Black hole non-uniqueness from spacetime topology

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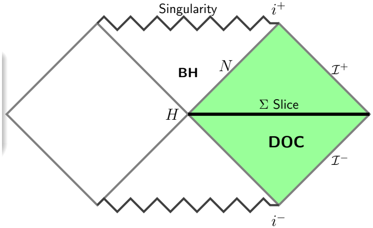
July 13 2016

Gravitational Solitons in $D = 4$

- Do *globally* regular, stationary AF spacetimes with **finite energy** exist?
- Lichnerowicz Thm: Einstein-Maxwell solitons *forbidden* $\Rightarrow$ 'no solitons without horizons' (Lichnerowicz, Gibbons, see also Anderson)
- Closely related to 4d BH uniqueness - **topological censorship** $\pi_1(\Sigma) = 0 \Rightarrow \Sigma$ has trivial topology (Friedman, Schleich, Witt)



DOC: $\langle M_{ext} \rangle = \mathbb{R} \times \Sigma$ is simply connected

Gravitational Solitons in $D > 4$

- Lichnerowicz thm in vacuum holds and in Einstein-Maxwell assuming no topology in exterior .

- $D > 4$ sp-t topology can support solitons. Simple connectedness weaker $\Rightarrow$ can have $H_p(\Sigma) \neq 0$, $p \geq 2$.

- $p = 2$: 'bubbles': supported by magnetic flux of 2-form F. Many examples in $D = 5$ supergravity (Bena, Warner...)
  Can show stationary Killing field K satiesfies $|K|_g^2 \leq 0$ globally, with $|K|_g^2 = 0$ on timelike hypersurfaces $\Rightarrow$ **evanescent** horizons. No ergoregion (Gibbons, Warner 14).

First law of Soliton Mechanics in $D = 5$ (HK,Lucietti 14)

- Consider $\mathbb{R} \times U(1)^2$-invariant spacetimes (g, F^I, X^A). Assume arbitrary $H_2(\Sigma)$.
  Strategy: express fields in terms of globally defined scalar potentials on **2d orbit space** $\mathcal{B} = \Sigma \setminus U(1)^2$.
- Variations $(\delta g, \delta F^I, \delta X^A)$ satisfy linearized field eqn + non obvious work

$$\boxed{M = \frac{1}{2}\sum_{[C]} \Psi[C] q[C] \qquad \delta M = \sum_{[C]} \Psi[C] \delta q[C]}$$

$[C]$ is a basis for 2-cycles with associated potential $\Psi[C]$ and

$$q[C] = \frac{1}{4\pi}\int_C F$$

Implications for black hole non-uniqueness

- Orbit space $\mathcal{B}$ encodes 'rod structure' (characterizes fixed points sets of $U(1)^2$ action) and determines Σ (Hollands, Yazadjiev 08)
  Known solutions have simplest possible rod structure $\Rightarrow$ **simplest topology** of Σ compatible with H
  1. Myers-Perry: $\Sigma = \mathbb{R} \times S^3$
  2. Black rings: $\Sigma = (S^2 \times D^2) - \{pt\}$ (Alaee,HK,Martniez & Chrusciel)
  3. Black lens ?

- Existence of BH with 2 cycles in the DOC represent a **gross violation of black hole uniqueness** beyond black rings (i.e. M, J, Q and H no longer characterize a black hole)

First law of black hole and soliton mechanics

- Consider sp-t with horizon and 2-cycles. In addition to bubbles there are non-contractible discs D with boundary S^1 attached to H with flux

$$\mathcal{Q}[D] = \frac{1}{4}\int_D \left(i_\xi \star F + \frac{1}{2} F \Phi_H \right)$$

- Leads to generalized first law (HK,Lucietti 14)

$$\boxed{\delta M = \kappa \delta A + \Omega_i \delta J_I + \Phi_H \delta Q + \sum_{[C]} \mathcal{Q}[C] \delta \Phi[C] + \sum_{[D]} \mathcal{Q}[D] \delta \Phi[D])}$$

and $\Phi[C]i, \Phi[D]$ are certain potentials. Note that role of charge/potential switched

- yields general derivation of 1st law for black rings (Copsey, Horowtiz) extra term due to ‘dipole charges’ contained in disc term

An explicit supersymmetric example

- Consider class of BPS 'Gibbons-Hawking' solutions of minimal supergravity with metric and Maxwell field (g, F).

$$\mathrm{d}s^2 = -f^2(\mathrm{d}t + \omega)^2 + f^{-1}\left[H^{-1}(\mathrm{d}\psi + \chi_i \mathrm{d}x^i)^2 + H\mathrm{d}x^i\mathrm{d}x^i\right]$$

Locally specified by 4 harmonic functions H, K, L, M on $\mathbb{R}^3$. f, ω, χ and F locally determined from this data (Gauntlett et al 01)

- Choose harmonic functions with 3 poles along $z-$axis

$$H = \frac{\alpha_0}{|\mathbf{x}|} + \frac{\alpha_0}{|\mathbf{x} - \mathbf{x_1}|} + \frac{\alpha_0}{|\mathbf{x} - \mathbf{x_2}|} \quad \text{and AF requires } \sum_{i=0}^{2} \alpha_i = 1$$

similarly for K, L, M.

- **Strategy:** Remove singular behaviour near poles. Fix $|\mathbf{x}| \to \mathbf{0}$ to correspond to event horizon, $\mathbf{x_1}, \mathbf{x_2}$ smooth timelike points

An explicit supersymmetric example (HK, Lucietti 14)

- **Summary:** constructed family of BPS asymptotically flat black holes with $H \simeq S^3$ specified by $(Q, J_1, J_2, q[C])$.

- **Properties:**
  1. near-horizon geometry is **globally isometric** to that of BMPV (Reall 01)
  2. exterior region contains a 2-cycle supported by magnetic flux $q[C] \neq 0$ and non-contractible disc with 'dipole' $q[D] \neq 0$. Can show $\Sigma \simeq (\mathbb{R}^4 \# (S^2 \times S^2)) \setminus B^4$.
  3. Stationary Killing field K goes *null* (never spacelike) on a timelike hypersurface in the DOC.

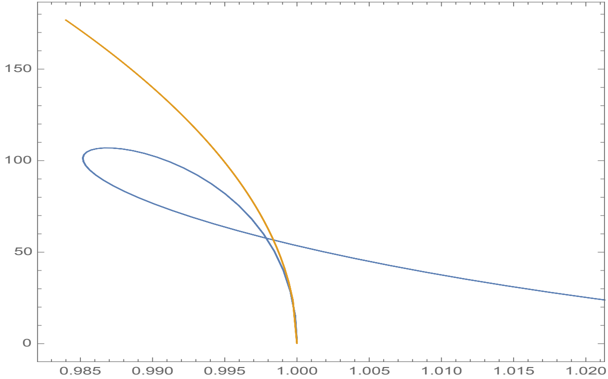
Black Hole non-uniqueness (HK, Lucietti 14, current work)

- BMPV: $H \simeq S^3$, specified by $(Q, J = J_1 = J_2)$ (Breckenridge et al. 96)
  Define
  $$\eta \equiv 1 - 6\sqrt{3}\pi \cdot \frac{J^2}{Q^3}$$
  BMPV exists only for $\eta > 0$.

- Can prove that in our family of black holes there is an **open set of regular solutions with** $J_1 = J_2$ and $\eta > 0$. These can have **same** conserved charges and NHG as BMPV but are **distinct**. Also have solutions with $\eta \leq 0$.

- Uniqueness theorem states that BMPV is the **only** BPS black hole with $H \simeq S^3$ (!) (Reall 01).
  Assumes K is timelike in the exterior.

Phase diagram for S^3 black holes



a_H vs j. Orange curve and blue curve represent BMPV and BMPV+bubble respectively. There is a region of phase space where a branch of the latter black hole has **larger entropy**.

Summary

- Black holes spacetimes containing 2-cycles in domain of outer communication exist.
  We have found first explicit examples. Expect non-BPS generalizations.

- Implications for black hole non-uniqueness and string theory microstate counting
  cf. evidence for new branches of 5D BPS black holes from microscopic counting (Haghighat, Murthy, Vafa, Vandoren 15)