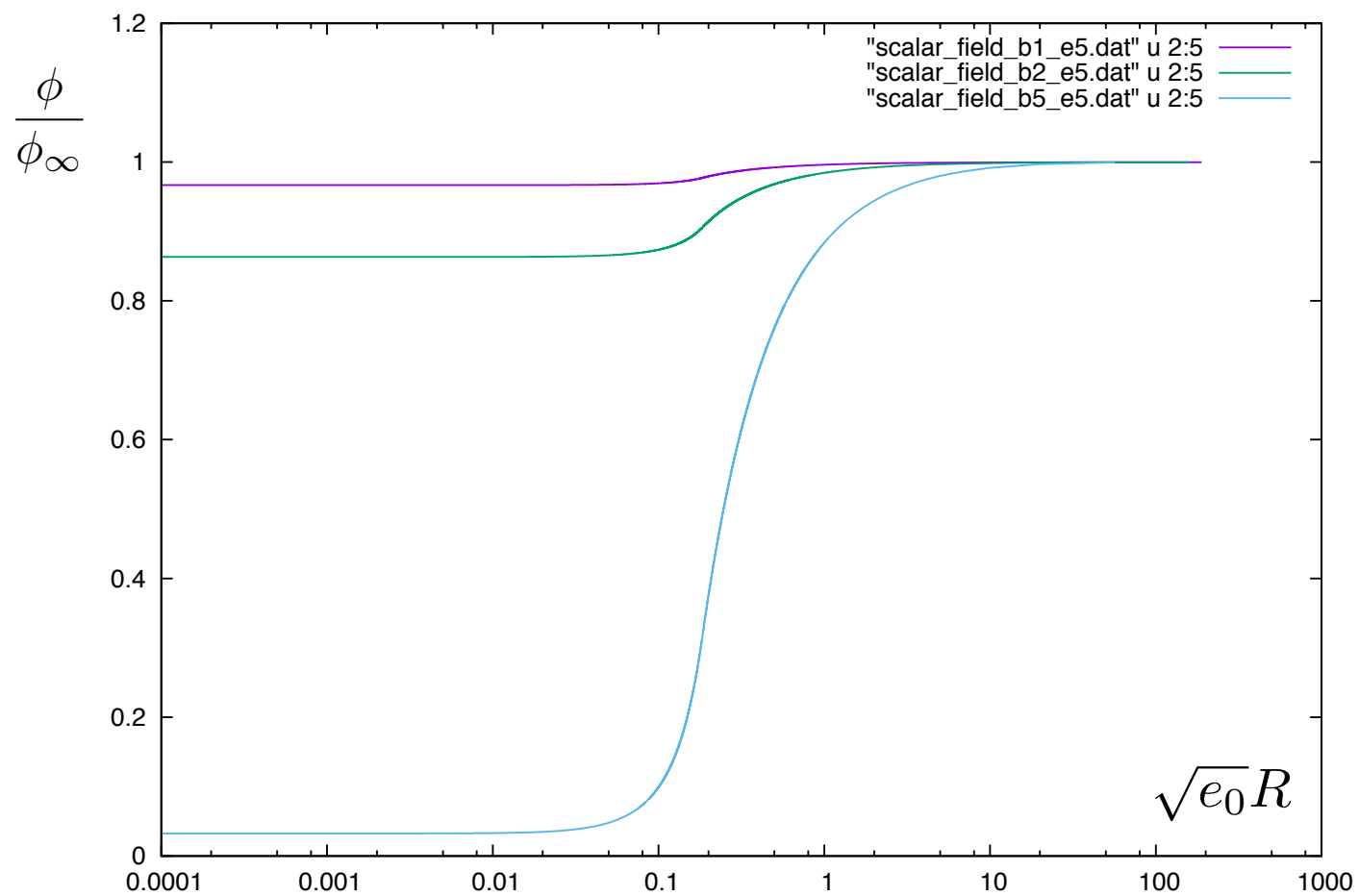
$$\frac{p}{p_0}$$





$$\beta = 1, 2, 5$$

$$e_0 = 10^5$$



# Conclusions & Future perspectives

- Chameleon models provide a potentially viable screening mechanism with the scalar field changing with the compactness of the star and with the coupling strength.
- Strong coupling ( $\beta > 1$ ) not consistent with CMB cosmological constraints. Need of an extra field  $\Rightarrow$  *Why Chameleon?*
- However the necessary coupling strength might have a back-reaction effect on the compactness of the star that will give strong constraints on the model (possibly ruling out). Consider a non constant coupling? Strong fine tuning!
- More realistic star models (e.g. polytropes) need to be considered to quantify the real effects of the field on realistic stars/compact objects.  $\Rightarrow$  *A substantial change of the stellar structure could be observable (change of compactness).*
- The screening from the dark matter halo around the star need to be considered for realistic astrophysical cases.
- Stability of the solution is also an issue to consider.
- *IM, P. S. Corasaniti, P. Ferreira, D. Mota (2016)* - in progress