



**CHALMERS**

# Models for Self-Gravitating Photon Shells and Geons

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# 1 Geons and the Einstein-Vlasov system

## 1.1 Geons

- In 1955 Wheeler introduced the concept of a **geon** (gravitational electromagnetic entity): a self-consistent, singularity free, solution of the Einstein-Maxwell system.
- Wheeler studied numerically so called idealized spherically symmetric geons. These are spherically symmetric on a time average.
- Wheeler found thin shell solutions with the property that  $2m/r \approx 8/9$ .
- The **massless** Einstein-Vlasov system models a photon gas and solutions to this system might be considered as *alternative models* for geons. The aim of this talk is to present an existence result of static spherically symmetric solutions to this system.

## 1.2 The Einstein-Vlasov system

Consider a spacetime  $(\mathcal{M}, g)$  and define the mass-shell of massless particles

$$\mathcal{P} = \{(x, p) \in T\mathcal{M} : g(p, p) = -m^2 = 0, p \text{ future directed}, p \neq 0\}.$$

The null geodesics of the metric  $g$  are the projections onto spacetime of the curves  $(x(s), p(s)) \in \mathcal{P}$  defined in local coordinates by

$$\frac{dx^\alpha}{ds} = p^\alpha, \quad \frac{dp^\alpha}{ds} = -\Gamma_{\beta\gamma}^\alpha p^\beta p^\gamma$$

The particle density function  $f : \mathcal{P} \rightarrow \mathbb{R}_+$  is required to solve the Vlasov equation

$$p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma_{\beta\gamma}^a p^\beta p^\gamma \frac{\partial f}{\partial p^a} = 0.$$

### 1.3 The Einstein-Vlasov System

The particle density function  $f : \mathcal{P} \rightarrow \mathbb{R}_+$  gives rise to an energy momentum tensor

$$T^{\mu\nu}(x) = \int_{\mathcal{P}_x} f p^\mu p^\nu \mu_{\mathcal{P}_x}.$$

It couples the Vlasov equation to Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

yielding the Einstein-Vlasov system.

## 1.4 Results on static and stationary massive solutions

- Existence of compactly supported spherically symmetric static solutions was first established by Rein and Rendall in 1993.
- Existence of axially symmetric static and stationary solutions was shown by A., Kunze and Rein in 2011 and in 2014 respectively.
- For any spherically symmetric matter model satisfying the energy condition  $p_r + 2p_T \leq \rho$ , and  $p_r \geq 0$ , the inequality

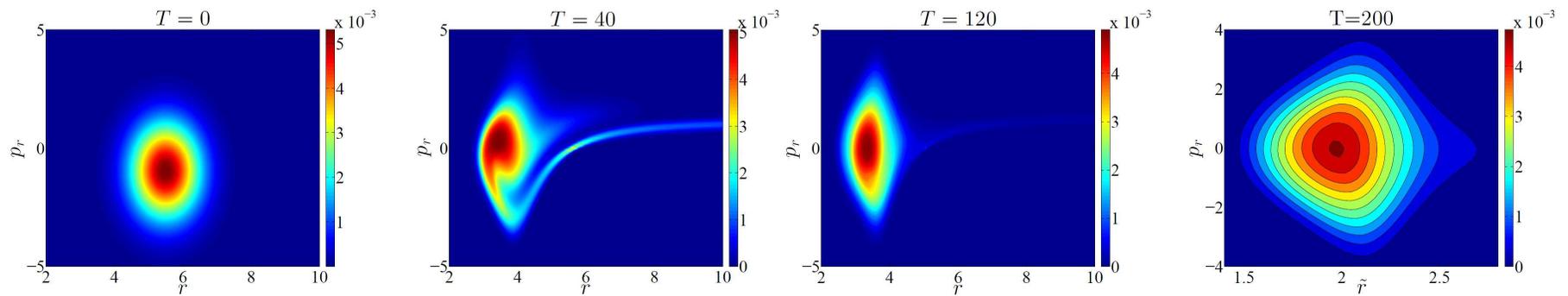
$$\sup_{r \geq 0} \frac{2m(r)}{r} \leq \frac{8}{9}$$

holds (A. 2008). Shell solutions have the property that  $2m(r)/r \rightarrow \frac{8}{9}$  as an infinitely thin shell is approached (A. 2007). The energy condition is satisfied for the spherically symmetric geons studied by Wheeler.

- Spherically symmetric, arbitrary thin, shell solutions to the massive Einstein-Vlasov system exist (A. 2007). This is the key result for showing existence of static solutions in the massless case.

## 1.5 Massless case - critical collapse

A numerical work on critical collapse for the massless Einstein-Vlasov system by Akbarian and Choptuik from 2014 was one reason that motivated us to try to show existence of massless static solution.



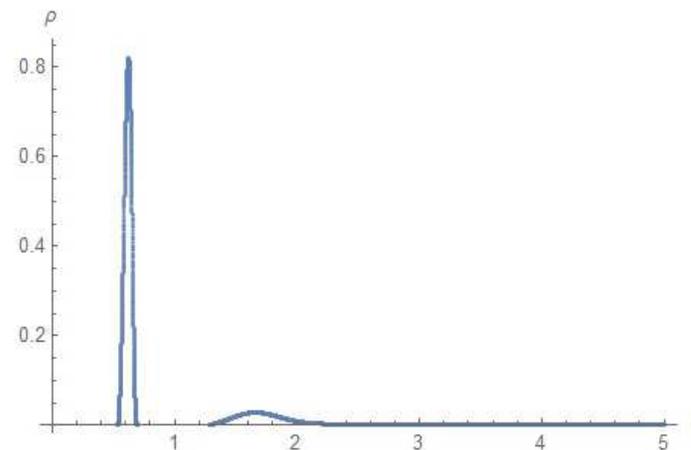
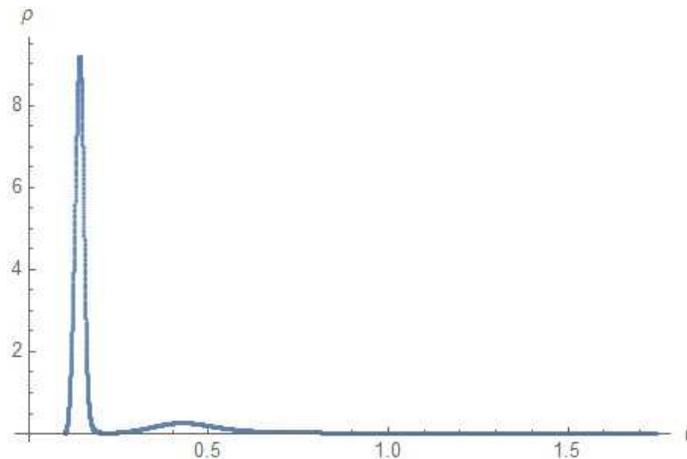
Source: A. AKBARIAN & M. CHOPTUIK, *Critical collapse in the spherically-symmetric Einstein-Vlasov model*, Phys. Rev. D **90**, 104023 (2014).

Remark: The range of the radial coordinate in the last frame is reduced.

## 2 Main results

## 2.1 Structure of solutions

- In the massive case a large variety of solutions has been constructed numerically.
- In the massless case a numerical study indicate that the features of the solutions are similar except that all solutions have infinitely extended "atmospheres".



## 2.2 The main result

**Theorem.** (*A., Fajman and Thaller '15*)

*There exist static, spherically symmetric, asymptotically flat solutions to the massless Einstein-Vlasov system, with compactly supported matter quantities. These solutions have the property that*

$$\frac{4}{5} < \sup_{r \in [0, \infty)} \frac{2m(r)}{r} < \frac{8}{9}.$$

## 2.3 Conjecture

**Conjecture.** *Let  $m(r)$  be the Hawking mass of a solution of the massless Einstein-Vlasov system and  $r \in [0, \infty)$  be the areal radius. Then a necessary condition for the existence of massless, asymptotically flat solutions is*

$$\Gamma := \sup_{r \in [0, \infty)} \frac{2m(r)}{r} \geq 0.7.$$

*In particular, we conjecture that the photon shells are necessarily highly relativistic.*

**Remark:** Originally we conjectured the lower bound 0.8 rather than 0.7. Carsten Gundlach then found that the value could be decreased. The conclusion is nevertheless that photon shells are necessarily highly relativistic whereas massive shell solutions do not need to be.

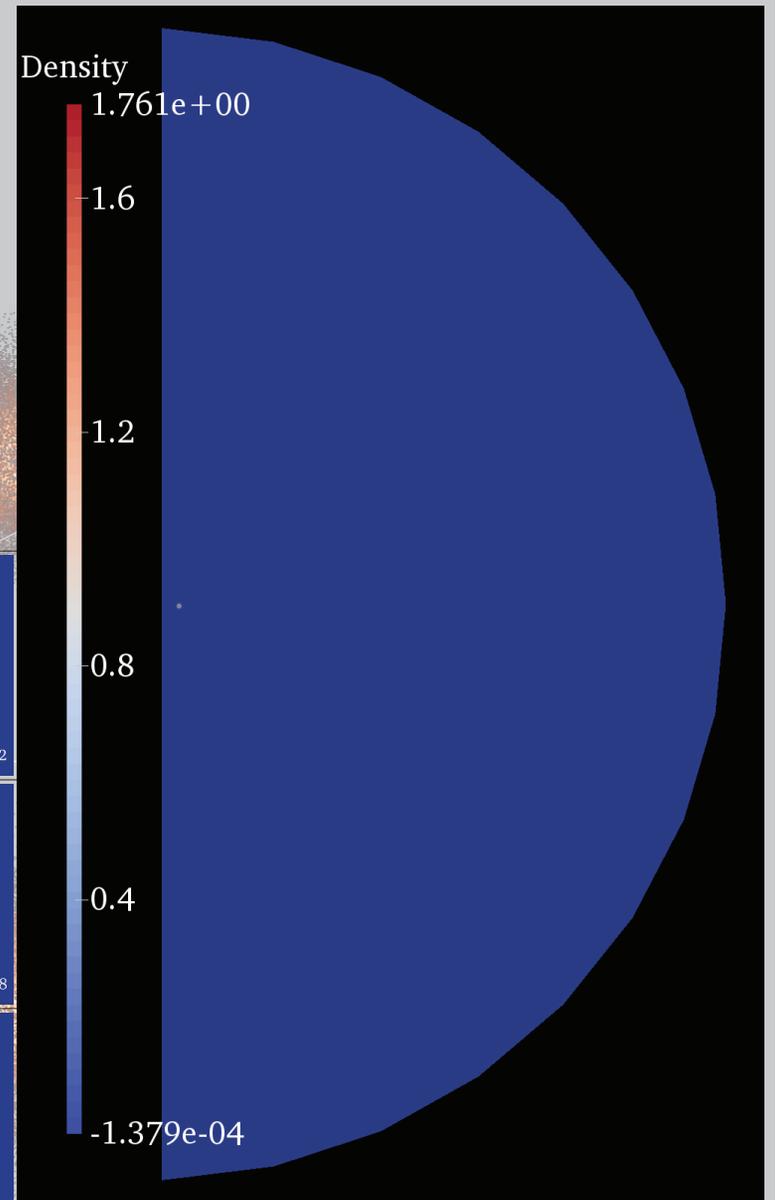
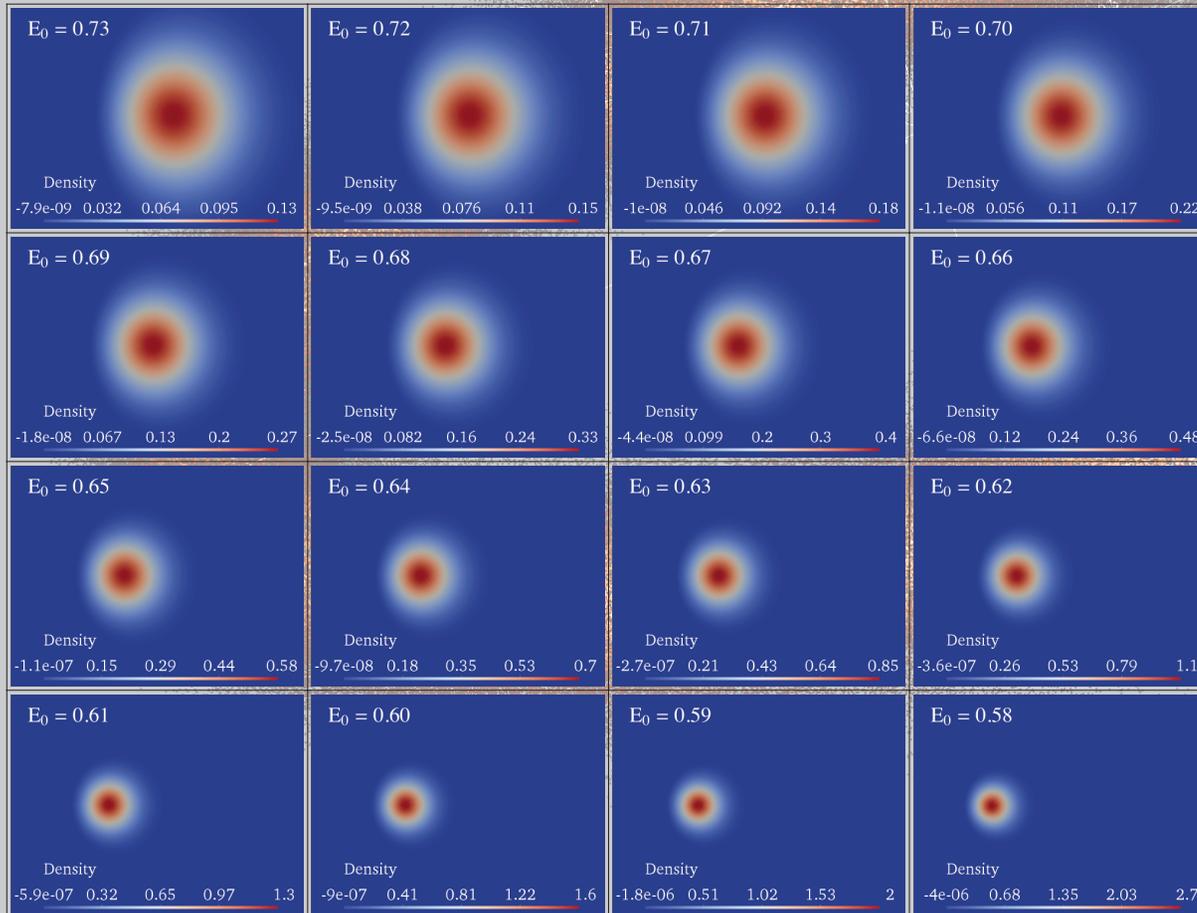
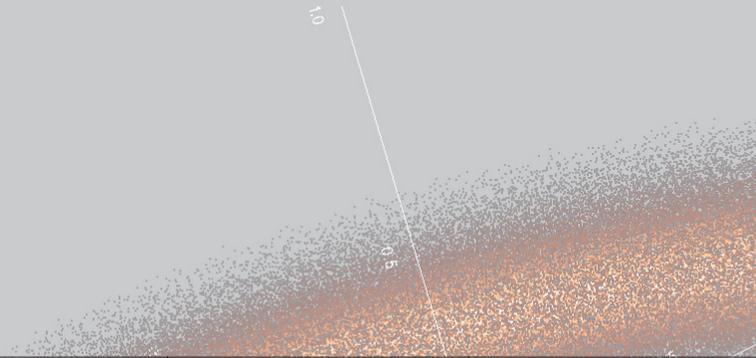
## 2.4 Remarks

- The structure of the solutions for the Einstein-Maxwell system obtained numerically by Wheeler 1955, and the solutions we construct, are thus very similar and consist of thin shell solutions for which  $2m/r \approx 8/9$ .
- Pure gravitational geons have been studied by Brill and Hartle 1964, and more rigorously by Anderson and Brill 1997. The structure of these solutions are again thin shell solutions for which  $2m/r \approx 8/9$ .

### 3 Stability and axisymmetric toroidal solutions

- The spherically symmetric static solutions are likely unstable.
- The original idea of Wheeler concerned toroidal geons which he argued might be stable or have longer life-times of existence.
- In a recent numerical work (together with Ames and Logg) we have constructed axisymmetric stationary solutions of the Einstein-Vlasov system in the massive case, in particular we have found highly relativistic toroidal solutions which give rise to ergoregions.

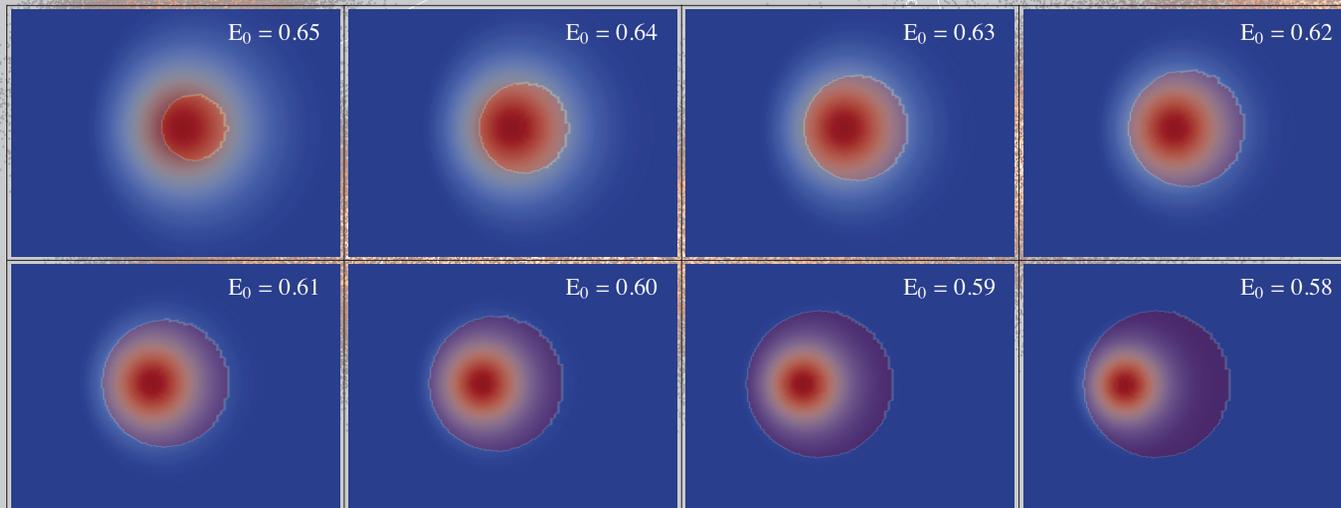
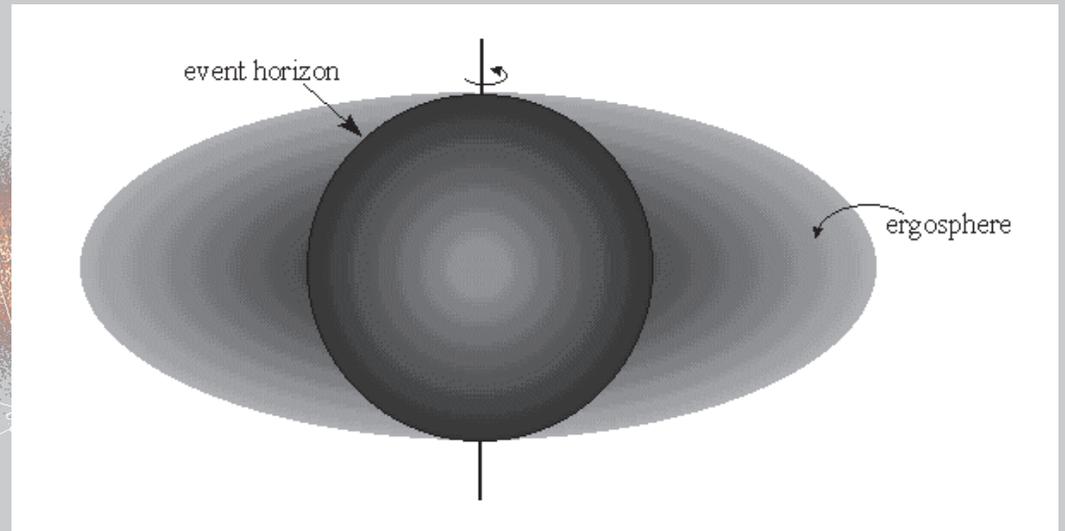
# Relativistic Toroids



# Relativistic Toroids : Existence of Ergoregions

Ergoregion is where the time-like Killing vector becomes spacelike

$$g\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial t}\right) > 0$$



## Massless toroidal solutions

- We are presently investigating if such toroidal solutions also exist in the massless case.
- Abrahams, Cook, Shapiro and Teukolsky '94 investigated the stability of toroidal solutions in the massive case and found evidence that all toroidal solutions they had constructed (without ergoregions) were stable.
- Hence, if massless toroidal solutions exist there is some hope that they are stable.

**Thank you!**