The Cost to Cancel the Big Lambda

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Plan:

- The CC problem – a brief reminder
- Earlier proposals $\rightarrow$ S. Weinberg's no-go theorem
- Loophole in the no-go theorem – success and failure
- Non-local theories – Big Constant out Small Constant In
- Topics not discussed:
  (1) Quantum instability of de Sitter space
  (recent works by A.M. Polyakov; earlier works: I. Antoniadis & E. Mottola; N. Tsamis & R. Woodard,...)
  (2) Multiplicity of vacua and the landscape
  (see a review by J. Polchinski)

Dichotomy:

- Cosmological Constant Problem – the old problem of Particle Physics and Cosmology (W. Pauli)

  Zel'dovich, 1967: A cutoff $\mu \sim 1\mathrm{GeV}$, obtained a huge value.

  Modern view: The natural value for the vacuum energy density

$$\rho_{vac} \gtrsim 10^{60} \rho_{observed}$$

- Both Particle Physics and Cosmology are very successful at a high precision level

  Yet, CC receives contributions from physics at various scales, each much greater than $\rho_{observed}$. "Local" vacuum energy does gravitate: particle physics effects, e.g., $< \bar{q}q >$ condensate contribution to the proton mass (measured), as well as to CC.

Einstein's equations

$$G_{\mu\nu} = \Lambda \, g_{\mu\nu} + 8\pi \, G_N \, T_{\mu\nu}^{\mathrm{dm,m,rad,..}}$$

Λ doesn't redshift (this defines it). Λ is power-sensitive to short distance physics at diverse scales; its natural value way too large. Dark energy could be small part of it, or could be something esle.

The scale of Dark Energy, $10^{-33}\ eV$, might be a stable scale where GR is modified – technical naturalness

$$G_{\mu\nu} = \left(10^{-33}\,\mathrm{eV}\right)^2 X_{\mu\nu} + 8\pi \, G_N \, T_{\mu\nu}^{\mathrm{dm,m,rad,..}}$$

Goals: eliminate big Λ, get technically natural DE

1980s, motivated by axion, search for adjustment mechanisms:
(Dolgov; Wilczek, Zee; Peccei, Sola, Wetterich;.....)

$$\rho_{vac}\sqrt{g} \to (\rho_{vac} + V(\phi))\sqrt{g} \to 0 \quad ???$$

A multitude of proposals failed, general obstruction

S. Weinberg's 1987 a no-go theorem: GR plus cosmological constant plus a conventional field theory, no Poincaré invariant solution can be obtained without fine tuning.

No adjustment mechanism is possible!

Loopholes in the no-go theorem:

Constant fields were assumed to preserve Poinceré symmetry.
This might be too restrictive: coordinate dependent background fields. Naively, this would break Poincareé invariance, however, one could think of cases when there is still remaining $ISO(3,1)$:
For example, a symmetry breaking pattern

$$ISO(3,1)_1 \times ISO(3,1)_2 \to ISO(3,1)_{\mathrm{Observ}}$$

The background fields:

$$g_{\mu\nu} = \eta_{\mu\nu}, \qquad \partial_\mu \phi^a = \delta^a_\mu$$

Another (similar) example with Galileon symmetry:

$$Gal_{\mathrm{int}} \to ISO(3,1)_{\mathrm{Observ}}$$

CC can be cancelled in such models, but other problems arise (see, e.g., de Rham, GG, Heisenberg, Pirtskhalava...)

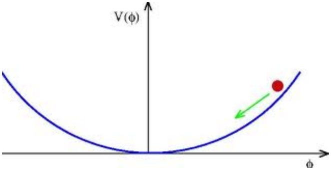
More unconventional theories: Eliminating the big CC, *Tseytlin '90*

The modified action principle:

$$\bar{S} = \frac{S}{V_g} = \frac{1}{V_g}\int d^4x\sqrt{g}\left(\frac{1}{2}R + L(g,\psi_n)\right)$$

where $V_g = \int d^4x\sqrt{g}$. Any constant shift, $L \to L + \Lambda$, gives rise to a shift of the new action by the same constant, $\bar{S} \to \bar{S} + \Lambda$, that does not affect equations of motion.

Subtracts a constant from a scalar potential



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Eliminates the "future value" of the stress tensor

The Einstein equations:

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R = T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T\,, \qquad R + T = 0\,.$$

Tseytlin's proposal for the trace equation:

$$R + T = \langle T\rangle - 2\langle g^{\mu\nu}\frac{\partial L}{\partial g^{\mu\nu}}\rangle$$

where $< \cdots >$ denotes a certain space-time average defined as follows:

$$\langle\cdots\rangle \equiv \frac{\int d^4x\sqrt{g}(\cdots)}{\int d^4x\sqrt{g}} \equiv \frac{[\cdots]}{V_g}\,. \qquad (1)$$

Local quantities are affected by global ones – non-locality

This non-locality is operative only for vacuum energy, nothing else

Problems with the loops, Tseytlin '90

The $1/V_g$ factor gives an effective rescaling of the Planck's constant, $\hbar \to \hbar V_g$

$$\bar{S}_{Ren} = \frac{1}{V_g} \int d^4x \sqrt{g} \left(\frac{1}{2} R + L(g, \psi_n) + V_g L_1(g, \psi_n) + \mathcal{O}(V_g^2) \right)$$

where $L_1, L_2, ..$ contain all possible terms consistent with diffeomorphism and internal symmetries. This ruins the solution!

Same could be seen by defining an extended action:

$$\bar{S}_{q,\lambda} = \frac{1}{q} \int d^4x \sqrt{g} \left(\frac{1}{2} R + L \right) + \lambda (V_g - q) \,,$$

and writing down the path integral for gravity as follows

$$Z_g = \mathrm{const} \int d\mu(g)\, dq\, d\lambda \exp\left(\frac{i}{\hbar} \bar{S}_{q,\lambda} \right) \,,$$

Dealing with the loop problems: *GG '14*

The main idea – global bigravity:

$$A = \frac{V_f}{V_g} S + \int d^D y \sqrt{f} \left(\frac{M_f^{D-2}}{2} R(y) + c_0 M^D \cdots \right)$$

where $f_{AB}(y)$ is another metric, and $V_f = \int d^D y \sqrt{f(y)}$.

The CC of our universe renormalizes CC in the other universe

$$\Delta A_{CC} = \frac{V_f}{V_g} \int d^4 x \sqrt{g} \Lambda = \int d^D y \sqrt{f} \Lambda$$

1. Our vacuum energy curves the other space-time; hence no old CC problem in our universe

2. If $V_f >> V_g$, then, $\hbar \to \hbar (V_g / V_f)$ loop effects suppressed

Defining the path integral for quantized SM:

$$Z(g, J_n) \sim \int d\mu(\tilde{\psi}_n)\exp\left(i\int d^4x\sqrt{g}\left(\mathcal{L}(g,\tilde{\psi}_n)+J_n\tilde{\psi}_n\right)\right)$$

The metric g is an external field, and so are the sources, J_n's. Then, the effective Lagrangian $L(g, \psi_n)$ is defined as a Legandre transform of $W(g, J_n) = -i\ln Z(g, J_n)$; D. Anselmi '06; also earlier works on the In-In formalism, Jordan '85 and refs therein:

$$\int d^4x\sqrt{g}L(g,\psi_n) \equiv Re\left(W(g, J_n) - \int d^4x\sqrt{g}J_n\psi_n\right)$$

where $\sqrt{g}\psi_n \equiv -i\delta\ln Z(g, J_n)/\delta J_n$. The obtained quantum effective action is (the real part of) a 1PI action. All the quantum corrections due to non-gravitational interactions are already taken into account in the effective Lagrangian L.

We define an *extended* action:

$$\bar{A}_{q,\lambda} = \frac{1}{q}\int d^4x\sqrt{g}\left(\frac{1}{2}R + L\right) + \lambda\left(\frac{V_g}{V_f} - q\right) + S_f$$

and the path integral for gravity as follows

$$Z_g \sim \int d\mu(g)\, d\mu(f)\, dq\, d\lambda \exp\left(i\bar{A}_{q,\lambda}\right)$$

where we also integrates w.r.t. the *parameters* q and λ. This can be rewritten in terms of the path integral for the SM fields Z_{SM}:

$$Z_g \sim \int d\mu(g) d\mu(f)\, dq\, d\lambda \left(e^{iS_{\mathrm{EH}}}\, Z_{\mathrm{SM}}(g, \psi_n)\right)^{\frac{1}{q}}\; e^{i\lambda\left(\frac{V_g}{V_f} - q\right) + iS_f}$$

The SM loops done in a conventional way, gravity loops via an unconventional prescription specified above.

The f-universe can be exactly supersymmetric, described, for instance, by unbroken AdS supergravity.

The new terms do not affect the trace equations, except that they just introduce a overall multiplier V_f. Thus, the cosmological constant is eliminated from the g-universe. There is, however, a new equation due to variation w.r.t. f:

$$M_f^{D-2}(R_{AB}(y) - \frac{1}{2} f_{AB} R(y)) = f_{AB}(\bar{S} + c_0 M^D) + \cdots . \qquad (2)$$

The right hand side contains a vacuum energy generated in our universe, $\bar{S} = \frac{[E_{vac}^4]}{V_g} = E_{vac}^4$, as well as that of the f-universe.

According to our construction, the net energy density is negative, so that the f-universe has an AdS curvature. If so, then $V_f = \infty$.

Still need to produce $V_f/V_g >>> 1$; use massive gravity – and its extensions – instead of GR in the g-universe:

GR Extended by Mass and Potential Terms

Previous no-go statements invalid: *de Rham, GG, '10*

The Lagrangian of the theory: *de Rham, GG, Tolley, '10*

Using $g_{\mu\nu}(x)$ and 4 scalars $\phi^a(x)$, $a = 0, 1, 2, 3$, define

$$\mathcal{K}^\mu_\nu(g,\phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\tilde{f}_{\alpha\nu}} \qquad \tilde{f}_{\alpha\nu} \equiv \partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

The Lagrangian is written using notation $tr(\mathcal{K}) \equiv [\mathcal{K}]$:

$$\mathcal{L} = M^2_{\rm pl}\sqrt{g}\left(R + m^2\left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right)\right)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2] \sim det_2(\mathcal{K})$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \sim det_3(\mathcal{K})$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4] \sim det_4(\mathcal{K})$$

Strongly coupled, UV completion/extension needed.

Cosmology of pure massive gravity. No flat FRW solution: *D'Amico, de Rham, Dubovsky, GG, Pirtskhalava, Tolley, '11*
Exception: Open FRW selfaccelerated universe, *Gumrukcuoglu, Lin, Mykohyama 11*, regretfully, this is unstable
Pseudo-homogeneous selfaccelerated solutions: In the dec limit: *de Rham, GG, Heisenberg, Pirtskhalava*. Exact solution: *Koyama, Niz, Tasinato (1,2,3), M. Volkov; L. Berezhiani, et al; ...*
For instance, *Koyama,Niz,Tasinato*:

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega^2)$$

while, ϕ^0 and ϕ^ρ, are inhomogeneous functions. Selfacceleration is a generic feature of this theory, however, vanishing of the kinetic terms for some of the 5 modes is also a common feature of these solutions – too bad! Anisotropic solutions and fluctuations: Gumrukcuoglu, Lin, Mukohyama, '12.

More complex solutions are OK (Mukohyama et al.), or else extensions beyond pure massive gravity are needed for cosmology.

Extensions of massive gravity (subjective and incomplete list):

Extended Quasidilaton: De Felice, Mukohyama, '13; Mukohyama, '13; De Felice, Gümrükcüoglu, Mukohyama, '13, Mukohyama, 14; GG, Kimura, Pirtskhalava, '14,'15

Bigravity: Hassan, R.A. Rosen, '11, ... Cosmology e.g., De Felice, Gümrükcüoglu, Mukohyama, Tanahashi, Tanaka, 14, ...
Extended and Generalized Massive Gravities: GG, Hinterbichler, Khoury, Pirtskhalava, Trodden, 13; Gümrükcüoglu, Hinterbichler, Lin, Mukohyama, Trodden 13; de Rham, Keltner, Tolley, 14, ...

Minimal Theory of massive gravity (Lorentz violating): De Felice, Mukohyama, '15,16

Thus, the g-universe has dS metric, and f-universe has AdS metric. $q = V_g/V_f \to 0$, hence quantum gravity corrections in the g-universe are determined by positive powers of the parameter, $\hbar q \to 0$. Quantum gravity is present only in the f-universe!

Could f and the fiducial metric, $\tilde{f}$, be related?
GG and Siqing Yu, '15: The f-universe as AdS_5

$$ds^2 = f_{AB}dy^A dy^B = \frac{l^2}{z^2}\left(\eta_{ab}dy^a dy^b + dz^2\right), \quad a = 0,1,2,3; A = a,5$$

The AdS boundary coordinates x^μ, $\mu = 0,1,2,3$. Parametrization of the boundary located at $z = 0$, $y^a = \phi^a(x)$

$$ds^2 = \frac{l^2}{z^2}\left(\tilde{f}_{\mu\nu}dx^\mu dx^\nu + dz^2\right), \quad \tilde{f}_{\mu\nu} = \partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab}$$

The fiducial metric, $\tilde{f}_{\mu\nu}$, as a non-dynamical pullback of the 5D AdS metric

$$A = \frac{V_f}{V_g}S_{mGR}(g,\tilde{f}) + S_{AdS_5}(f)$$

This removes our CC into the 5D AdS space (need a small hierarchy between 5D and 4D CC's, as before), and gives rise to dark energy via massive gravity or it extensions.

Conclusions:

- The big cosmological constant can be eliminated via a nonlocal mechanism. The cost is high –space-time nonlocality. The proposed action is stable w.r.t. quantum gravity loop corrections. Embedding in SUGRA.

- Dark energy can be accommodated by various means, but not by means of CC. Using massive gravity and its extensions has virtues of: (a) ascribing origin to the fiducial metric, (b) removing the quantum strong coupling problem.

- Possible observational consequences from non-locality – no tensor mode from inflation since $\hbar q \to 0$; observational consequences from massive gravity and its extensions.