Entanglement and the architecture of spacetime

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Session D1 - Loop quantum gravity and spin foams

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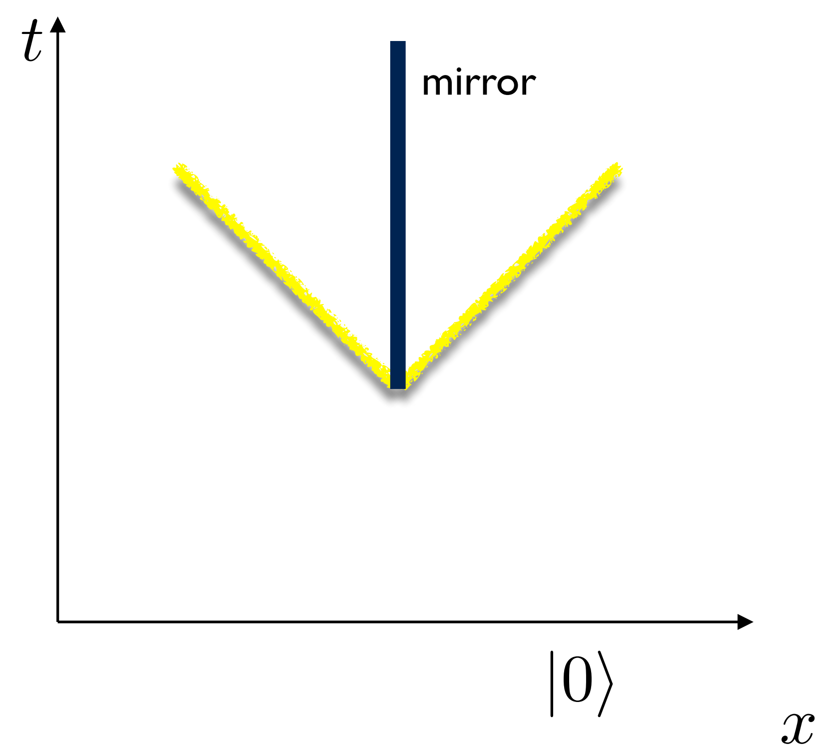
Entanglement is important to reconstruct a spacetime geometry

- Cutting out entanglement in QFT in curved space results in large back-reaction

A.Anderson and B. DeWitt, Found. Phys. (1986)
"Does the Topology of Space Fluctuate?"



- Disentangling the quantum gravity d.o.f. of two spacetime regions results in a pinching off of spacetime

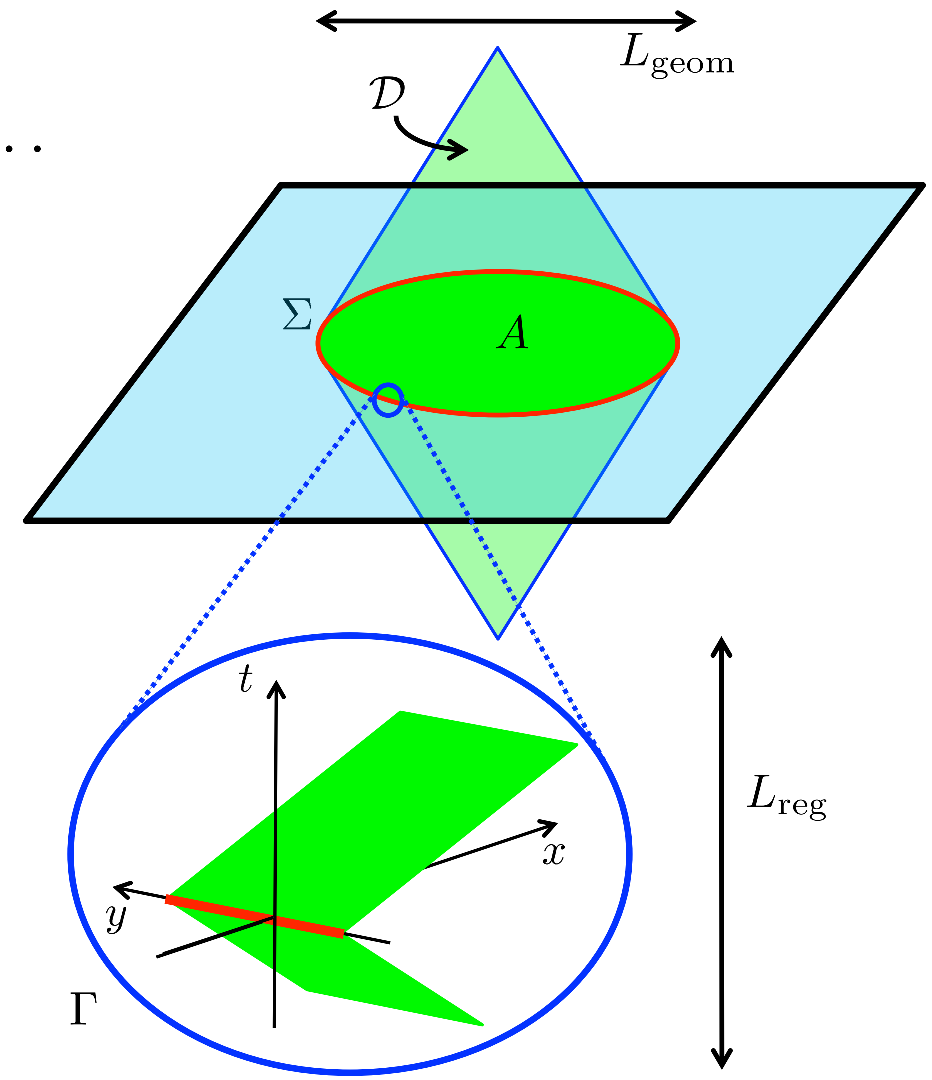
M. Van Raamsdonk, GRG (2010)
“Building up spacetime with quantum entanglement”

Entanglement is the architecture of a spacetime geometry

- Entanglement entropy as a probe of the architecture of spacetime
  Area-law not generic, property of semiclassical states

E.B. and R.Myers, CQG (2012)
"On the Architecture of Spacetime Geometry"

$$S_A(|0\rangle) = \; 2\pi \, \frac{\text{Area}(\partial \text{A})}{L^2_{Planck}} \; + \; \ldots$$



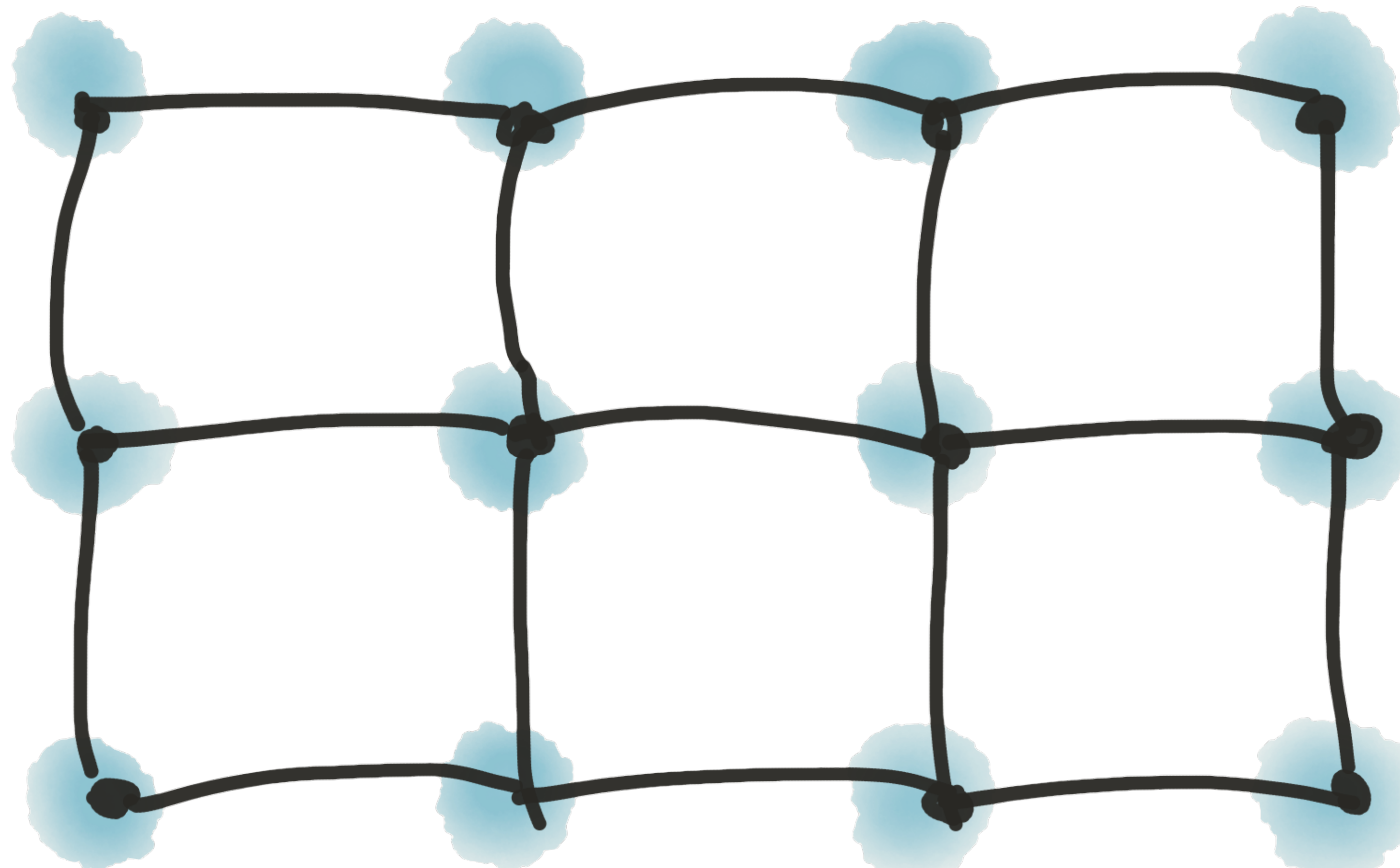
Arguments from: Black Hole thermodynamics (Bekenstein, Hawking)
Holographic entanglement (Ryu, Takayanagi; Hubeny, Rangamani, Takayanagi)
Loop Quantum Gravity (E.B.)

Plan:

- defining the entanglement entropy in LQG
- squeezed vacua in LQG and area-law states
- perspectives

Loop quantum gravity and oscillators

Bosonic representation of LQG [Girelli-Livine 05, Tambornino, Dupuis, Bonzom,…]



two oscillators per seed $a_i^A \ , \ a_i^{A\dagger}$

spin from oscillators $$|j,m\rangle = \frac{(a^{0\dagger})^{j+m}}{\sqrt{(j+m)!}}\frac{(a^{1\dagger})^{j-m}}{\sqrt{(j-m)!}}|0\rangle$$

[Schwinger '52]

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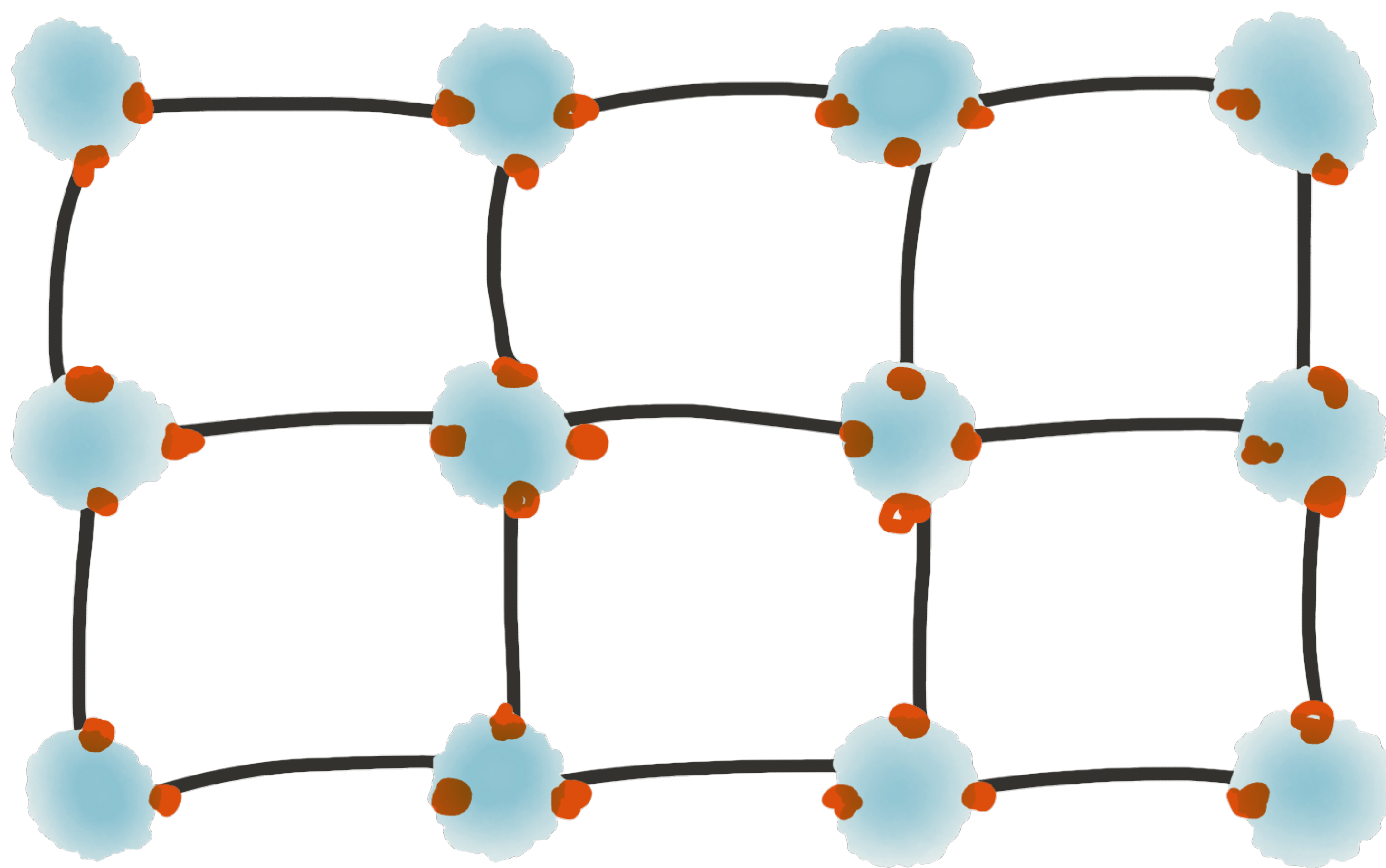

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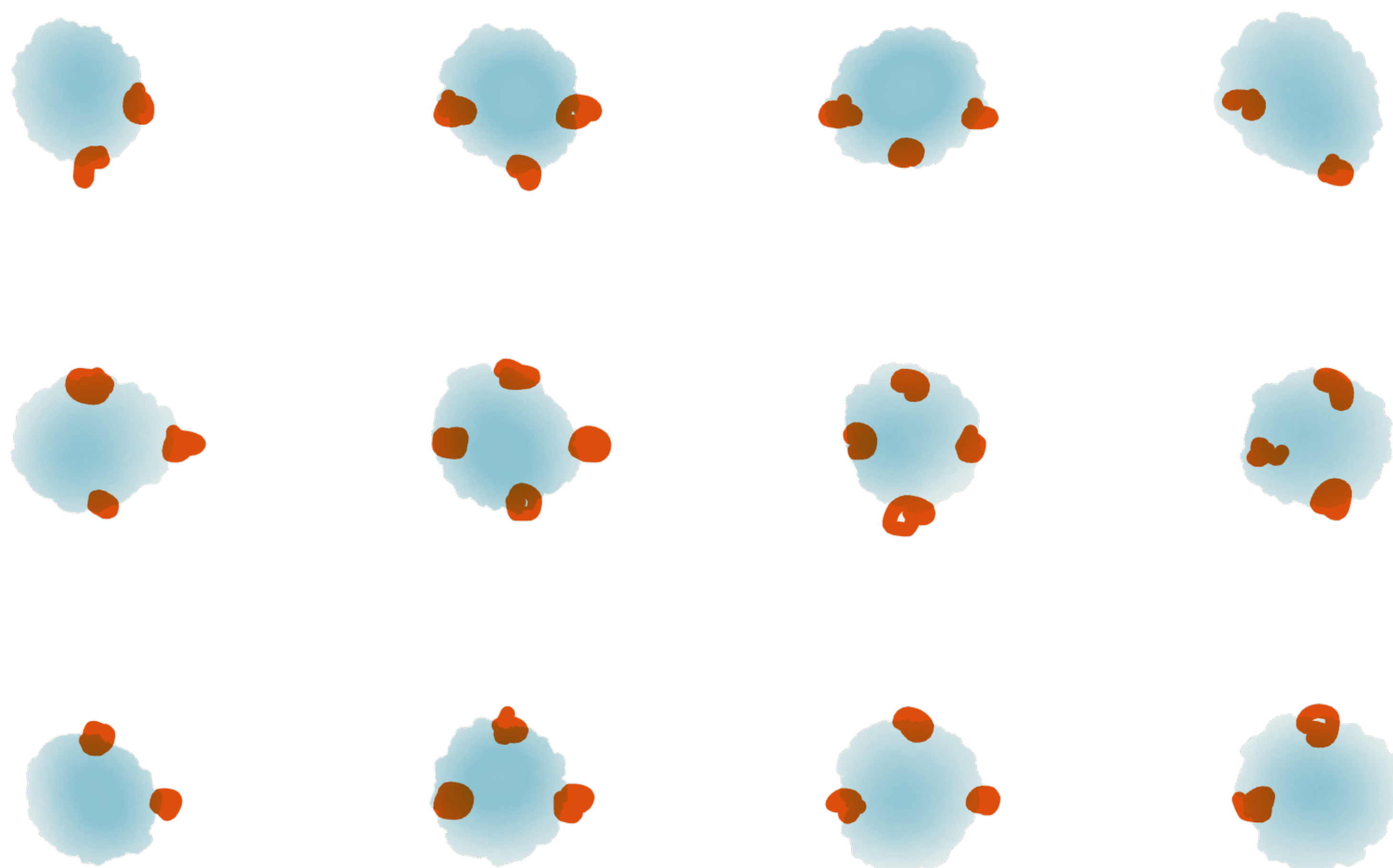

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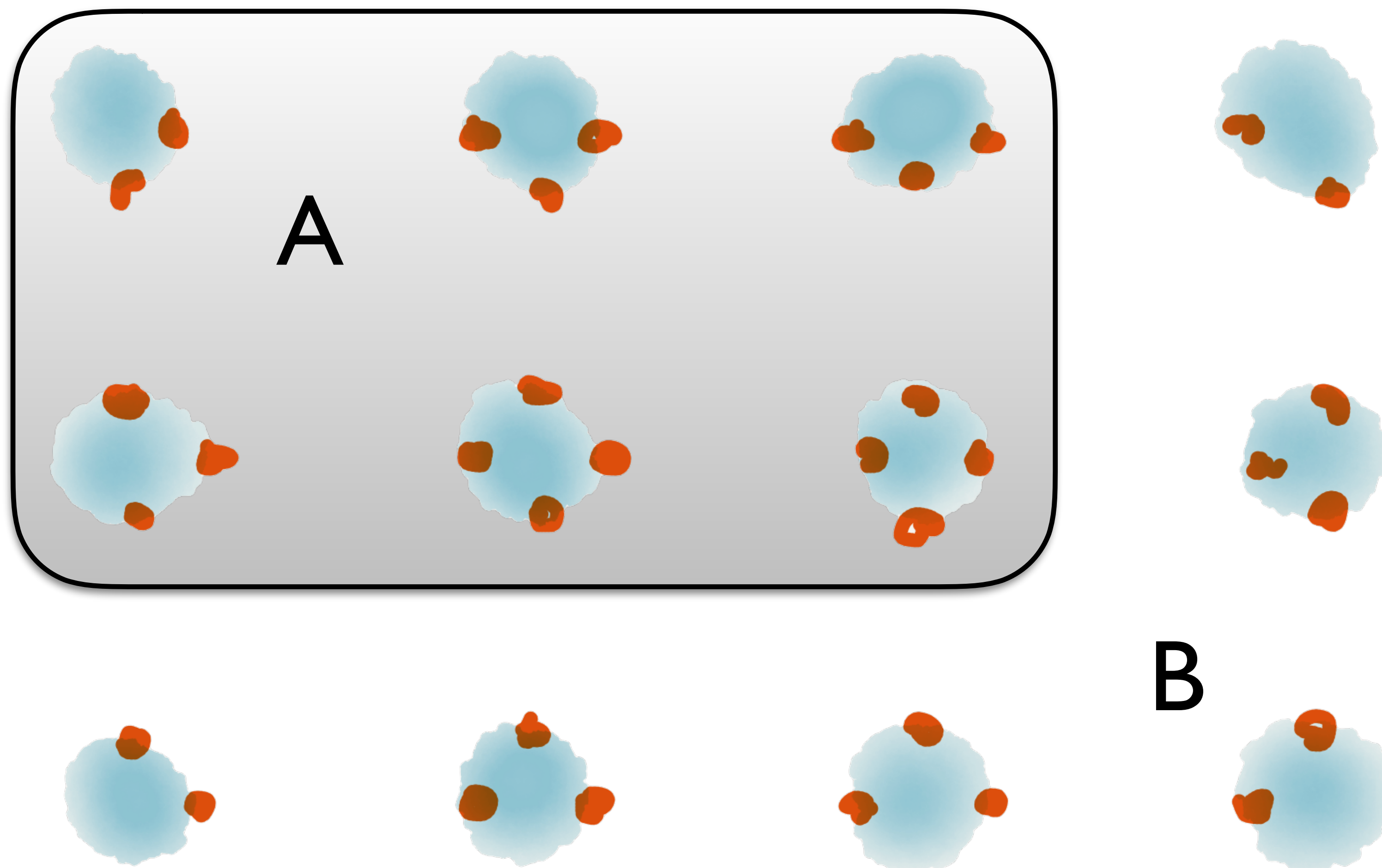
[Schwinger '52]

Subsystem A = collection of oscillators [EB, Guglielmon, Hackl, Yokomizo 2016]



Entanglement entropy

$$S_A(|s\rangle) \equiv -\mathrm{Tr}_A\left(\rho_A \log \rho_A\right)$$

where

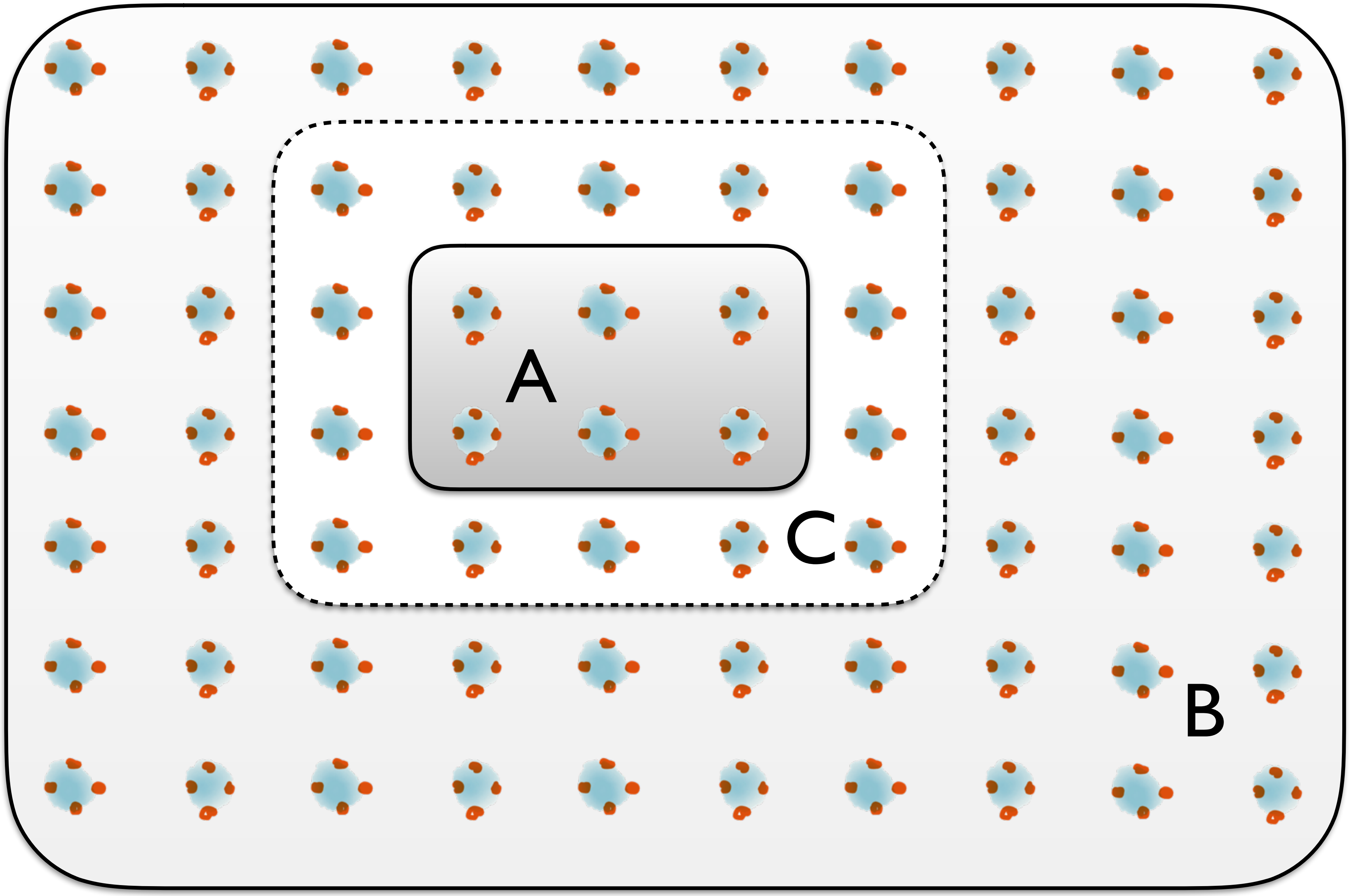
$$\rho_A = \mathrm{Tr}_B(|s\rangle\langle s|)$$

Defining entanglement in loop quantum gravity II

Subsystem A = collection of oscillators

Buffer zone C

[EB, Guglielmon, Hackl, Yokomizo 2016]



Entanglement Entropy between macroscopic d.o.f.

$$S_{A,\mathrm{macro}}(|s\rangle) \equiv \frac{1}{2} H(\rho_{AB}|\rho_A \otimes \rho_B) = \frac{1}{2}\mathrm{Tr}(\rho_{AB}\log\rho_{AB} - \rho_{AB}\log(\rho_A \otimes \rho_B))$$

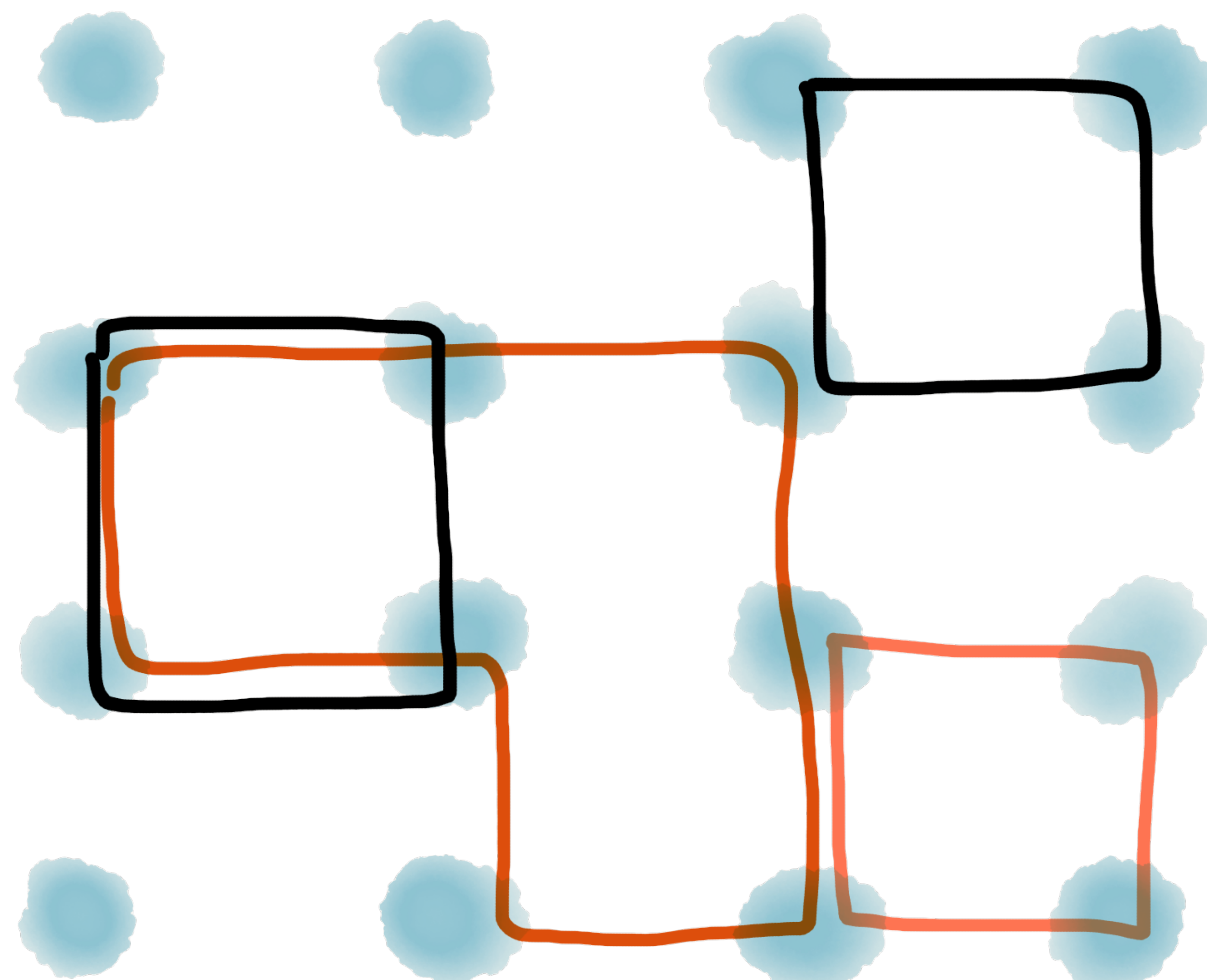
$$= \frac{1}{2}(S_A + S_B - S_{AB})$$

Loop expansion and the area law

multi-loop $\Phi = \{\alpha_1, \alpha_2, \ldots\}$

multi-loop state $:W_\Phi : |0\rangle = F^\dagger_\Phi |0\rangle$ where $F_\Phi = \prod_{\alpha\in\Phi} \Big(\prod_{\langle i,j\rangle \in \alpha} \epsilon_{AB}\, a_i^A a_j^B \Big)^{m_\alpha}$

loop-expansion of a state $|\Gamma, s\rangle = \sum_\Phi c_\Phi(s)\; F^\dagger_\Phi |0\rangle$



long-range correlation and area law
require $c_{\Phi_1\cup\Phi_2}(s) \neq c_{\Phi_1}(s)\, c_{\Phi_2}(s)$

See J.Guglielmon's talk in D1

[EB-Guglielmon-Hackl-Yokomizo 1605.05356]

Squeezed vacua in LQG

Squeezed vacua are labeled by a spinor-correlation matrix γ_{ij}^{AB}

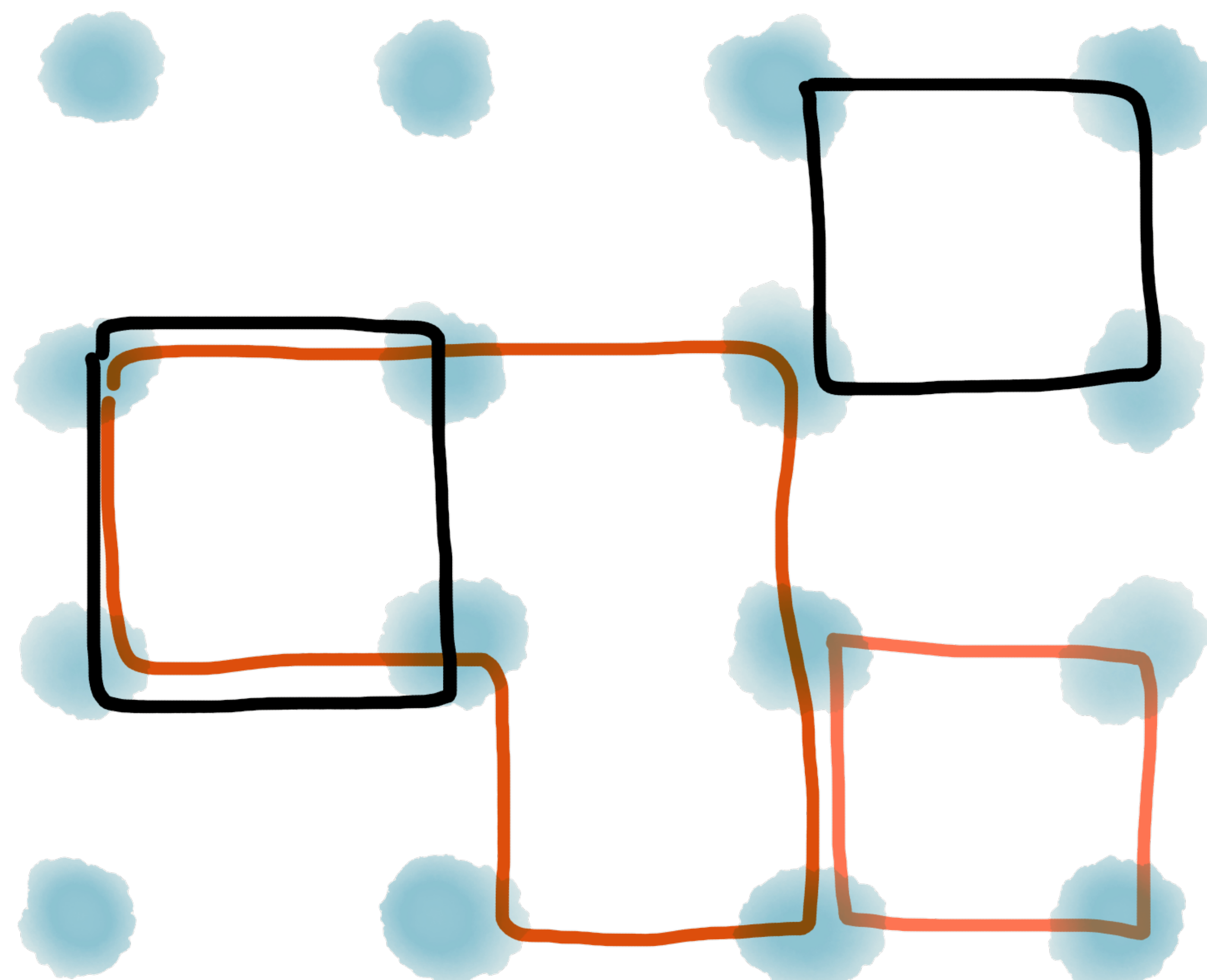
Loop expansion $$|\Gamma,\gamma\rangle = \sum_\Phi c_\Phi(\gamma)\; F_\Phi^\dagger |0\rangle$$

with $$c_\Phi(\gamma) = \frac{1}{\prod_\ell (2j_\ell)!\;\prod_n (j_n+1)!} \int \bar{Z}_\Phi\; e^{-z_i^A \bar{z}_A^i + \frac{1}{2}\gamma_{ij}^{AB}\, z_A^i\, z_B^j}\; \prod_{i,A} \frac{dz_i^A \wedge d\bar{z}_i^A}{\pi}$$

$$Z_\Phi = \prod_{\alpha\in\Phi} \Big(\prod_{\langle i,j\rangle \in\alpha} \epsilon_{AB}\, z_i^A z_j^B \Big)^{m_\alpha}$$

long-ranged matrix γ_{ij}^{AB}

leads to long-range correlations
in spin-spin, W-W,...
and to area law



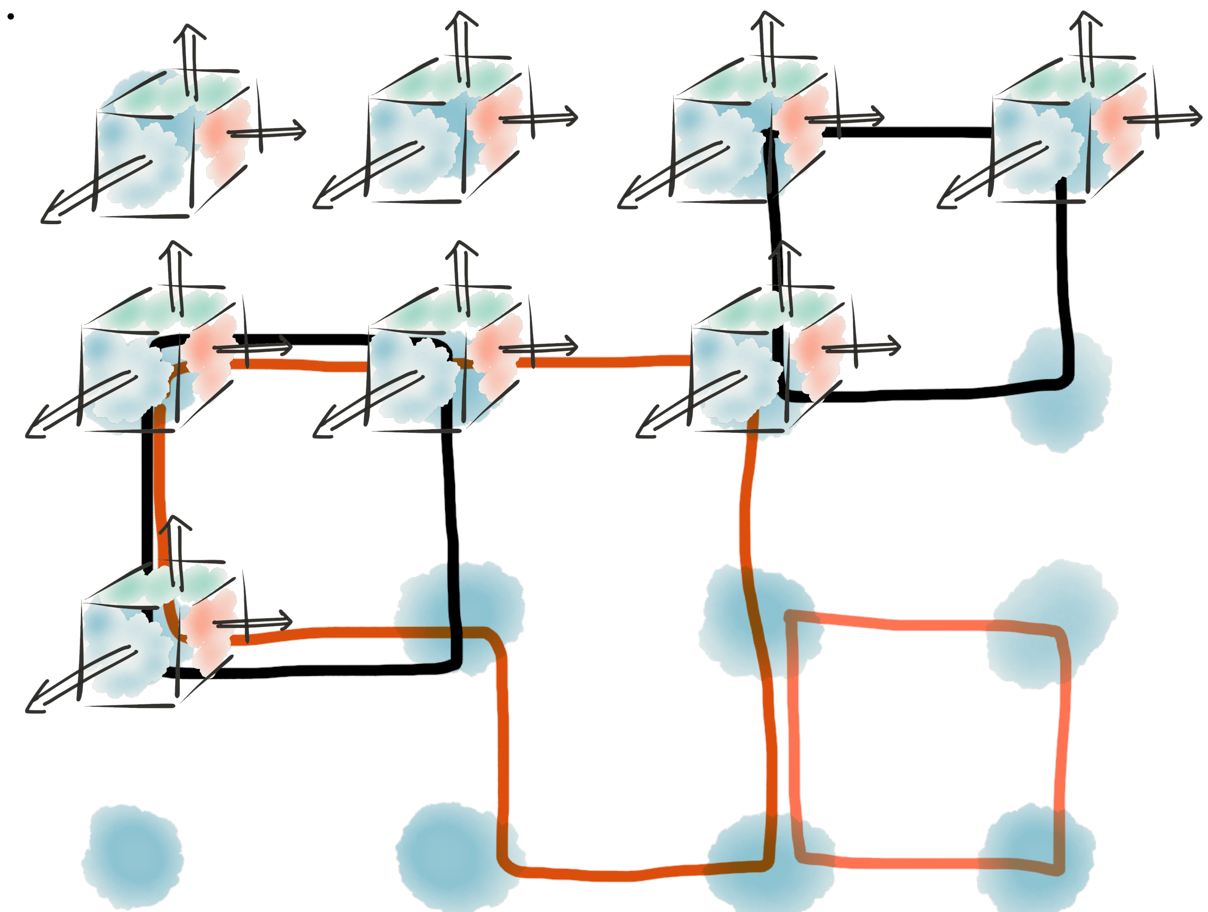
[EB-Guglielmon-Hackl-Yokomizo 1605.05356]

<u>Correlations in a small squeezing expansion</u> $\lambda \ll 1$

$$|\Gamma,\gamma\rangle = |0\rangle + \lambda^4 \sum_{\square} c_{\square}\, F^\dagger_{\square}\, |0\rangle +$$
$$+\,\lambda^6 \Big(\sum_{\square\square} c_{\square\square}\, F^\dagger_{\square\square}|0\rangle + \sum_{\square} c_{\square}\, F^\dagger_{\square}|0\rangle + \sum_{\square} c_{\square}\, F^\dagger_{\square}|0\rangle \Big)$$
$$+\,\lambda^8 \Big(\sum_{\square\,\square} c_{\square\,\square}\, F^\dagger_{\square} F^\dagger_{\square}|0\rangle + \dots \Big) \;+ O(\lambda^{10}).$$

Correlations at the same node or nearby nodes encode the expectation value of local geometric operators and provide a classical background.

Correlations between distant nodes encode quantum fluctuations over that background.



See N.Yokomizo's talk in D1

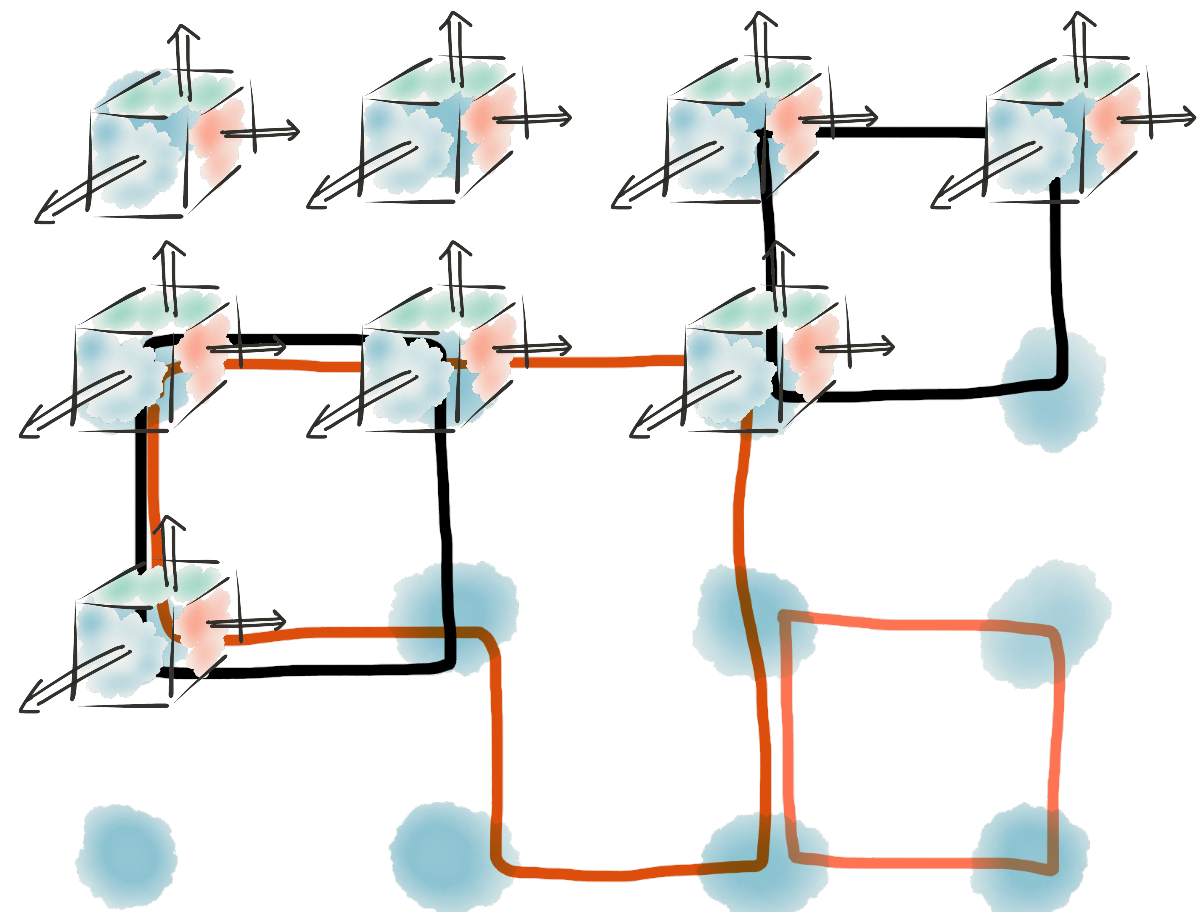
$$\gamma^{AB}_{(m\mu)(n\nu)} = \lambda\, \big(\delta_{mn} + \varepsilon\, f_{mn}\big)\, \epsilon_{CD}\, \hat{z}^C_\mu\, \hat{z}^D_\nu\;\; \epsilon^{AB}\,.$$

Perspectives

- class of states that satisfy the area-law for the entanglement entropy

$$S_{A,\mathrm{macro}}(|\Gamma,\gamma\rangle) = \alpha \,\frac{\mathrm{Area}(\partial \mathrm{A})}{L^2_{Planck}} + \ldots$$

- correlations between macroscopic observables



- graviton propagator and boundary states in spin-foams

- new approximation scheme for physical states

$$\langle\Gamma,\gamma|H_{\boxplus}|\Gamma,\gamma\rangle = 0\,, \qquad \langle\Gamma,\gamma|H_{\boxplus}\,H_{\boxplus'}|\Gamma,\gamma\rangle = 0 \qquad \forall\ \boxplus,\boxplus'.$$