Thinking outside the truncation: new hair for holographic superconductors

Gavin S. Hartnett

work with Jorge Santos and Harvey Reall, *work in progress*

Wednesday 13^{th} July, 2016

GR21
Columbia University

Holographic Superconductors

Originally proposed by Hartnoll, Herzog, Horowitz (2008)

Consider 4D theory of Einstein-Maxwell-charged scalar

$$S = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2 - |\mathcal{D}\phi|^2 - V(\phi)\right), \qquad \mathcal{D}_\mu = \nabla_\mu - iqA_\mu\,.$$

$$ds^2 = -fdt^2 + \frac{dr^2}{g} + r^2\left(dx_1^2 + dx_2^2\right), \quad \phi = \phi(r)\,. \tag{1}$$

For appropriate (q, V), two classes of static, $\mathbb{R}^2$-invariant solutions:

1 AdS-Reissner Nordstrom ($\phi = 0$)

2 charged AdS black hole with scalar hair ($\phi \neq 0$)

Extensions and Successes

By considering more complicated matter content and geometries, this approach has been extended in many ways, by many authors

Some examples:

- Drude behaviour
- non-linear conductivity
- lattice effects
- cuprate-based superconductors
- momentum relaxation
- anisotropies/disorder

Extensions and Successes

By considering more complicated matter content and geometries, this approach has been extended in many ways, by many authors

Some examples:

- Drude behaviour
- non-linear conductivity
- lattice effects
- cuprate-based superconductors
- momentum relaxation
- anisotropies/disorder

  Bottom-up framework

Top-down framework

It is interesting and important to consider these models in the context of precise AdS/CFT dualities

- bottom-up dualities are not fully specified or defined
- bottom-up models only retain subset of full 10/11D degrees of freedom

Top-down framework

It is interesting and important to consider these models in the context of precise AdS/CFT dualities

- bottom-up dualities are not fully specified or defined
- bottom-up models only retain subset of full 10/11D degrees of freedom
- AdS/CMT dualities derivable from 10D string theory/11D M-theory
  - have the KK degrees of freedom
  - not freely specified – constrained by compactification

Top-down framework

It is interesting and important to consider these models in the context of precise AdS/CFT dualities

- bottom-up dualities are not fully specified or defined
- bottom-up models only retain subset of full 10/11D degrees of freedom
- AdS/CMT dualities derivable from 10D string theory/11D M-theory
  - have the KK degrees of freedom
  - not freely specified – constrained by compactification
- Does requiring an AdS/CMT duality to be “top-down” lead to any important consequences/restrictions/effects?

Top-down framework

Consider 11D SUGRA

$$2\kappa_{11}^2 S = \int d^{11}x\sqrt{-g}\Big(R - \frac{1}{2}|G_4|^2\Big) + \frac{1}{6}\int C_3 \wedge G_4 \wedge G_4\,,$$

Top-down framework

Consider 11D SUGRA

$$2\kappa_{11}^2 S = \int d^{11}x\sqrt{-g}\Big(R - \frac{1}{2}|G_4|^2\Big) + \frac{1}{6}\int C_3 \wedge G_4 \wedge G_4\,,$$

simple Freund-Rubin solution:

$$ds_{11}^2 = ds^2(AdS_4) + 4\,\underbrace{\left[\eta \otimes \eta + ds^2(\mathbb{CP}^3)\right]}_{S^7}, \qquad G_4 = -3\mathrm{vol}(AdS_4)\,.$$

Top-down framework

Consider 11D SUGRA

$$2\kappa_{11}^2 S = \int d^{11}x\sqrt{-g}\Big(R - \frac{1}{2}|G_4|^2\Big) + \frac{1}{6}\int C_3 \wedge G_4 \wedge G_4\,,$$

simple Freund-Rubin solution:

$$ds_{11}^2 = ds^2(AdS_4) + 4\,\underbrace{\left[\eta\otimes\eta + ds^2(\mathbb{CP}^3)\right]}_{S^7}, \qquad G_4 = -3\mathrm{vol}(AdS_4)\,.$$

Top-down embedding of Holographic Superconductor Gauntlett, Sonner, Wiseman (2009)

$$ds_{11}^2 = e^{-4U}ds_4^2 + 4e^{2U}ds^2(\mathbb{CP}^3) + 4e^{-4U}(\eta + A_1)\otimes(\eta + A_1)$$

$$\begin{aligned} G_4 =& -3e^{-12U}(1-|\chi|^2)\mathrm{vol}_4 + \frac{3i}{4}e^{-12U}\star_4(\chi^* D\chi - \chi D\chi^*)\wedge(\eta + A_1) \\ &+ F_2\wedge J + \sqrt{3}\left[\chi(\eta + A_1)\wedge\Omega - \frac{i}{4}D\chi\wedge\Omega + cc\right] \end{aligned}$$

$$F = dA_1, \quad J = dA/2, \quad e^{6U} = 1 - \frac{3}{4}|\chi|^2$$

Top-down framework

Resulting non-linear 4D effective action:

$$S = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2 - h(\chi)|D\chi|^2 - V(\chi)\right)$$

Example of what's called a consistent truncation:

- only subset of all KK modes are retained
- Remarkable that this exists at all!

Expanded around $\chi = 0$,

$$S = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2 - |D\tilde{\chi}|^2 + 2|\tilde{\chi}|^2\right)$$

$\tilde{\chi}$ has mass $M^2 = -2$ ($\Delta = 1, 2$) and $q = 4$

Top-down framework

Resulting non-linear 4D effective action:

$$S = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2 - h(\chi)|D\chi|^2 - V(\chi)\right)$$

Example of what's called a consistent truncation:

- only subset of all KK modes are retained
- Remarkable that this exists at all!

Expanded around $\chi = 0$,

$$S = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2 - |D\tilde{\chi}|^2 + 2|\tilde{\chi}|^2\right)$$

$\tilde{\chi}$ has mass $M^2 = -2$ $(\Delta = 1, 2)$ and $q = 4$

> $\tilde{\chi}$ is a **pseudo-scalar**: $\Delta = 2$ required by supersymmetry
> Breitenlohner, Freedman (1982)

Top-down framework

Resulting non-linear 4D effective action:

$$S = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2 - h(\chi)|D\chi|^2 - V(\chi)\right)$$

Example of what's called a consistent truncation:

- only subset of all KK modes are retained
- Remarkable that this exists at all!

Expanded around $\chi = 0$,

$$S = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2 - |D\tilde{\chi}|^2 + 2|\tilde{\chi}|^2\right)$$

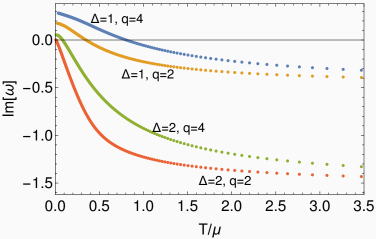
$\tilde{\chi}$ has mass $M^2 = -2$ $(\Delta = 1, 2)$ and $q = 4$

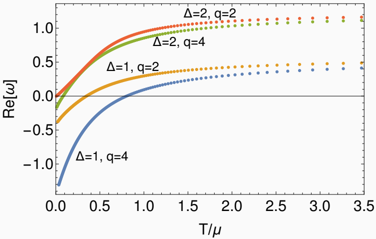
$\tilde{\chi}$ is a **pseudo-scalar**: $\Delta = 2$ required by supersymmetry
Breitenlohner, Freedman (1982)
$\Rightarrow$ unstable to developing scalar hair!

Superconducting Instabilities

near 2nd order phase transition, non-linearities negligible:
$(\mathcal{D}^2 - M^2)\tilde{\chi} = 0$

- $\tilde{\chi} = \exp(-i\omega t)\varphi(r)$
- $T > T_c$: only phase is Reissner Nordstrom (stable quasi-normal mode)
- $T = T_c$: linearized zero mode
- $T < T_c$: hairy BH phase (linear instability)

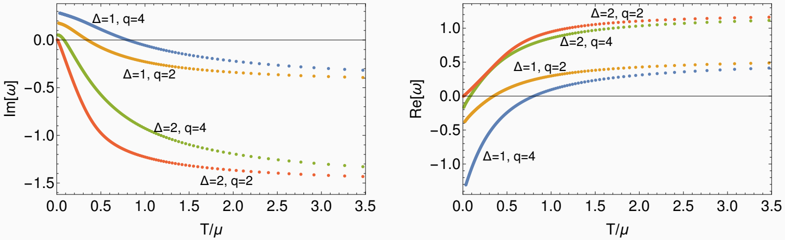




Superconducting Instabilities

near 2nd order phase transition, non-linearities negligible:
$(\mathcal{D}^2 - M^2)\tilde{\chi} = 0$

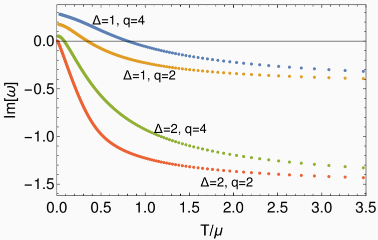
- $\tilde{\chi} = \exp(-i\omega t)\varphi(r)$
- $T > T_c$: only phase is Reissner Nordstrom (stable quasi-normal mode)
- $T = T_c$: linearized zero mode
- $T < T_c$: hairy BH phase (linear instability)

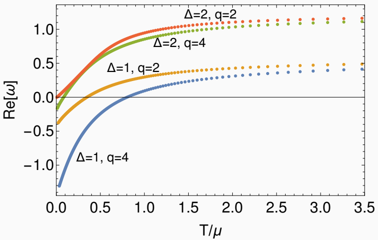


$\boxed{\text{smaller } \Delta\text{, larger } q \Rightarrow \text{stronger instability (higher } T_c)}$

Superconducting Instabilities

Δ	charge q	T_c/μ	found by
1	4	0.803688	
1	2	0.347821	
2	4	0.0831536	Gauntlett, Sonner, Wiseman (2009)
2	2	0.00707295	

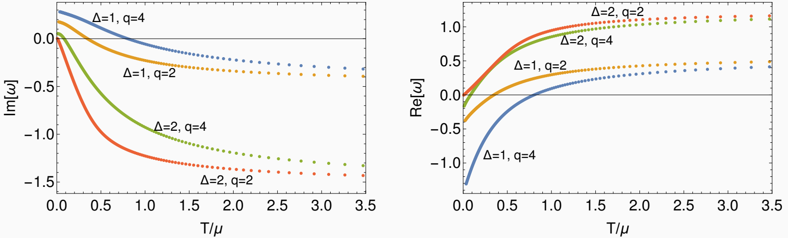




$\boxed{\text{smaller } \Delta, \text{ larger } q \Rightarrow \text{stronger instability (higher } T_c)}$

Superconducting Instabilities

Δ	charge q	T_c/μ	found by
1	4	0.803688	
1	2	0.347821	Donos, Gauntlett (2011)
2	4	0.0831536	Gauntlett, Sonner, Wiseman (2009)
2	2	0.00707295	



$\boxed{\text{smaller } \Delta\text{, larger } q \Rightarrow \text{stronger instability (higher } T_c\text{)}}$

General Perturbations of 11D Embedding

- Are there any other instabilities?

General Perturbations of 11D Embedding

- Are there any other instabilities?
- we considered perturbations around 11D AdS-RN$\times S^7$ background

$$\delta R_{AB} - \frac{1}{6}\delta G_{(A|CDE}G_{|B)}{}^{CDE} + \frac{1}{144}h_{AB}G^2 + \frac{1}{4}G_{(A|ECD}G_{B)P}{}^{CD}h^{EP} + \frac{g_{AB}}{144}\left(2G\cdot\delta G - 4G_{CDEF}G_G{}^{DEF}h^{CG}\right) = 0\,,$$

$$d\star\delta G_4 + d(\delta\star)G_4 - G_4\wedge\delta G_4 = 0\,.$$

- in general won't fit in consistent truncation

General Perturbations of 11D Embedding

- Are there any other instabilities?
- we considered perturbations around 11D AdS-RN$\times S^7$ background

$$\delta R_{AB} - \frac{1}{6}\delta G_{(A|CDE}G_{|B)}{}^{CDE} + \frac{1}{144}h_{AB}G^2 + \frac{1}{4}G_{(A|ECD}G_{B)P}{}^{CD}h^{EP} + \frac{g_{AB}}{144}\left(2G\cdot\delta G - 4G_{CDEF}G_G{}^{DEF}h^{CG}\right) = 0\,,$$

$$d\star\delta G_4 + d(\delta\star)G_4 - G_4\wedge\delta G_4 = 0\,.$$

- in general won't fit in consistent truncation
- need to separate out the $\mathbb{CP}^3$ dependence: use charged $\mathbb{CP}^3$ harmonics (previously used in perturbations of Myers-Perry BHs)
- charged: $(\mathcal{L}_\eta - im)\delta g_{AB} = 0$
- from 4D perspective perturbation has charge $q = m$

General Perturbations of 11D Embedding

- Among many different $\mathbb{CP}^3$ harmonics, found one remarkably simple perturbation
- Out of δg_{AB}, δG_4 and their derivatives, constructed gauge invariant scalar Ψ obeying $\left(\mathcal{D}^2 - M^2\right)\Psi = 0$, with $M^2 = -2$ and $q = 4$

General Perturbations of 11D Embedding

- Among many different $\mathbb{CP}^3$ harmonics, found one remarkably simple perturbation
- Out of δg_{AB}, δG_4 and their derivatives, constructed gauge invariant scalar Ψ obeying $\left(\mathcal{D}^2 - M^2\right)\Psi = 0$, with $M^2 = -2$ and $q = 4$
- Ψ is a **scalar**, (rather than pseudo-scalar), so

General Perturbations of 11D Embedding

- Among many different $\mathbb{CP}^3$ harmonics, found one remarkably simple perturbation
- Out of δg_{AB}, δG_4 and their derivatives, constructed gauge invariant scalar Ψ obeying $\left(\mathcal{D}^2 - M^2\right)\Psi = 0$, with $M^2 = -2$ and $q = 4$
- Ψ is a **scalar**, (rather than pseudo-scalar), so
- $\Delta = (1, 2)$

General Perturbations of 11D Embedding

- Among many different $\mathbb{CP}^3$ harmonics, found one remarkably simple perturbation
- Out of δg_{AB}, δG_4 and their derivatives, constructed gauge invariant scalar Ψ obeying $\left(\mathcal{D}^2 - M^2\right)\Psi = 0$, with $M^2 = -2$ and $q = 4$
- Ψ is a **scalar**, (rather than pseudo-scalar), so
- $\Delta = (1, 2)$

Δ	charge q	T_c/μ	found by
1	4	0.803688	GSH, Reall, Santos
1	2	0.347821	Donos, Gauntlett (2011)
2	4	0.0831536	Gauntlett, Sonner, Wiseman (2009)
2	2	0.00707295	

Dominant instability/first mode to go unstable!!

Implications & Further Directions

- previous studies of this embedding constructed non-linear hairy solutions based off of sub-dominant instabilities

Implications & Further Directions

- previous studies of this embedding constructed non-linear hairy solutions based off of sub-dominant instabilities
  - these are sub-leading saddles, not relevant in thermodynamic limit
  - need to backreact Ψ & construct non-linear solution!

Implications & Further Directions

- previous studies of this embedding constructed non-linear hairy solutions based off of sub-dominant instabilities
  - these are sub-leading saddles, not relevant in thermodynamic limit
  - need to backreact Ψ & construct non-linear solution!
- seems unlikely that Ψ fits into a consistent truncation, in which case:

Implications & Further Directions

- previous studies of this embedding constructed non-linear hairy solutions based off of sub-dominant instabilities
  - these are sub-leading saddles, not relevant in thermodynamic limit
  - need to backreact Ψ & construct non-linear solution!
- seems unlikely that Ψ fits into a consistent truncation, in which case:
  - non-linear phase necessarily involves infinite set of KK modes
  - seems unusual from a condensed matter perspective $\rightarrow$ implications for AdS/CMT?
  - familiar from attempts to model d-wave superconductivity holographically, Hartnett, Horowitz (2013), Kim, Taylor (2013)

Implications & Further Directions

- previous studies of this embedding constructed non-linear hairy solutions based off of sub-dominant instabilities
  - these are sub-leading saddles, not relevant in thermodynamic limit
  - need to backreact Ψ & construct non-linear solution!
- seems unlikely that Ψ fits into a consistent truncation, in which case:
  - non-linear phase necessarily involves infinite set of KK modes
  - seems unusual from a condensed matter perspective $\rightarrow$ implications for AdS/CMT?
  - familiar from attempts to model d-wave superconductivity holographically, Hartnett, Horowitz (2013), Kim, Taylor (2013)
  - somewhat unusual: "lumpy" black holes do not often dominate grand canonical ensemble (c.f. Santos/Way/Dias talks)

Implications & Further Directions

- Application to landscape of superconducting membranes (Denef, Hartnoll (2009): we specifically looked at the case $KE_6 = \mathbb{CP}^3$, how universal is this scalar Ψ (depends on spectrum of charged harmonics on KE_6)

Thank You!