Kerr black holes with hair

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- In the next decade or so, VLBIs will be able to resolve the center of galaxies.
- It is therefore an excellent time to consider alternatives to the paradigmatical Kerr black holes of Einstein's general relativity.

Public service announcement

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- The solutions I will discuss:
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    - can provide distinct, and unique, phenomenology.

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- Most of those black holes have little to do with astrophysics.
- See review by Herdeiro and Radu from 2015.

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- For a Lagrangian of the form:

$$\mathcal{L} = \frac{1}{4}R - \frac{1}{2}\nabla_a\Phi\nabla^a\Phi - V\left(\Phi\right),$$

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- with the following assumptions:
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  3. scalar field inherits the isometries of the metric.

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- Kerr black holes with scalar hair [Herdeiro, Radu 2014]

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- We are interested in rotating solutions and they can be described by the following metric and field ansatz:

$$ds^2 = -e^{2F_0}dt^2 + e^{2F_1}\left(dr^2 + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta \left(d\varphi - W dt\right)^2 ,$$

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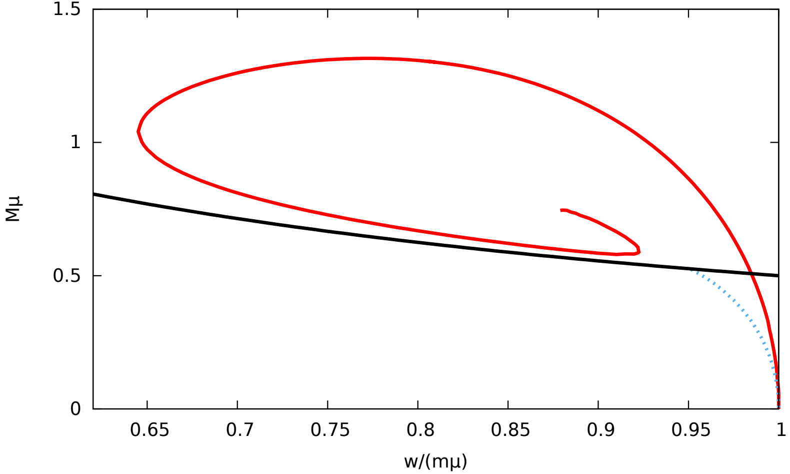
- They have a conserved Noether-charge, Q, and for boson stars $J = mQ$, where J is the angular momentum of the boson star.

Boson stars



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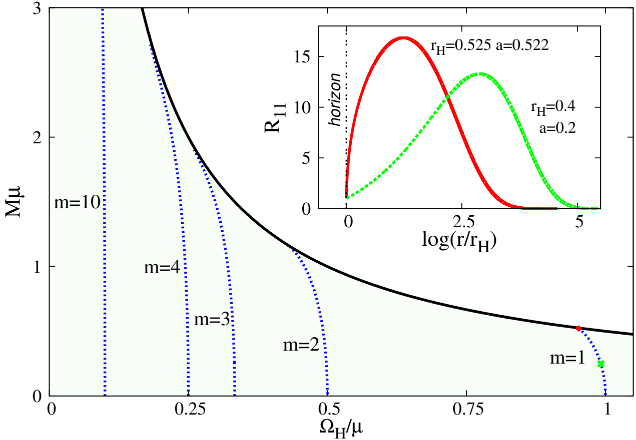
  - At the threshold of the superradiant states, $w_R = w_c$, we find true bound states: scalar-clouds. [J. Degollado, C. Herdeiro, HR *. . . to appear . . .*]

Scalar clouds around Kerr black holes



Kerr black holes with scalar hair

- It is "simple" to inject a black hole to the center of a boson star:

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta \,(d\varphi - W dt)^2$$

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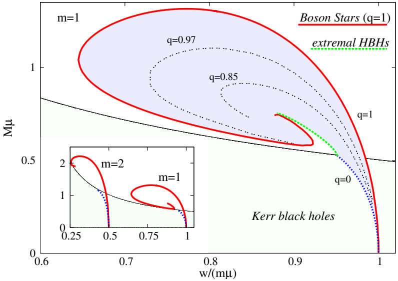
- We find the solutions numerically



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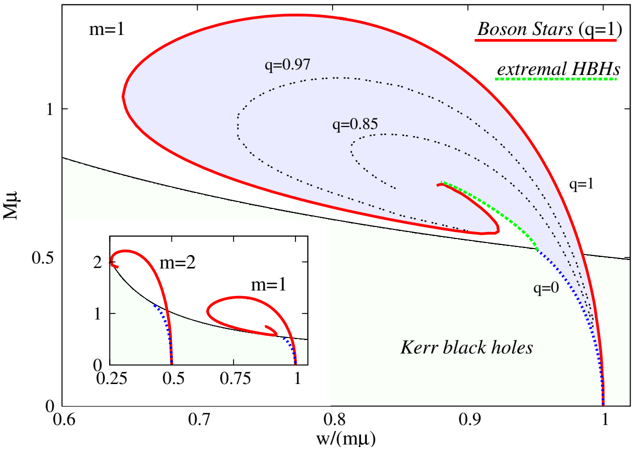
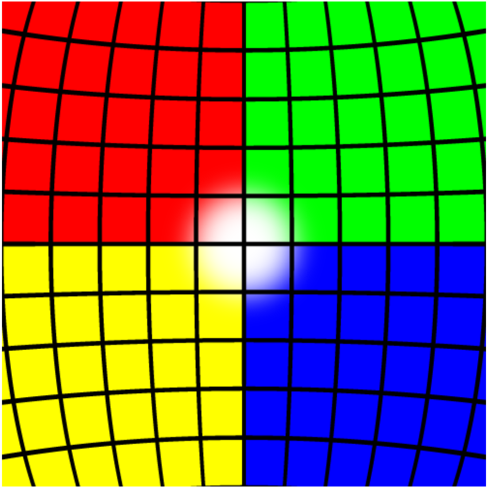
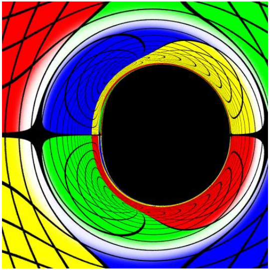


Backwards ray-tracing



Shadows of KBHsSH: I



	$M_{\rm ADM}$	$M_{\rm H}$	$J_{\rm ADM}$	$J_{\rm H}$	$\frac{M_{\rm H}}{M_{\rm ADM}}$	$\frac{J_{\rm H}}{J_{\rm ADM}}$	$\frac{J_{\rm ADM}}{M^2_{\rm ADM}}$	$\frac{J_{\rm H}}{M^2_{\rm H}}$
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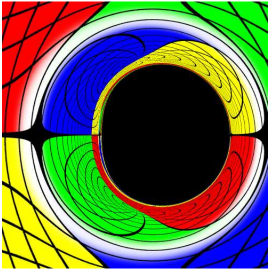

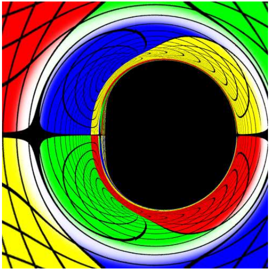

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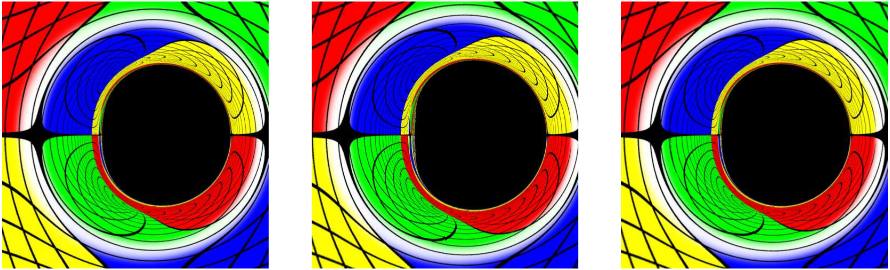

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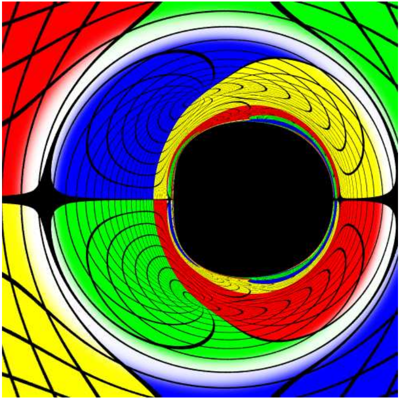

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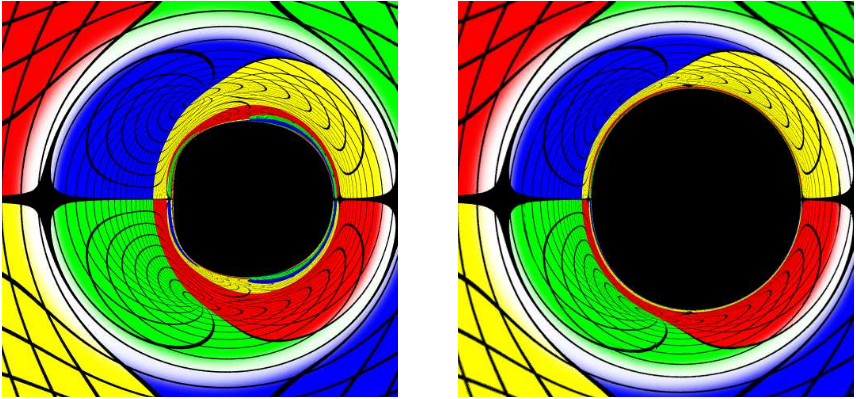

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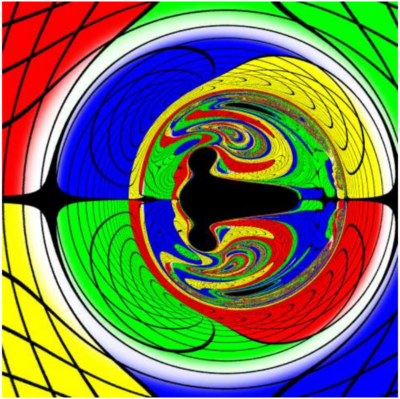

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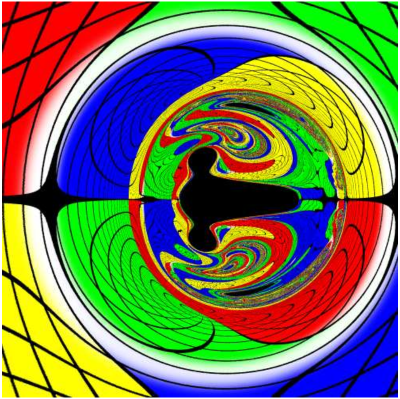

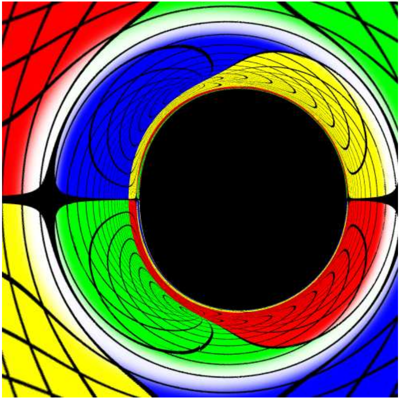

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- However, this increase in mass comes solely from the scalar field and not the central black hole.
- Therefore, these solutions are “hairier but not heavier”.

Proca hair

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- Since Proca fields can “suffer” from superradiance around Kerr black holes and rotating Proca stars have been shown to exist [Brito, Cardoso, Herdeiro, Radu (2015)]

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- One notable difference is that their energy densities possess a second local maximum.

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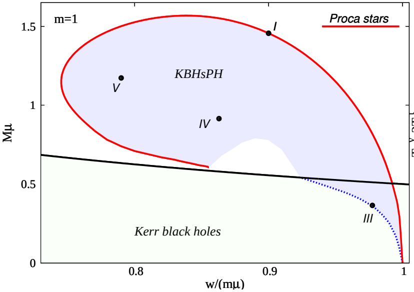

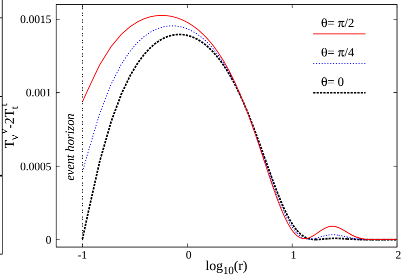

Kerr-Newman black holes with scalar hair

J. Delgado, C. Herdeiro, E. Radu, HR, ...*to appear*...(2016)

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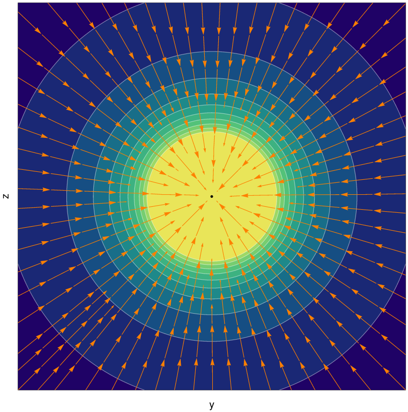
- For example, the gyromagnetic ratio of these configurations is $g < 2$, only reaching the limit for vanishing hair.

Electric field lines

Kerr-Newman



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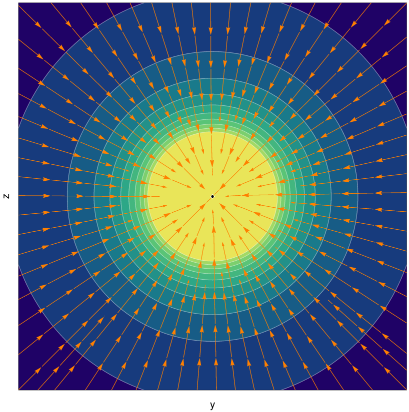

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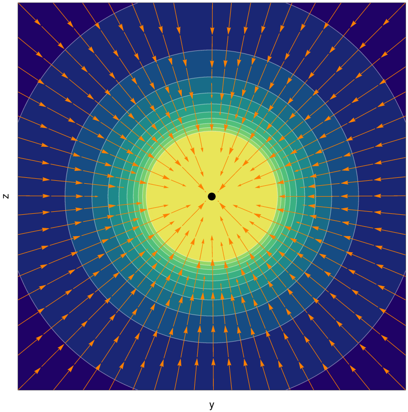

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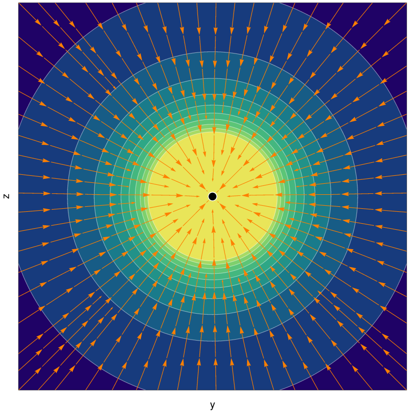
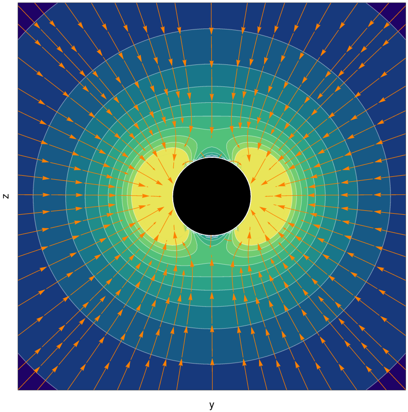
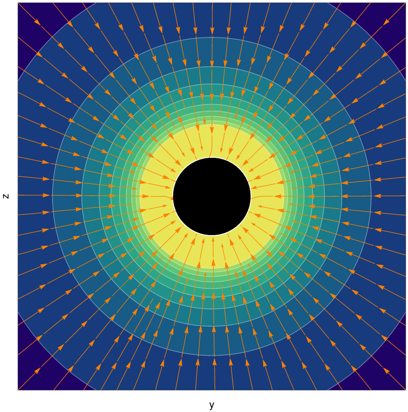


Electric field lines

Kerr-Newman



KNBHsSH



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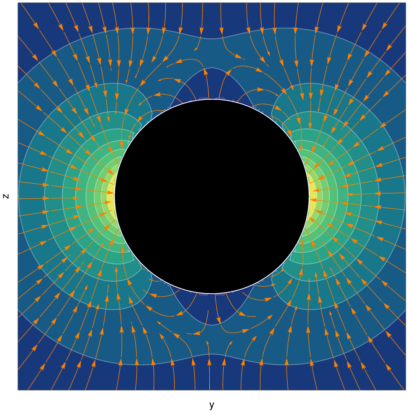

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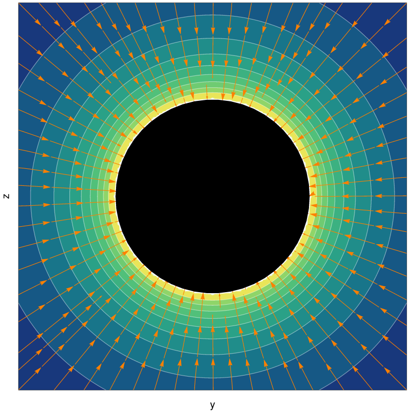

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  - consider other fields, such as Proca fields. [C. Herdeiro, E. Radu, HR (2016)]
  - These generalizations hint of a more general mechanism where the "synchronization condition" $w = m\Omega_H$ generates hairy black holes.

Future work

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- Perform time evolutions of these solutions to study their stability and dynamics (e.g. collisions).