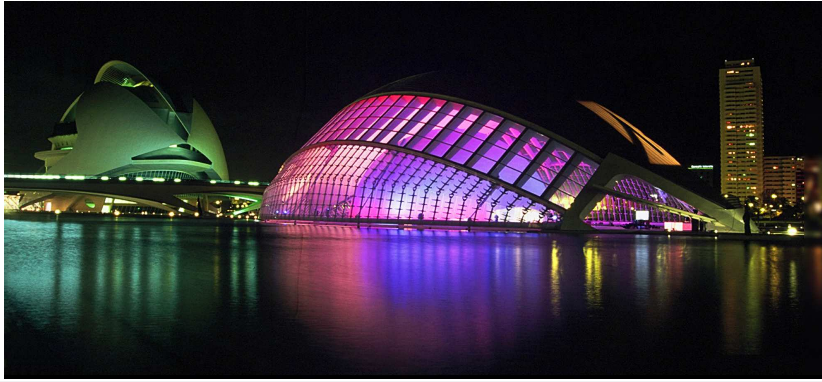

Geodesically complete black hole space-times in arbitrary dimension

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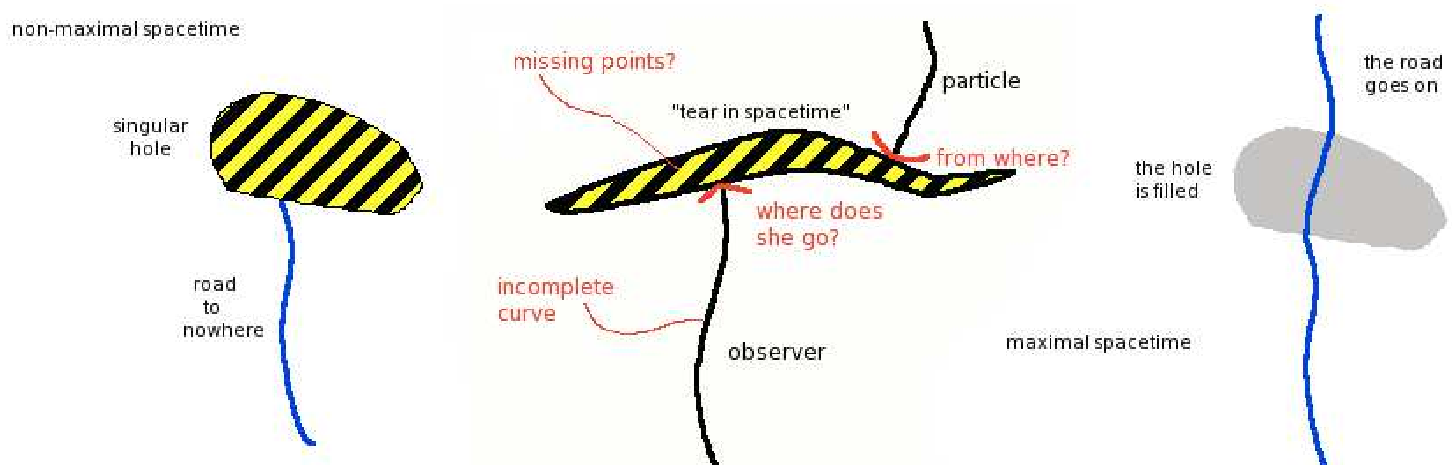
based on arXiv:1507.07763 [hep-th], PRD92(2015)044018 with

D. Bazeia, L.Losano, D. Rubiera-Garcia, and A. Sanchez-Puente

Motivations

- Geodesic completeness is a basic criterion to determine whether a space-time is singular or not.



See Geroch, Ann.Phys.(1968)

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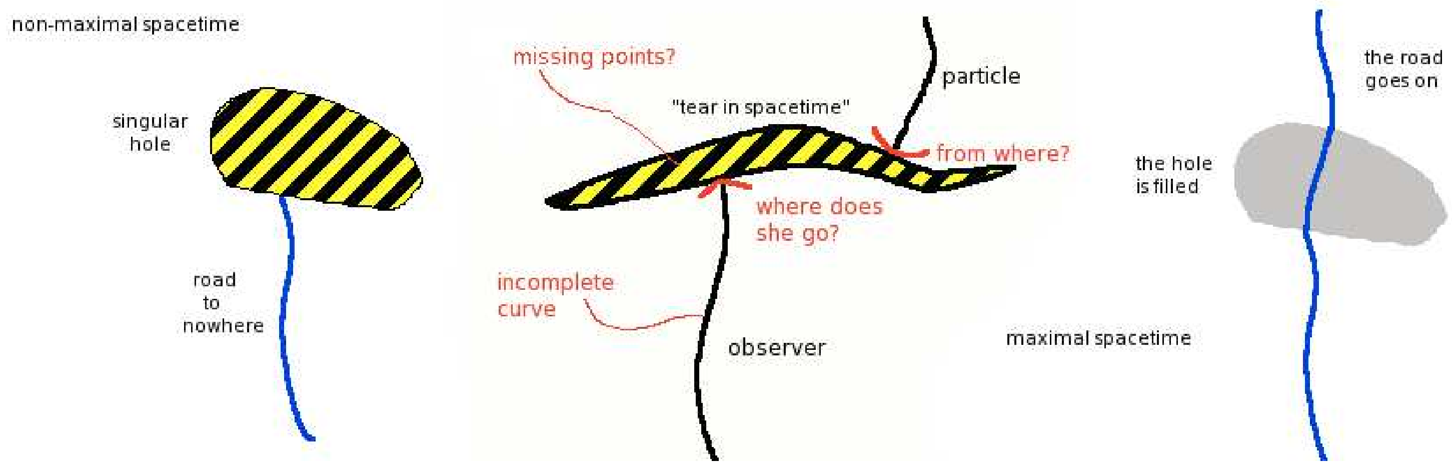

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- Incomplete geodesics and curvature divergences are two logically independent concepts that typically appear together in black hole scenarios.
  - This correlation has spreaded the view that theories with bounded curvature invariants could lead to nonsingular space-times.
  - Despite many efforts in different directions, singularities are still an important open question in black hole physics.

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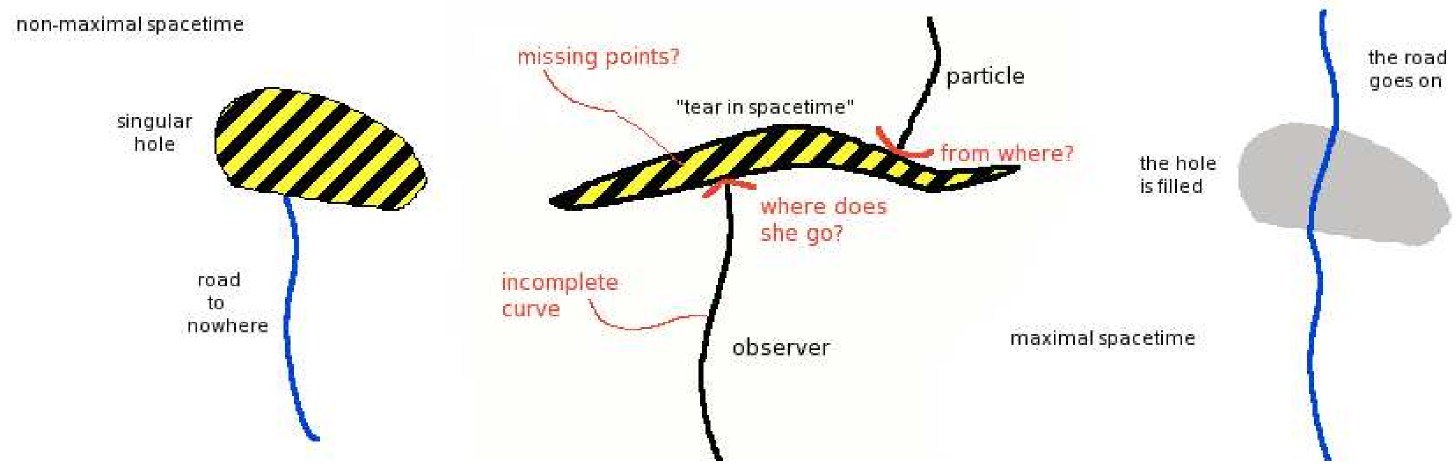

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  - This correlation has spreaded the view that theories with bounded curvature invariants could lead to nonsingular space-times.
  - Despite many efforts in different directions, singularities are still an important open question in black hole physics.
- In this talk I will provide examples of geodesically complete, and hence nonsingular, BH space-times which, nonetheless, exhibit curvature divergences.

Our framework: metric-affine gravity

- In the metric-affine (or Palatini) formalism, one assumes that $g_{\mu\nu}$ and $\Gamma^{\alpha}_{\beta\gamma}$ are independent entities: $S = \int d^n x \sqrt{-g} L[g_{\mu\nu}, \Gamma^{\alpha}_{\beta\gamma}] + S_{matter}[g_{\mu\nu}, \psi_m]$

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$$\delta S=\int d^n x\left[\sqrt{-g}\left(\frac{\delta L}{\delta g^{\mu\nu}}-\frac{L}{2}g_{\mu\nu}\right)\delta g^{\mu\nu}+\sqrt{-g}\,\frac{\delta L}{\delta\Gamma^{\alpha}_{\beta\gamma}}\,\delta\Gamma^{\alpha}_{\beta\gamma}\right]+\delta S_{matter}$$

$$\delta g^{\mu\nu}\Rightarrow\frac{\delta L}{\delta g^{\mu\nu}}-\frac{L}{2}g_{\mu\nu}=8\pi G T_{\mu\nu}$$

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- Metric approach:

The relation $\delta \Gamma^{\alpha}_{\beta\gamma} = \frac{g^{\alpha\rho}}{2} \left[\nabla_\beta \delta g_{\rho\gamma} + \nabla_\gamma \delta g_{\rho\beta} - \nabla_\rho \delta g_{\beta\gamma} \right]$ implies

$$\frac{\delta L}{\delta \Gamma^{\alpha}_{\beta\gamma}} \delta \Gamma^{\alpha}_{\beta\gamma} = \left\{ g^{\alpha\mu} \frac{\delta L}{\delta \Gamma^{\alpha}_{\lambda\nu}} - \frac{g^{\alpha\lambda}}{2} \frac{\delta L}{\delta \Gamma^{\alpha}_{\mu\nu}} \right\} \nabla_\lambda \delta g_{\mu\nu}$$ and leads to

$$\delta g^{\mu\nu} \Rightarrow \left(\frac{\delta L}{\delta g^{\mu\nu}} - \frac{L}{2} g_{\mu\nu} \right) + \nabla_\lambda \left[g_{\gamma\nu} \frac{\delta L}{\delta \Gamma^{\mu}_{\lambda\gamma}} - g_{\beta\mu} g_{\gamma\nu} g^{\alpha\lambda} \frac{\delta L}{\delta \Gamma^{\alpha}_{\beta\gamma}} \right] = 8\pi G T_{\mu\nu}$$

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- Metric and **Palatini** variations generally lead to different field equations.

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  - This is a Born-Infeld type extension of General Relativity.
  - GR is recovered at low energies: (here $\Lambda_{eff} = \frac{\lambda-1}{\varepsilon}$)

$$\lim_{\varepsilon\to 0} S = \frac{1}{2\kappa^2}\int d^nx\sqrt{-g}\left[R-2\Lambda_{eff}+\frac{\varepsilon R^2}{4}-\frac{\varepsilon}{2}R_{\mu\nu}R^{\mu\nu}+\ldots\right]+S_m$$

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The space-time metric $g_{\mu\nu}$ and the auxiliary metric $h_{\mu\nu}$ are related by a matter-induced deformation $\Omega^{\mu}{}_{\alpha}$.

Black Holes with charge in BI gravity

- The equations using $q_{\mu\nu}$ take on a very familiar form:

$$R^{\mu}{}_{\nu}(q) = \frac{\kappa^2}{|\Omega|^{1/2}}\left(\mathcal{L}_{BI}\delta^{\mu}_{\nu} + T^{\mu}{}_{\nu}\right), \text{ where } \begin{cases} \mathcal{L}_{BI} = \frac{|\Omega|^{1/2}-\lambda}{\kappa^2\varepsilon} \\ |\Omega|^{\frac{1}{2}}(\Omega^{-1})^{\mu}{}_{\nu} = \lambda\delta_{\mu}{}^{\nu} - \kappa^2\varepsilon T_{\mu}{}^{\nu} \end{cases}$$

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- Coupling BI gravity to a static, spherically symmetric electric field one finds:
  - Deformation matrix:

$$\hat{\Omega} = \begin{pmatrix} \Omega_{+}\hat{I}_{2\times 2} & \hat{0}_{n\times 2} \\ \hat{0}_{2\times n} & \Omega_{-}\hat{I}_{n\times n} \end{pmatrix}, \text{ where } \begin{cases} \Omega_{+} = \frac{\left(\lambda+z^{-2(n-2)}\right)^{\frac{2}{n-2}}}{\left(\lambda-z^{-2(n-2)}\right)^{\frac{n-4}{n-2}}} \\ \Omega_{-} = \left(\lambda - z^{-2(n-2)}\right)^{\frac{2}{n-2}} \end{cases}$$

  - Line element: $ds^2 = -\frac{A(z)}{\Omega_{+}}dt^2 + \frac{1}{A(z)\Omega_{+}}dx^2 + r^2(x)d\Omega^2_{(n-2)}$
  - Other definitions:
    - $A(z) = \left[1 - \frac{2M(z)}{r}\frac{1}{\Omega_{-}^{1/2}}\right]$, $\frac{M_z}{\delta_1 M_0} = -z^{d-2}\left(\frac{\Omega_{-}-1}{\Omega_{-}^{1/2}}\right)\left(\lambda + \frac{1}{z^{2(d-2)}}\right)$.
    - $r_q^{2(d-3)} \equiv \frac{\kappa^2 q^2}{(4\pi)}$, $l_\varepsilon^2 \equiv -\varepsilon$, $r_c{}^{2(d-2)} \equiv l_\varepsilon^2 r_q^{2(d-3)}$, $r \equiv r_c z$
    - $\delta_1 \equiv \frac{(d-3)r_c^{d-1}}{2M_0 l_\varepsilon^2}$. [See arXiv:1507.07763 [hep-th] for details!]

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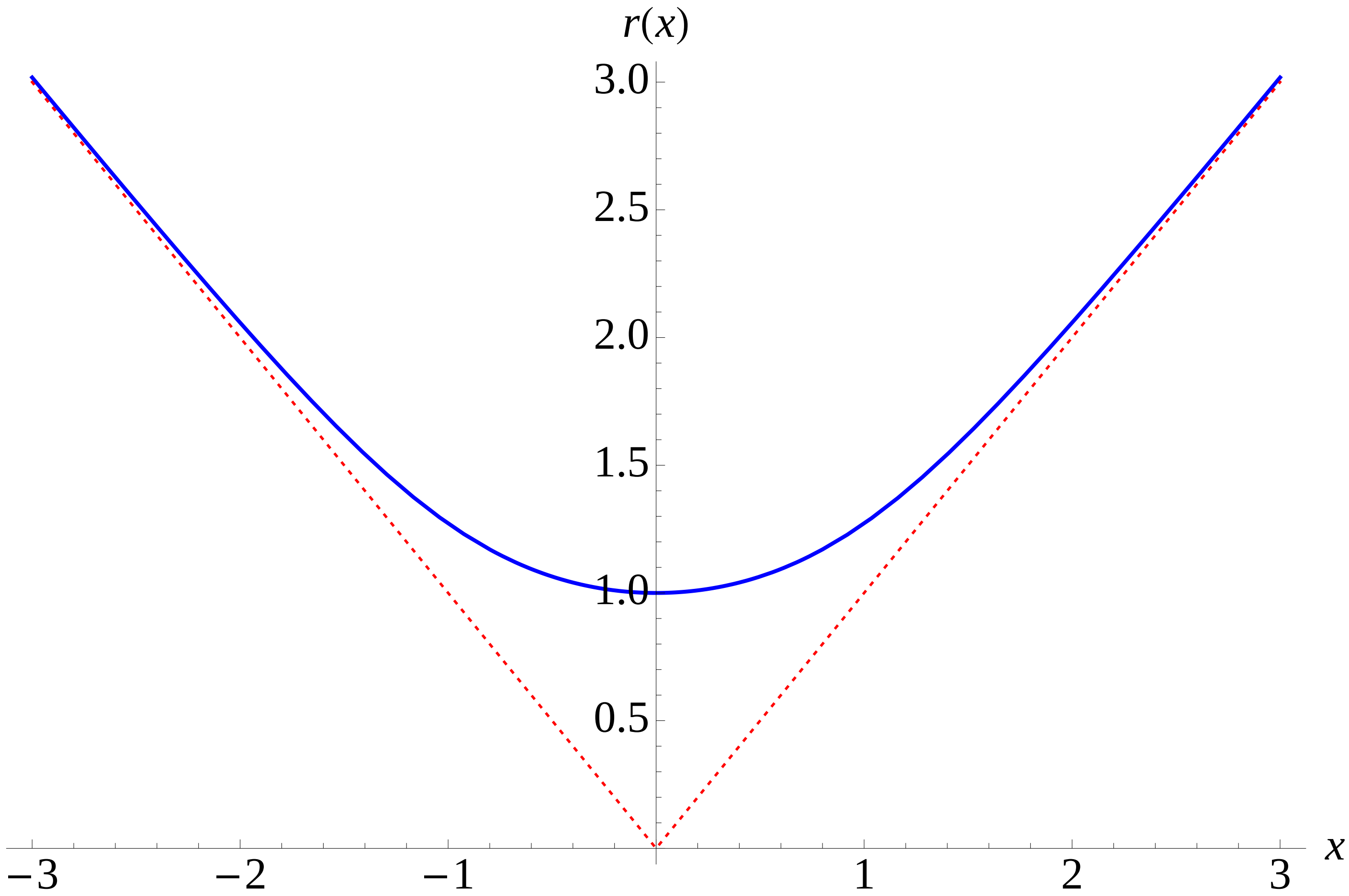
- When $z \gg 1$ (or $r \gg r_c$) the GR solutions are quickly recovered.

Wormhole structure

- The radial sector is given by $r^2(x) = \frac{x^2+\sqrt{x^4+4r_c^4}}{2}$, with a minimum at $x = 0$. This is reminiscent of a wormhole geometry.



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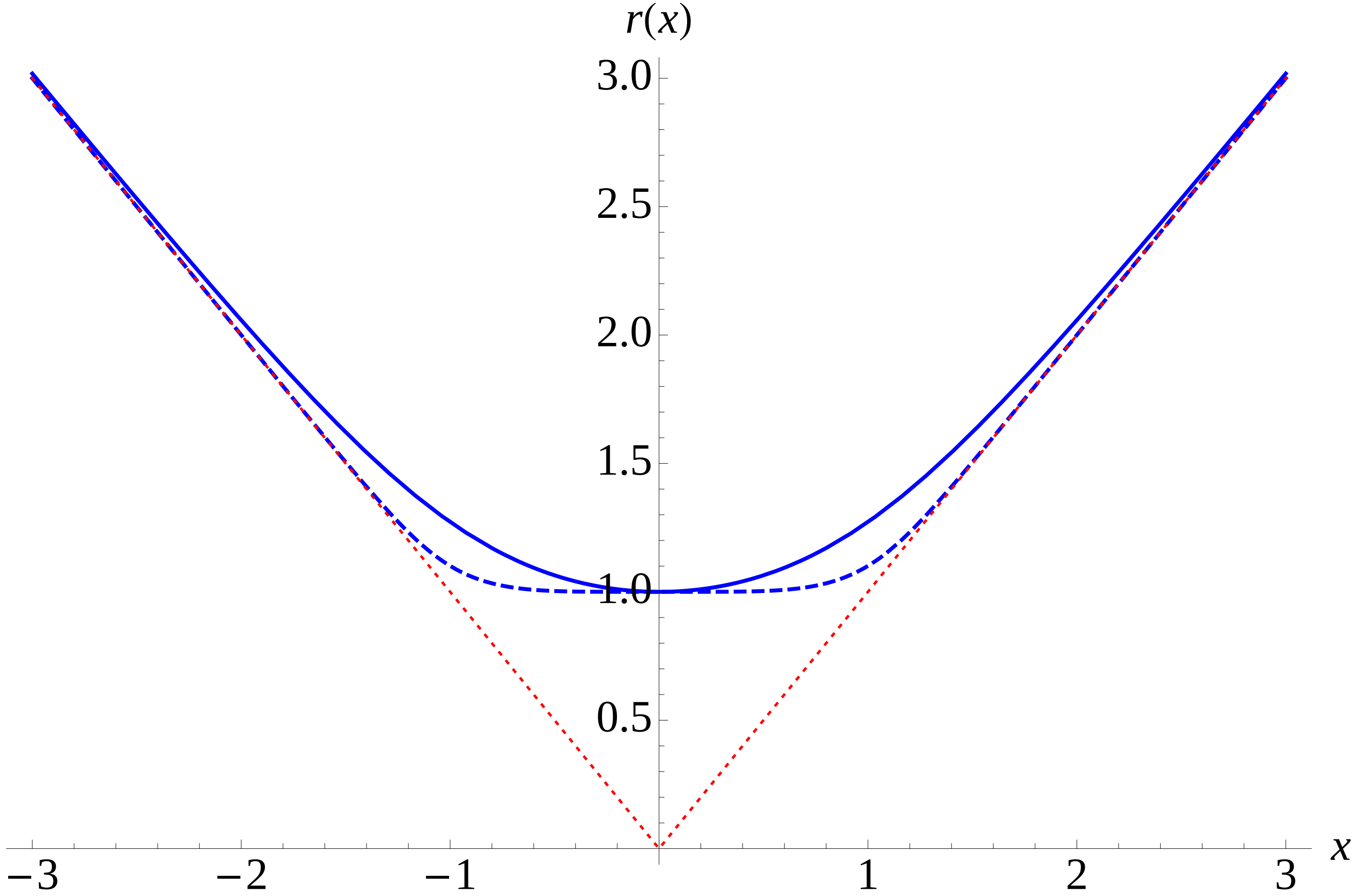

- $D = 4$ (solid), $D = 7$ (dashed)

Black Holes as Geons

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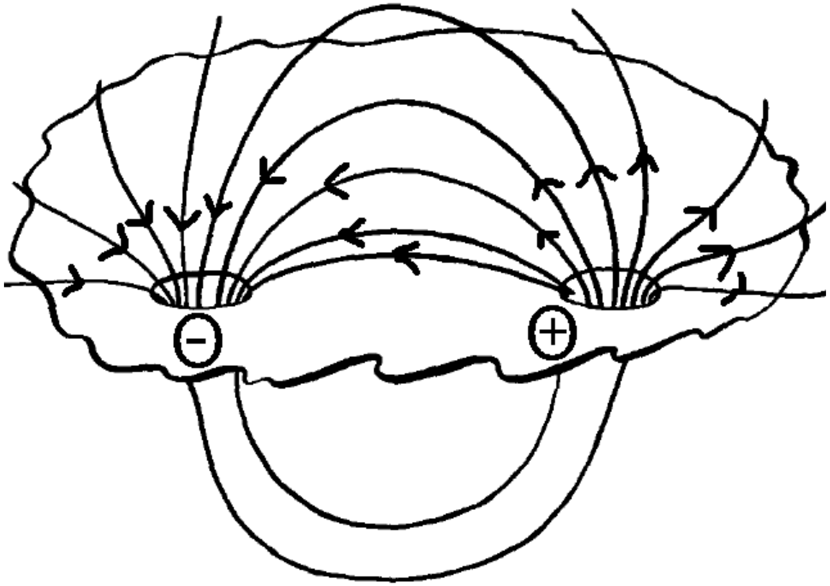
- If there is a hole at the center, where are the sources???
- The lines of force of the electric field enter through one of the wormhole mouths and exit through the other creating the illusion of a negatively charged object on one side and a positively charged object on the other.



- The locally measured electric charge is defined by the flux $\Phi \equiv \int_S *F = 4\pi q$ through any hypersurface S enclosing a wormhole mouth.
- There is no need for sources in this scenario of self-gravitating fields.
- Wheeler (1955) coined the term **geon** for regular, self-gravitating fields.

Geodesic completeness in Born-Infeld

- The equation that governs the evolution of geodesics in this space-time is:

$$\frac{1}{\Omega_+^2}\left(\frac{dx}{d\tau}\right)^2 = E^2 - V_{eff} \quad , \text{ with } \quad V_{eff} \equiv \left(\kappa + \frac{L^2}{r^2}\right)\frac{A(r)}{\Omega_+} \, .$$

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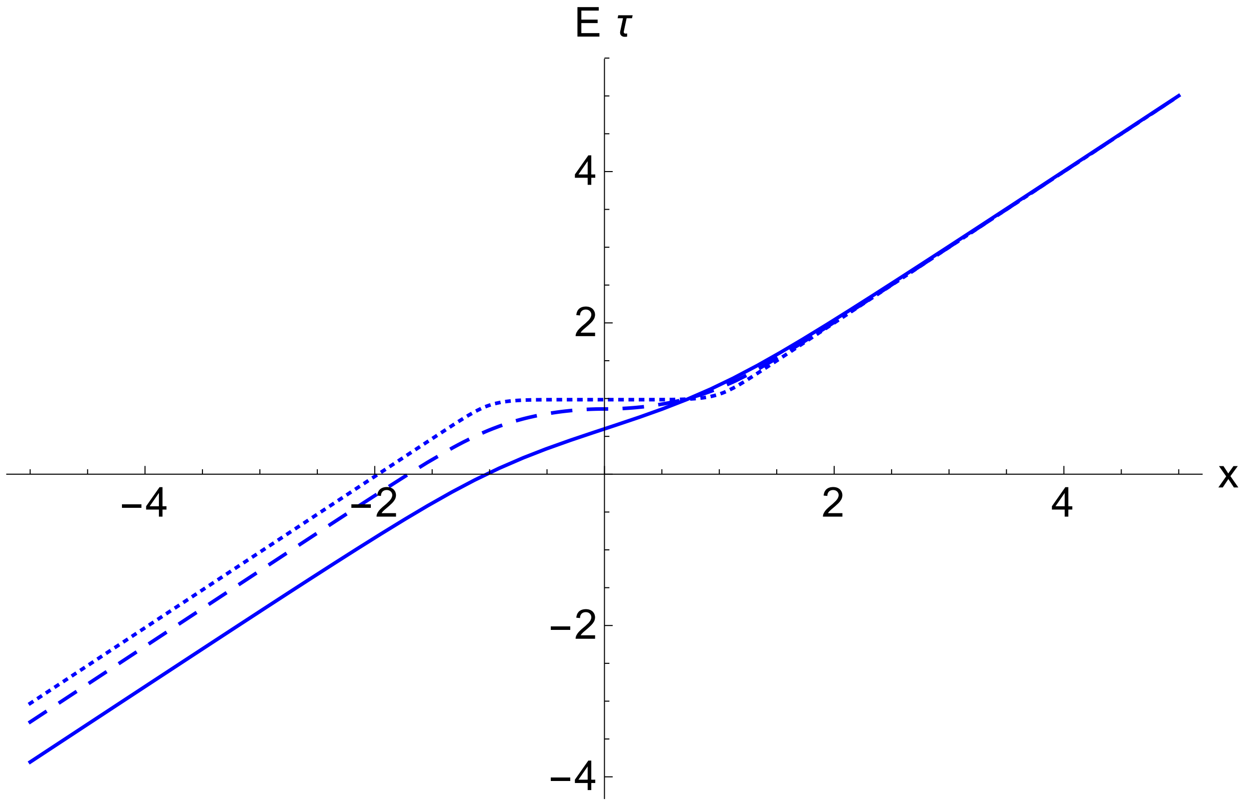

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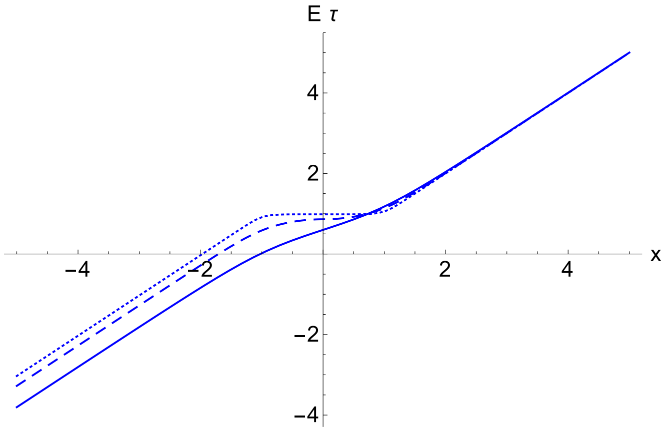

- Null and time-like geodesics with $L \neq 0$: $V_{eff} \propto \frac{(\delta_1 - \delta_d)}{|x|}$.
  - When $\delta_1 > \delta_d$ geodesics bounce before reaching the wormhole.
  - When $\delta_1 < \delta_d$ the wormhole is reached and crossed: WH case:

    $(\tau(x) - \tau_0) \propto x|x|^{d-\frac{7}{2}}$. This guarantees that $\tau(x) \in]-\infty, +\infty[$.

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- In metric-affine extensions of GR:
  - Central singularity of charged black holes replaced by a wormhole.
  - These wormholes have been discovered, not designed.
  - The WH guarantees the extendibility of geodesics.

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  - Curvature pathologies appear as a “reason” for the incompleteness.
- In metric-affine extensions of GR:
  - Central singularity of charged black holes replaced by a wormhole.
  - These wormholes have been discovered, not designed.
  - The WH guarantees the extendibility of geodesics.
- In general, curvature divergences arise at the wormhole throat.
  - This puts forward that geodesic incompleteness and curvature divergences are logically independent concepts.
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Summary and Conclusions

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- Conclusion:

The **avoidance of singularities** can be achieved with simple models in classical geometric scenarios with independent metric and affine structures.

Thanks