New Insights into Quantum Gravity from Holography

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Why holography?

Early motivation came from the Bekenstein-Hawking entropy of black holes:

$$S_{BH} = A / 4$$

Diffeomorphism invariance implies that the Hamiltonian is defined by a surface integral at infinity. If the spectrum of black holes is discrete, one can (in principle) determine the state by measuring the energy at infinity.

(Balasubramanian, Marolf, Rozali)

Holography is not just a property of black holes, but should be a general property of quantum gravity:

Everything that happens in a region of space can be described by degrees of freedom living on the boundary.

('t Hooft and Susskind)

Advantages of anti-de Sitter for holography

- A static slice in AdS has constant negative curvature, so the spheres at large radius are much bigger than usual
- The conformal boundary at infinity is timelike
- Black hole thermodynamics is better behaved in AdS since the negative curvature acts like a confining box (Hawking and Page)

Gauge/gravity duality

(Maldacena; Gubser, Klebanov, Polyakov; Witten)

With anti-de Sitter boundary conditions, string theory (which includes gravity) is completely equivalent to a (nongravitational) gauge theory living on the boundary at infinity.

When string theory is weakly coupled, gauge theory is strongly coupled, and vice versa.

AdS can be written

$$ds^2 = r^2[-dt^2 + dx_i dx^i] + \frac{dr^2}{r^2}$$

The gauge theory lives on the Minkowski spacetime at r = ∞.

Scaling symmetry: $r \rightarrow ar$, $(t, x_i) \rightarrow (t/a, x_i/a)$ so small r corresponds to large distances or low energy in the gauge theory.

Attempt to disprove this conjecture:

A 4D theory should have many more degrees of freedom than a 3D theory. So let's compare the entropy at high temperature:

3D thermal gas: $E \sim T^3 V$, $S \sim T^2 V$

4D thermal gas: $E \sim T^4 V$, $S \sim T^3 V$

BUT in a theory with gravity, at high T this gas will collapse to form a large black hole.

The planar black hole has metric

$$ds^2 = r^2[-f(r)dt^2 + dx_i dx^i] + \frac{dr^2}{r^2 f(r)}$$

where $$f(r) = \left(1 - \frac{r_0^3}{r^3}\right)$$

The Hawking temperature is $T \sim r_0$.

The total energy is $E \sim r_0^3 V \sim T^3 V$.

The entropy is $S \sim A \sim r_0^2 V \sim T^2 V$.

So the 3+1 BH energy and entropy are exactly like a thermal gas in 2+1 dimensions.

Early Evidence

- Symmetries agree
- Gauge theory analogs of all massless string modes are known
- Gauge theory analogs of many massive string modes have been found (strings arise as long chains of operators)
- Many interactions have been shown to agree

Many calculations agree

- Microscopic derivation of black hole entropy
- Partition functions
- Expectation value of Wilson loops
- Renormalization group flow
- …

Gauge/gravity duality provides a background independent formulation of quantum gravity.

Only the asymptotic metric is fixed. The metric in the interior is free to fluctuate.

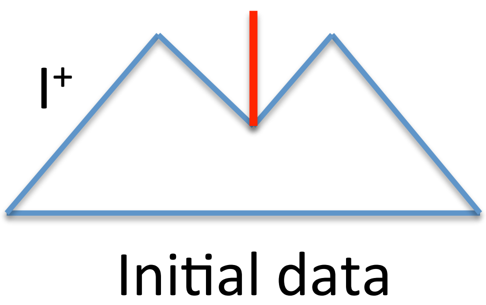
A powerful feature of gauge/gravity duality is that statements that are easy to establish on one side often imply highly nontrivial results about the dual theory.

For example, the fact that black hole evaporation must be unitary follows immediately from unitary evolution of the dual gauge theory.

Cosmic censorship

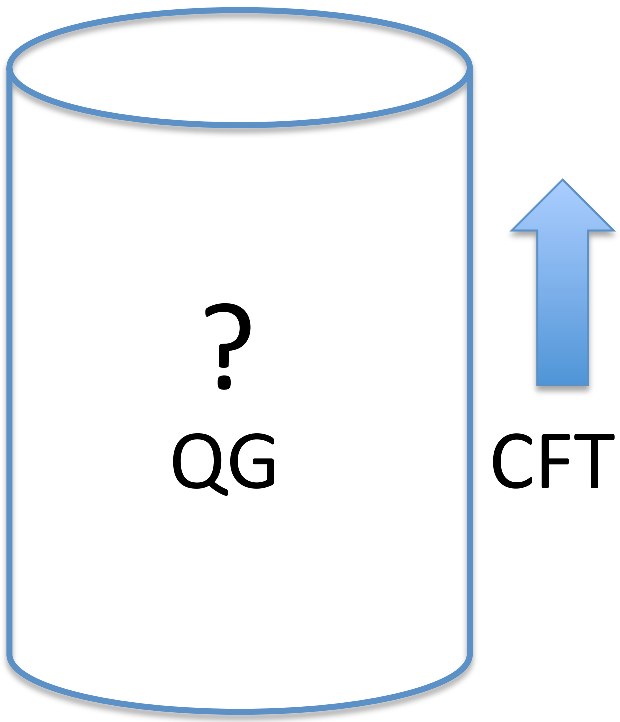
Classical GR conjecture: Generic, asymptotically flat, initial data has a maximal evolution that contains a complete null infinity.

i.e. this can't happen.



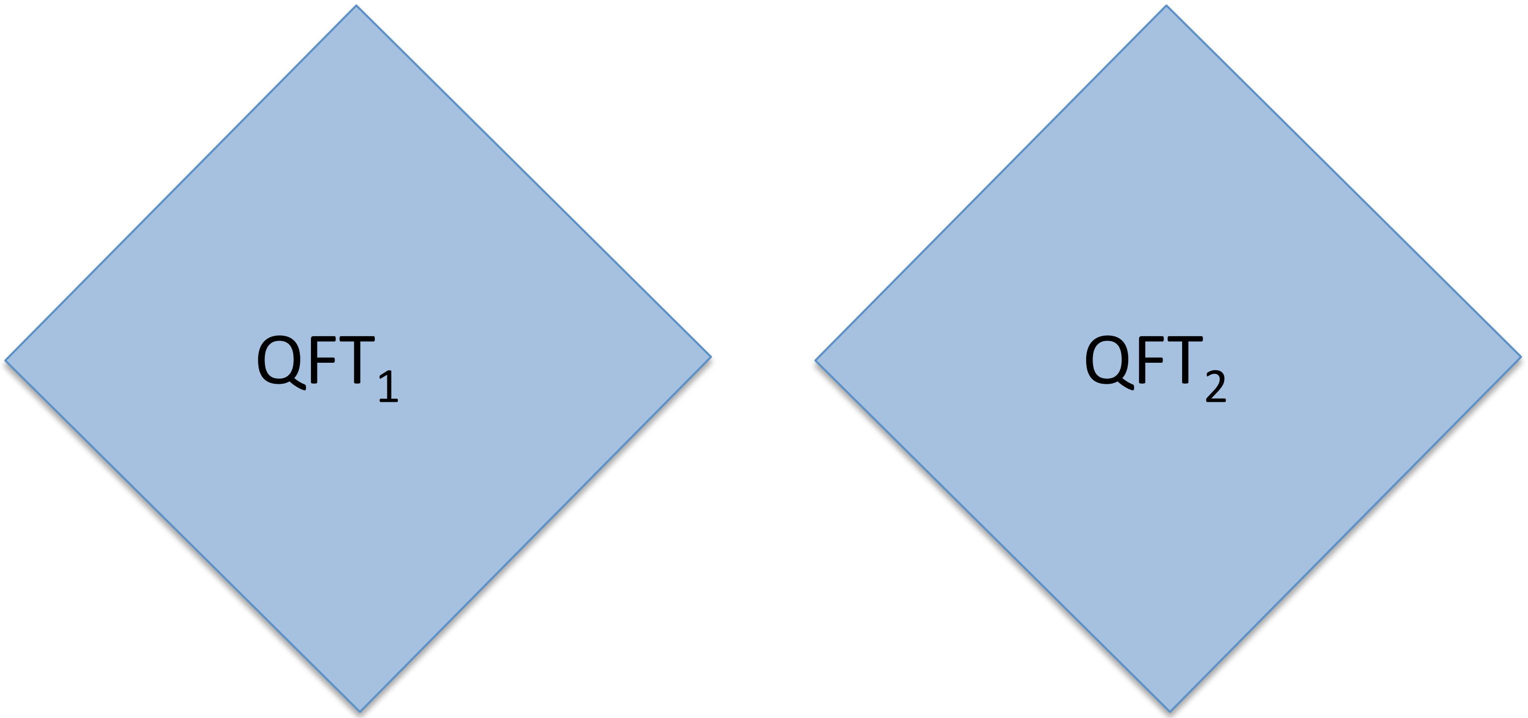
If cosmic censorship fails, it was hoped that quantum gravity would resolve the singularity so evolution continues.

In holography we know that this is true! Regardless of what happens in a localized region in the interior, evolution in the CFT on the boundary continues.

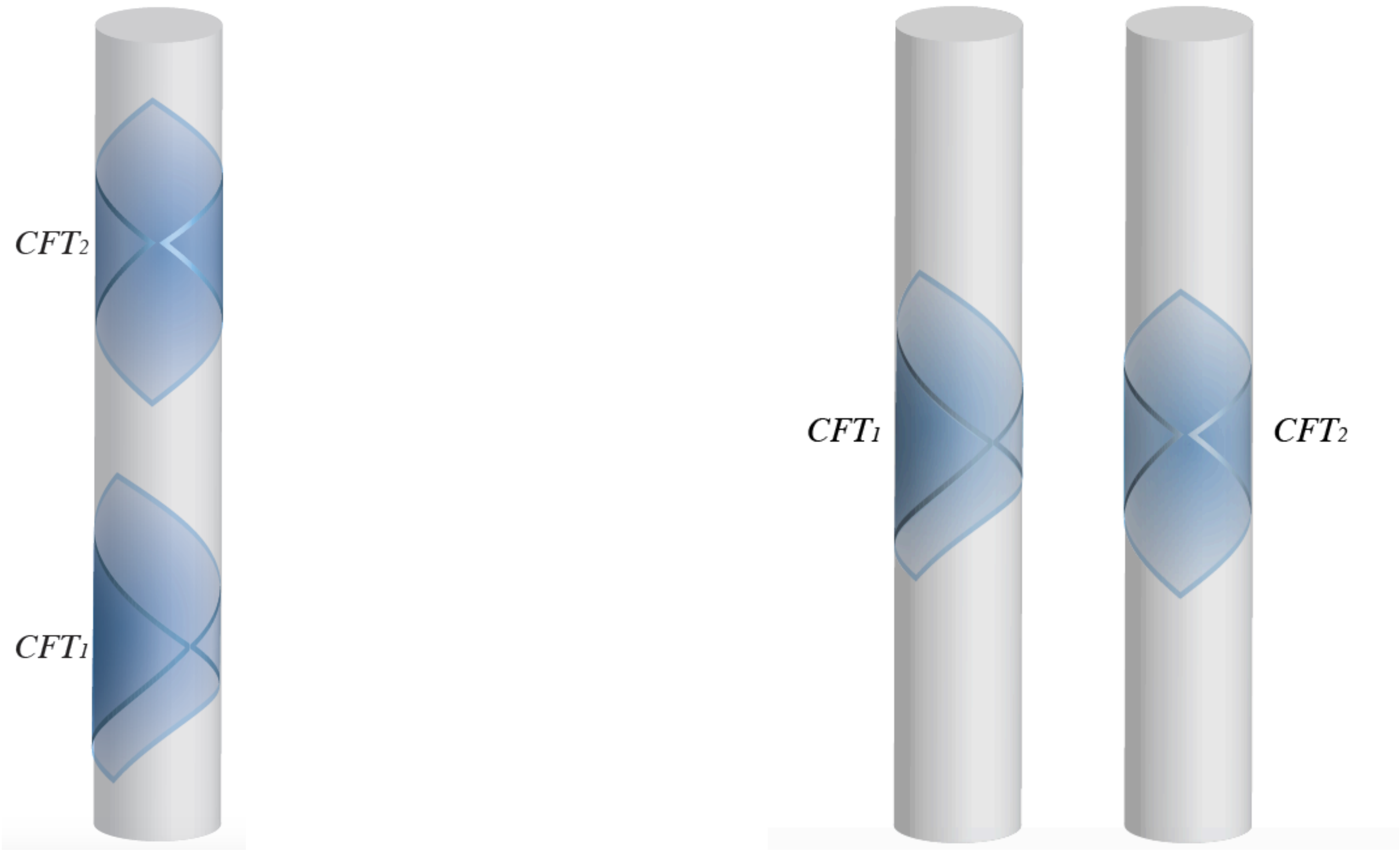


When can two QFTs communicate?

Usually, two QFTs on separate spacetimes cannot send signals to one another



Two copies of a CFT on Minkowski space can be mapped either to one static cylinder or two.

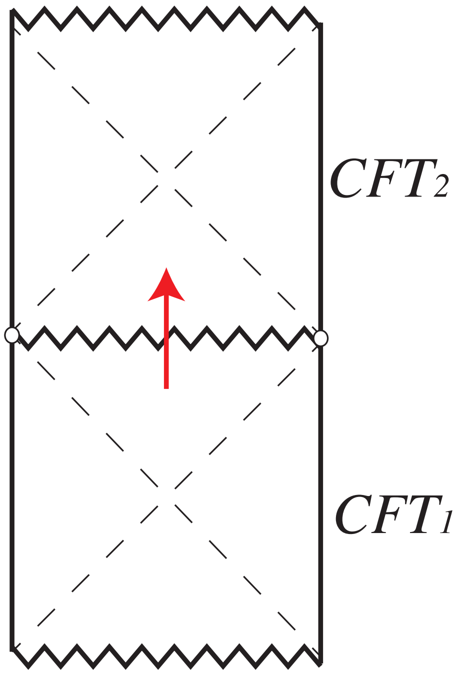


Definition: Two field theories are <u>independent</u> if the joint Hilbert space of the system can be written as a tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ and all operators are sums of $\Phi_1 \otimes I_2, \; I_1 \otimes \Phi_2$

No Transmission Principle (NTP):
Two independent QFTs cannot transmit signals to one another.

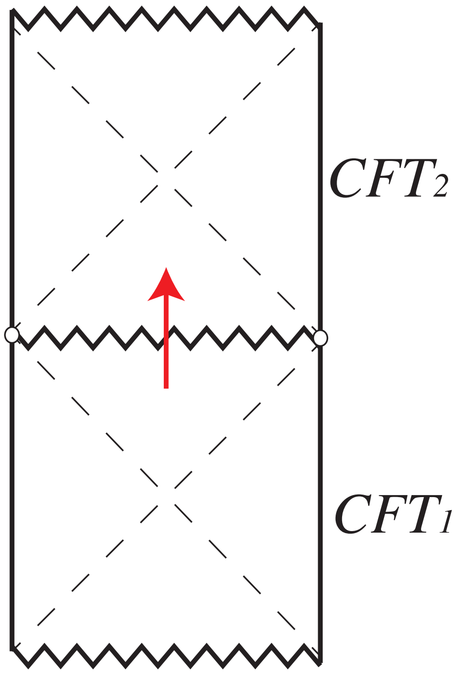
Application to holography: If two independent QFTs have gravity duals, then no signals can be transmitted between their bulk duals.

No evolution through black holes



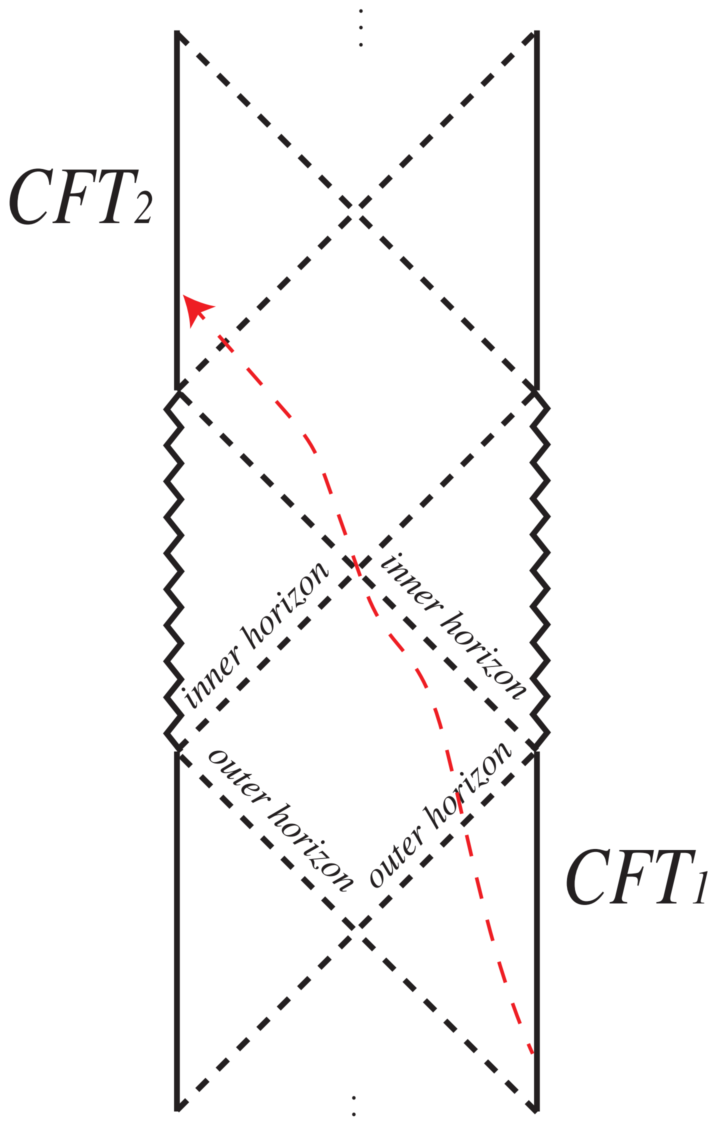
Could quantum gravity resolve the singularity and allow signals to emerge in another asymptotically AdS spacetime?

No evolution through black holes



Could quantum gravity resolve the singularity and allow signals to emerge in another asymptotically AdS spacetime?

No. The CFTs are independent, and this would violate the NTP.



A charged AdS black hole seems to violate the NTP even classically.

But the inner horizon is known to be unstable. Signals cannot get through classically.

NTP implies that signals cannot get through even in full quantum gravity.

Application to singular CFTs

Some CFTs cannot be evolved past a certain time.

If evolution on the boundary stops, then evolution in the bulk must stop as well.

There must be a cosmological singularity classically, which quantum gravity cannot resolve into a bounce.

What is a singular CFT?

Consider CFTs on maximally conformally extended spacetimes.

We assume the spacetime is topologically S x R, where S is compact, and can be foliated by a one parameter family of Cauchy surfaces S_t.

Definition: A standard conformal frame is one in which the volume of S_t is bounded from above and below by nonzero constants for all t.

Definition: A CFT is singular if evolution ends in finite time in a standard conformal frame.

Examples of singular CFTs

1) Put any CFT on a spacetime with a cosmological singularity, e.g.,

$$ds^2 = -dt^2 + \sum_{i=1}^{d-1} t^{2p_i} dx_i^2$$

If $\Sigma\, p_i = 0$, this is in a standard conformal frame.

(Das, Michelson, Narayan, Trivedi; Engelhardt, Hertog, G.H.)

2) Destabilize a CFT by adding a potential unbounded from below, e.g.,

$$S = S_0 + k \int d^3x\ \mathcal{O}^3$$

(Hertog, G.H.; Craps, Hertog, Turok; Barbon, Rabinovici)

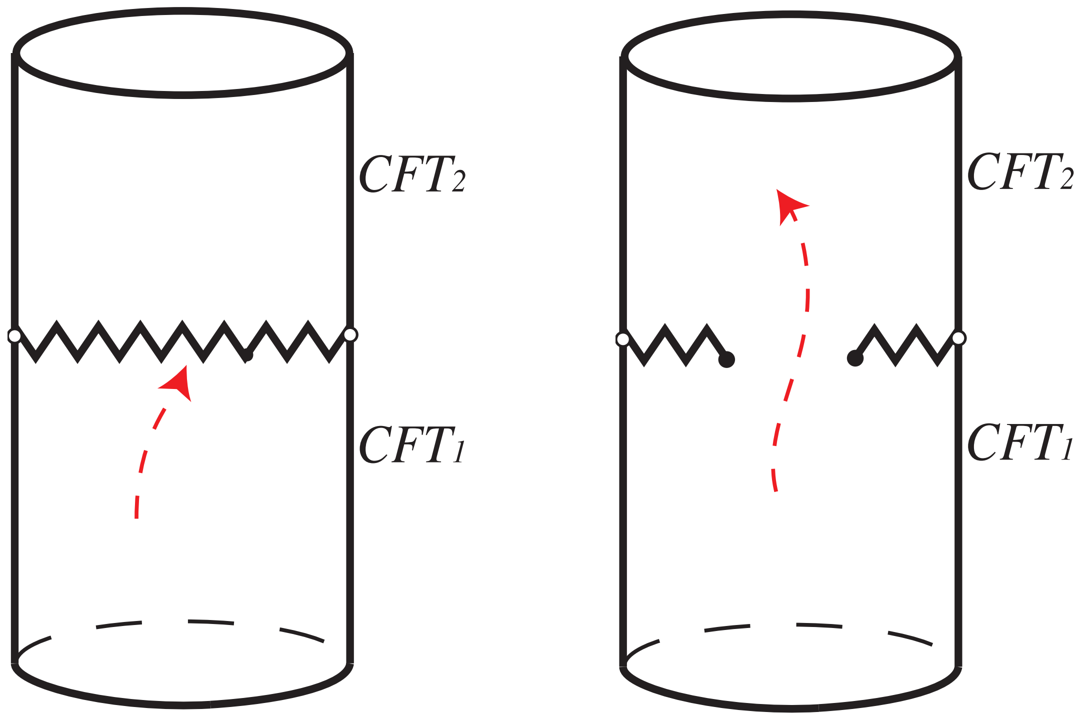
3) Add a relevant perturbation to a CFT on de Sitter

(Maldacena; Harlow, Susskind; Barbon, Rabinovici)

Classical SUGRA implications

In all known cases, there are bulk solutions with cosmological singularities.

The NTP implies this must always be the case.



Not allowed

If the boundary metric has a cosmological singularity, we get a purely geometric result which is like a new type of singularity theorem:

If the conformal boundary metric of an asymptotically AdS solution of low energy string theory has a cosmological singularity, then the bulk solution must also have a cosmological singularity.

This is not obvious. There are nonsingular bulk metrics with singular boundary metrics:

$$ds^2 = \frac{1}{z^2}\left[-dt^2 + \sum_{i=1}^{d-1}(t^2 + z^2)^{p_i} dx_i^2 + dz^2\right]$$

Comments

1) One cannot avoid our conclusions by adding couplings between the CFTs associated with different asymptotic regions, since that would violate causality.

2) One could make up a rule to identify a state in CFT_1 with one in CFT_2, but it would be extra input not contained in the original CFTs – bulk and boundary theories would not be equivalent.

3) There is no natural way to identify the states.

Summary

A number of nontrivial statements follow from simple assumptions about holography including:

1) A form of cosmic censorship holds in QG.

2) Cannot evolve through black holes to another asymptotically AdS region of spacetime.
3) Cannot evolve through certain cosmological singularities to another region of spacetime.