A group theoretic approach to shear-free radiating stars

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The spacetime

We consider the particular case of spherically symmetric, shear-free radiating stellar models. The line element for the interior for the spacetimes is given by

$$ds^2 = -A^2 dt^2 + B^2 \left[dr^2 + r^2 d\Omega^2 \right], \tag{1}$$

where A and B are metric functions of t and r, and $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$. The acceleration and and expansion are nonzero but the fluid is shear-free.

The energy momentum tensor has the form

$$T_{ab} = (\mu + p_\perp)\, u_a u_b + p_\perp g_{ab} + (p_\| - p_\perp)\chi_a \chi_b + q_a u_b + q_b u_a, \tag{2}$$

with heat flux and anisotropic stress. The fluid four-velocity $u^a = \frac{1}{A}\delta^a_0$ is comoving, χ^a is an unit four-vector along the radial direction ($u_a \chi^a = 0$), and the heat flow vector $q^a = (0, q, 0, 0)$ is radially directed ($u_a q^a = 0$).

The Einstein field equations for the heat conducting spherically symmetric anisotropic fluid (2) become

$$
\begin{aligned}
\mu &= \frac{3}{A^2}\frac{B_t^2}{B^2} - \frac{1}{B^2}\left(2\frac{B_{rr}}{B} - \frac{B_r^2}{B^2} + \frac{4B_r}{rB}\right), && \text{(3a)}\\
p_{\parallel} &= \frac{1}{A^2}\left(-2\frac{B_{tt}}{B} - \frac{B_t^2}{B^2} + 2\frac{A_t}{A}\frac{B_t}{B}\right) \\
&\quad + \frac{1}{B^2}\left(\frac{B_r^2}{B^2} + 2\frac{A_r}{A}\frac{B_r}{B} + \frac{2}{r}\frac{A_r}{A} + \frac{2}{r}\frac{B_r}{B}\right), && \text{(3b)}\\
p_{\perp} &= -\frac{2}{A^2}\frac{B_{tt}}{B} + 2\frac{A_t}{A^3}\frac{B_t}{B} - \frac{1}{A^2}\frac{B_t^2}{B^2} + \frac{1}{r}\frac{A_r}{A}\frac{1}{B^2} \\
&\quad + \frac{1}{r}\frac{B_r}{B^3} + \frac{A_{rr}}{A}\frac{1}{B^2} - \frac{B_r^2}{B^4} + \frac{B_{rr}}{B^3}, && \text{(3c)}\\
q &= -\frac{2}{AB^2}\left(-\frac{B_{rt}}{B} + \frac{B_r B_t}{B^2} + \frac{A_r}{A}\frac{B_t}{B}\right), && \text{(3d)}
\end{aligned}
$$

for the line element (1). The equations (3) describe the gravitational interactions in the interior of a shear-free spherically symmetric star with heat flux and anisotropic pressures.

The boundary of a radiating star divides the spacetime into interior and exterior regions. The interior spacetime (1) has to match across the boundary of the star to the Vaidya spacetime

$$ds^2 = -\left(1 - \frac{2m(v)}{R}\right) dv^2 - 2dvdR + R^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \qquad (4)$$

which is the exterior. Here the quantity $m(v)$ denotes the mass of the star as measured by an observer at infinity. Matching leads to the junction condition

$$(p_{\parallel})_{\Sigma} = (Bq)_{\Sigma}, \qquad (5)$$

where the hypersurface Σ defines the boundary of the radiating sphere.

The boundary of a radiating star divides the spacetime into interior and exterior regions. The interior spacetime (1) has to match across the boundary of the star to the Vaidya spacetime

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which is the exterior. Here the quantity $m(v)$ denotes the mass of the star as measured by an observer at infinity. Matching leads to the junction condition

$$(p_{\|})_{\Sigma} \;=\; (Bq)_{\Sigma}, \quad (5)$$

where the hypersurface Σ defines the boundary of the radiating sphere. The junction condition (5) together with the field equations (2) gives

$$2\frac{B_{rt}}{AB^2} + 2\frac{B_{tt}}{A^2B} - 2\frac{A_tB_t}{A^3B} - 2\frac{B_rB_t}{AB^3} - 2\frac{A_rB_r}{AB^3} - 2\frac{A_rB_t}{A^2B^2} - \frac{B_r^2}{B^4} + \frac{B_t^2}{A^2B^2} - 2\frac{A_r}{rAB^2} - 2\frac{B_r}{rB^3} = 0, \quad (6)$$

valid at the boundary of a shear-free radiating star.

Lie symmetry analysis

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DOI 10.1007/s10714-011-1268-5

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Regular Article - Theoretical Physics

A group theoretic approach to shear-free radiating stars

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Regular Article - Theoretical Physics

A group theoretic approach to shear-free radiating stars

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Radiating stars in the presence of electric charge

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We consider the general model of an accelerating, expanding and shearing radiating star in the presence of charge. Using a new set of variables arising from the Lie symmetries of differential equations we transform the boundary equation into ordinary differential equations. We present several new exact solutions for the master equation. A particular family of solution may be interpreted as a charged generalised Euclidean stars. This family admits a linear barotropic equation of state. In the uncharged limit we regain stellar models where proper and areal radii are equal, and its generalisation.

An nth order differential equation

$$F\left(r, t, A, B, A_r, B_r, A_t, B_t, A_{rr}, B_{rr}, A_{rt}, B_{rt}, A_{tt}, B_{tt}, \dots\right) = 0 \quad (7)$$

where $A = A(r,t)$ and $B = B(r,t)$, admits a Lie symmetry of the form

$$\begin{aligned} G &= \xi_1\left(r, t, A, B\right)\frac{\partial}{\partial r} + \xi_2\left(r, t, A, B\right)\frac{\partial}{\partial t} \\ &\quad + \eta_1\left(r, t, A, B\right)\frac{\partial}{\partial A} + \eta_2\left(r, t, A, B\right)\frac{\partial}{\partial B} \end{aligned} \quad (8)$$

provided that

$$G^{[n]}F\Big|_{F=0} = 0 \quad (9)$$

where $G^{[n]}$ is the nth prolongation of the symmetry G. The process is algorithmic and so can be implemented by computer algebraic packages.

Using `PROGRAM LIE` (Head 1993), we find the set of symmetries

$$
\begin{aligned}
G_1 &= -Af'(t)\frac{\partial}{\partial A} + f(t)\frac{\partial}{\partial t}, && \text{(10a)}\\
G_2 &= A\frac{\partial}{\partial A} + B\frac{\partial}{\partial B}, && \text{(10b)}\\
G_3 &= A\frac{\partial}{\partial A} + r\frac{\partial}{\partial r}, && \text{(10c)}
\end{aligned}
$$

where $f(t)$ is an arbitrary function of t.

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$$G_2 = A\frac{\partial}{\partial A} + B\frac{\partial}{\partial B}, \tag{10b}$$

$$G_3 = A\frac{\partial}{\partial A} + r\frac{\partial}{\partial r}, \tag{10c}$$

where $f(t)$ is an arbitrary function of t. We take a general linear combination

$$aG_1+bG_2+cG_3 = \left[c+b-af'(t)\right]A\frac{\partial}{\partial A}+bB\frac{\partial}{\partial B}+af(t)\frac{\partial}{\partial t}+cr\frac{\partial}{\partial r}, \tag{11}$$

to reduce the partial differential equation (6) into ordinary differential equations for further analysis. Note that a, b and c are arbitrary constants.

Then (11) gives the invariants

$$
\begin{aligned}
x &= \frac{\exp\left(\int^t \frac{dt}{af(t)}\right)}{r^{1/c}}, && \text{(12a)} \\
A &= \frac{h(x)}{f(t)} \exp\left(\int^t \frac{cdt}{af(t)} + \int^t \frac{bdt}{af(t)}\right), && \text{(12b)} \\
B &= g(x) r^{b/c}, && \text{(12c)}
\end{aligned}
$$

where $a \neq 0$ and $c \neq 0$. Note that g and h being arbitrary functions of x.

Then (11) gives the invariants

$$x = \frac{\exp\left(\int^t \frac{dt}{af(t)}\right)}{r^{1/c}}, \tag{12a}$$

$$A = \frac{h(x)}{f(t)} \exp\left(\int^t \frac{cdt}{af(t)} + \int^t \frac{bdt}{af(t)}\right), \tag{12b}$$

$$B = g(x) r^{b/c}, \tag{12c}$$

where $a \neq 0$ and $c \neq 0$. Note that g and h being arbitrary functions of x. Using the invariants (12) we can write (6) in the form

$$\begin{aligned}
&\left[2a^2 g x^{2b+2c+1}\left((b+c)g - xg'\right)\right] h' h^2 + 2acx^{b+c+2} g^2 g' h' h \\
&-2c^2 x^2 g^3 g' h' + \left[c^2 x g^2 \left(xg'^2 - 2g\left((b+c-1)g' - xg''\right)\right)\right] h \\
&-\left[2acg x^{b+c+1}\left(g\left(xg'' + g'\right) - xg'^2\right)\right] h^2 + \left[a^2 x^{2(b+c)}\left(xg' - bg\right)\right. \\
&\left.\times\left((b+2c)g - xg'\right)\right] h^3 = 0,
\end{aligned} \tag{13}$$

where primes denote differentiation with respect to to the new variable x.

To progress we make the assumption

$$g(x) = kh(x) \text{ and } y = \frac{h'}{h}. \tag{14}$$

Then the transformation (14) enables us to write (13) in the form

$$y' + \frac{2a^2bx^{2(b+c)} + acx^{b+c}\left(2ax^{b+c} - k\right) + (1-b)c^2k^2 - c^3k^2}{ckx\left(ck - ax^{b+c}\right)}y + \left(\frac{3ax^{b+c}}{2ck} + \frac{1}{2}\right)y^2 + \frac{a^2b(b+2c)x^{2(b+c-1)}}{2ck\left(ax^{b+c} - ck\right)} = 0. \tag{15}$$

Observe that (15) is a Riccati equation in the quantity y. Riccati equations can be transformed to second order linear equations.

We let

$$u(x) = \exp\left[\int^x \left(\frac{3ax^{b+c}}{2ck} + \frac{1}{2}\right) y(x)dx\right]. \tag{16}$$

Note that in equation (16) the term $\left(\frac{3ax^{b+c}}{2ck} + \frac{1}{2}\right)$ is the coefficient of the quantity y^2 in (15). Using (16) equation (15) is transformed to

$$u'' + \gamma(x)u' + \zeta(x)u = 0, \tag{17}$$

where

$$\begin{aligned}\gamma(x) &= \Big[6a^3bx^{3(b+c)} + a^2cx^{2(b+c)}\left(6ax^{b+c} + (5b-3)k\right) - c^3k^2 \\ &\quad \left(6ax^{b+c} + (b-1)k\right) + ac^2kx^{b+c}\left(5ax^{b+c} + (2-6b)k\right) \\ &\quad -c^4k^3\Big]\left[ckx\left(-3a^2x^{2(b+c)} + 2ackx^{b+c} + c^2k^2\right)\right]^{-1}, \qquad (18a)\\ \zeta(x) &= \frac{a^2b(b+2c)x^{2(b+c-1)}\left(3ax^{b+c} + ck\right)}{4c^2k^2\left(ax^{b+c} - ck\right)}. \qquad (18b)\end{aligned}$$

Therefore we have the remarkable feature that the second order nonlinear equation (13) has been transformed to the linear equation (17) via the transformations (14) and (16).

Case $b = -c$

If we set $b = -c$ then equation (17) becomes

$$x^2 u'' + xu' + \frac{a^2(3a+ck)}{4k^2(ck-a)} u = 0, \tag{19}$$

which is a simpler form. It is interesting to note that this case produces the Euler equation (19). We can integrate (19) to give

$$u(x) = \tilde{c}_1 \cosh\left(\frac{a\sqrt{3a+ck}}{2k\sqrt{a-ck}} \log(x)\right) + \tilde{c}_2 \sinh\left(\frac{a\sqrt{3a+ck}}{2k\sqrt{a-ck}} \log(x)\right), \tag{20}$$

where $\tilde{c}_1$ and $\tilde{c}_2$ are arbitrary constants of integration. Then from (16) we obtain

$$y(x) = \frac{ac\sqrt{3a+ck}}{\sqrt{a-ck}} \frac{c_1 x^{\frac{a\sqrt{3a+ck}}{k\sqrt{a-ck}}} - c_2}{x\left(3ax^c + ck\right)\left(c_1 x^{\frac{a\sqrt{3a+ck}}{k\sqrt{a-ck}}} + c_2\right)}, \tag{21}$$

where $c_1 = \tilde{c}_1 + \tilde{c}_2$ and $c_2 = \tilde{c}_1 - \tilde{c}_2$.

Hence we have the potentials

$$
\begin{aligned}
A &= \frac{1}{f(t)}\left(m\left[r^{-1/c}\exp\left(\int^t \frac{dt}{af(t)}\right)\right]^{\frac{a\sqrt{3a+ck}}{2k\sqrt{a-ck}}}\right. \\
&\qquad \left. +n\left[r^{-1/c}\exp\left(\int^t \frac{dt}{af(t)}\right)\right]^{-\frac{a\sqrt{3a+ck}}{2k\sqrt{a-ck}}}\right)^{\frac{2ck}{3a+ck}}, \qquad (22a) \\
B &= k\frac{f(t)}{r}A, \qquad (22b)
\end{aligned}
$$

where $m = c_1 c_3^{\frac{3a+ck}{2ck}}$ and $n = c_2 c_3^{\frac{3a+ck}{2ck}}$ are constants. This is a new solution to the master equation.

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&\qquad \left. +n\left[r^{-1/c}\exp\left(\int^t \frac{dt}{af(t)}\right)\right]^{-\frac{a\sqrt{3a+ck}}{2k\sqrt{a-ck}}}\right)^{\frac{2ck}{3a+ck}}, \qquad (22a) \\
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\end{aligned}
$$

where $m = c_1 c_3^{\frac{3a+ck}{2ck}}$ and $n = c_2 c_3^{\frac{3a+ck}{2ck}}$ are constants. This is a new solution to the master equation. The line element for this case is

$$
ds^2 = \left[\left(m\psi^{1/2} + n\psi^{-1/2}\right)^{\frac{2ck}{3a+ck}}\right]^2 \left(-\left[\frac{1}{f(t)}\right]^2 dt^2 + \left[\frac{k}{r}\right]^2 \left[dr^2 + r^2 d\Omega^2\right]\right), \qquad (23)
$$

where $\psi = \left[r^{-1/c}\exp\left(\int^t \frac{dt}{af(t)}\right)\right]^{\frac{a\sqrt{3a+ck}}{k\sqrt{a-ck}}}$.

The matter variables become

$$\mu = \frac{2\left(a^2 - ack + c^2k^2\right)\left(ck\left(m^2\psi^2 - 4mn\psi + n^2\right) - 6amn\psi\right)}{ck^3(a-ck)(3a+ck)\left(m\psi+n\right)^2\left(\psi^{-1/2}\left(m\psi+n\right)\right)^{\frac{4ck}{3a+ck}}},$$

$$p_{\|} = \frac{2a\left(6amn\psi - ck\left(m^2\psi^2 - 4mn\psi + n^2\right)\right)}{k^2(ck-a)(3a+ck)\left(m\psi+n\right)^2\left(\psi^{-1/2}\left(m\psi+n\right)\right)^{\frac{4ck}{3a+ck}}},$$

$$p_{\perp} = \frac{(a+ck)\left(\psi^{-1/2}\left(m\psi+n\right)\right)^{-\frac{4ck}{3a+ck}}\left(12amn\psi + ck\left(m\psi+n\right)^2\right)}{ck^3(3a+ck)\left(m\psi+n\right)^2\left(\psi^{-1/2}\left(m\psi+n\right)\right)^{\frac{4ck}{3a+ck}}},$$

$$q = \left[\frac{k}{r}\left(m\psi^{1/2} + n\psi^{-1/2}\right)^{\frac{2ck}{3a+ck}}\right]^{-1} p_{\|}.$$

From the above we generate the linear barotropic equation of state

$$p_{\|} = \lambda\mu, \quad \lambda = \frac{ack}{a^2 - ack + c^2k^2}, \tag{25}$$

provided that $k \neq -\frac{3a}{c}$.

Case: $k = -3a/c$ and $b = -c$

If we set $k = -3a/c$ and $b = -c$, then equation (13) becomes

$$24x^2hh'' - 24x^2h'^2 + 24xhh' + c^2h^2 = 0, \tag{26}$$

which is greatly simplified. Now (26) can be integrated to give

$$h(x) = \frac{nx^m}{\exp\left[\frac{c^2}{48}\log^2(x)\right]}, \tag{27}$$

where m and n are constants of integration.

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where m and n are constants of integration.
Hence we obtain the metric functions

$$A = \frac{n}{f(t)} \frac{\left[r^{-1/c}\exp\left(\int^t \frac{dt}{af(t)}\right)\right]^m}{\exp\left(\frac{c^2}{48}\log^2\left[r^{-1/c}\exp\left(\int^t \frac{dt}{af(t)}\right)\right]\right)}, \tag{28a}$$

$$B = -\frac{3a}{c}\frac{f(t)}{r}A. \tag{28b}$$

This is also a new solution to the master equation (6).

The line element for this case given by

$$ds^2 = \left[\frac{n\varphi^m}{\exp\left(\frac{c^2}{48}\log^2\varphi\right)}\right]^2 \left[-\left(\frac{1}{f(t)}\right)^2 dt^2 + 9\left(\frac{a}{cr}\right)^2 \left[dr^2 + r^2 d\Omega^2\right]\right], \tag{29}$$

where $\varphi = r^{-1/c}\exp\left(\int^t \frac{dt}{af(t)}\right)$.

The line element for this case given by

$$ds^2 = \left[\frac{n\varphi^m}{\exp\left(\frac{c^2}{48}\log^2\varphi\right)}\right]^2 \left[-\left(\frac{1}{f(t)}\right)^2 dt^2 + 9\left(\frac{a}{cr}\right)^2\left[dr^2 + r^2 d\Omega^2\right]\right], \tag{29}$$

where $\varphi = r^{-1/c}\exp\left(\int^t \frac{dt}{af(t)}\right)$.The matter variables become

$$\begin{aligned}
\mu &= \frac{13\left(c^4\log^2\varphi - 48mc^2\log\varphi + 24\left(c^2+24m^2\right)\right)\exp\left(\frac{1}{24}c^2\log^2\varphi\right)}{2592a^2n^2\varphi^{2m}},\\
p_{\|} &= \frac{\left(48mc^2\log\varphi - c^4\log^2\varphi - 24\left(c^2+24m^2\right)\right)\exp\left(\frac{1}{24}c^2\log^2\varphi\right)}{864a^2n^2\varphi^{2m}},\\
p_{\perp} &= \frac{\left(48mc^2\log\varphi - c^4\log^2\varphi + 48c^2 + 576m^2\right)\exp\left(\frac{1}{24}c^2\log^2\varphi\right)}{648a^2n^2\varphi^{2m}},\\
q &= \left[\frac{3a}{cr}\frac{n\varphi^m}{\exp\left(\frac{c^2}{48}\log^2\varphi\right)}\right]^{-1} p_{\|}.
\end{aligned}$$

We also have $p_{\|} = \lambda\mu, \quad \lambda = -\frac{3}{13}$, which is linear and barotropic.

Case: $b = \left(\pm\frac{\sqrt{3}}{3} - 1\right) c$

In this case equation (13) becomes

$$6gg'h' - 3\left(2gg'' + g'^2\right)h \pm 2\sqrt{3}g'h^2 = 0. \tag{31}$$

The advantage of (31) is that it is a Bernoulli equation in h. Here g is unspecified. We integrate (31) to obtain

$$h(t) = \pm\frac{\sqrt{3}}{2}\frac{g'\sqrt{g}}{\sqrt{g} + d}, \tag{32}$$

where d is a constant of integration.

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$$h(t) = \pm\frac{\sqrt{3}}{2}\frac{g'\sqrt{g}}{\sqrt{g}+d}, \tag{32}$$

where d is a constant of integration. Hence the potentials functions become

$$A = \pm\frac{\sqrt{3}}{2}\frac{g'\sqrt{g}}{\sqrt{g}+d}r^{\pm\frac{\sqrt{3}}{3}}, \tag{33a}$$

$$B = gr^{\pm\frac{\sqrt{3}}{3}-1}, \tag{33b}$$

which is a new exact solution for the shear-free model.

The line element for this model becomes

$$ds^2 = -\frac{3}{4}g'^2 r^{\pm 2\frac{\sqrt{3}}{3}} dt^2 + g^2 r^{\pm\frac{2\sqrt{3}}{3}-2}\left[dr^2 + r^2 d\Omega^2\right]. \qquad (34)$$

from (33). In the above we have set the arbitrary constant $d = 0$ without any loss of any generality.

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from (33). In the above we have set the arbitrary constant $d = 0$ without any loss of any generality. The matter variables become

$$\begin{aligned} \mu &= \frac{14 r^{\mp\frac{2}{\sqrt{3}}}}{3g^2}, \\ p_{\parallel} &= -\frac{4 r^{\mp\frac{2}{\sqrt{3}}}}{3g^2}, \\ p_{\perp} &= -\frac{r^{\mp\frac{2}{\sqrt{3}}}}{g^2}, \\ q &= \left[g r^{\pm\frac{\sqrt{3}}{3}-1}\right]^{-1} p_{\parallel}. \end{aligned}$$

This solution also satisfies the barotropic equation of state.

$$p_{\parallel} = \lambda\mu, \quad \lambda = -\frac{2}{7}, \tag{36}$$

which is linear.

Summary

- We developed the junction condition equation that relates the radial pressure to the heat flux which a highly nonlinear partial differential equation in the metric functions.
- We demonstrated that this equation admits three Lie point symmetries.
- Using the general linear combination of these symmetries we reduced the governing highly nonlinear partial differential equation to ordinary differential equations.
- By solving the reduced ordinary differential equations and transforming to the original variables we obtained exact solutions for the master equation.
- We present the line element explicitly in each case and show that our solutions obey the linear barotropic equation of state.

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Thank You!