Size and angular momentum of axisymmetric objects

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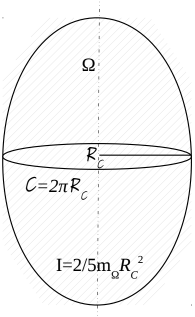
Axially symmetric objects

Rigid, axially symmetric, isolated, rotating object Ω. *Newtonian limit*.

Total energy $\Rightarrow$ $\mathcal{E} \approx \mathcal{E}_{rest} + \mathcal{E}_{bind} + \mathcal{E}_{rot}$

$$\mathcal{E}_{rot} = \frac{J_\Omega^2}{2I}, \qquad I := \text{moment of inertia} \approx c_1 m_\Omega \mathcal{R}_\mathcal{C}^2$$



We expect

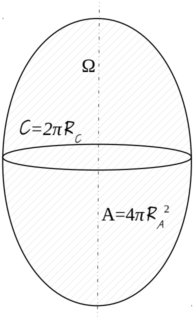
- $m_\Omega \lesssim \mathcal{R}/2$ (Hoop conjecture)

$$\mathcal{E} \gtrsim \mathcal{E}_{rest} + \mathcal{E}_{bind} + c_2 \frac{J_\Omega^2}{\mathcal{R}\mathcal{R}_\mathcal{C}^2}$$

Settings I

Initial data $(M, g, K; \mu, j)$, axially symmetric

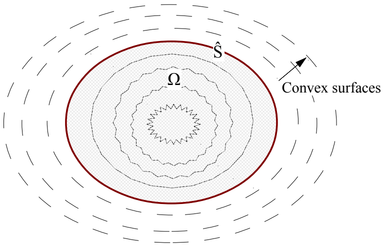
- object: open set $\Omega \subset M$.
- Positive matter density $\Rightarrow$ Weak energy condition.
- isolated $\Rightarrow$ Asymptotically flat
- total energy $\Rightarrow$ ADM mass.
- angular momentum $\Rightarrow$ Komar integral.
- measure of sizes $\Rightarrow$ $\mathcal{R}_A := \sqrt{\frac{A}{4\pi}}$, $\mathcal{R}_{\mathcal{C}} := \frac{\mathcal{C}}{2\pi}$



Settings II

Technical assumptions

- Initial data is maximal: $\mathrm{tr} K = 0$.
- There exists a smooth Inverse mean curvature flow of surfaces S_r on M going to infinity.
- There exists $\hat{r}$ such that S_r is convex for $r \geq \hat{r}$.



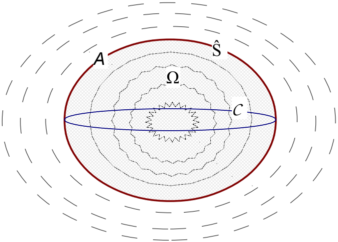
Result

$(M, g, K; \mu, j)$ initial data, maximal, AF, axially symmetric, satisfying the WEC. Assume there exists a smooth IMCF of surfaces S_r on M going to infinity, and $\hat{r}$ such that S_r is convex for $r \geq \hat{r}$. Then, taking $\Omega := int(S_{\hat{r}}) = int(\hat{S})$

$$m_{ADM} \geq m_\Omega + \frac{4J_\Omega^2}{5\mathcal{R}_A\mathcal{R}_\mathcal{C}^2}$$

$$m_\Omega := \int_\Omega \mu dS d\rho, \quad \rho := \sqrt{A/4\pi}$$

$$\mathcal{R}_A := \sqrt{\frac{A(\hat{S})}{4\pi}}, \qquad \mathcal{R}_\mathcal{C} := \frac{\mathcal{C}(\hat{S})}{2\pi}$$



Remarks $\quad m_{ADM} \geq m_\Omega + \frac{4J_\Omega^2}{5\mathcal{R}_A\mathcal{R}_\mathcal{C}^2}$

- Global inequality
  - cf. $J \lesssim \mathcal{R}^2$ by Dain, Khuri, Reiris.

- Linear in m_{ADM}, quadratic in J .
  - Newtonian limit $\mathcal{E} \gtrsim \frac{J^2}{\mathcal{R}\mathcal{R}_\mathcal{C}^2}$.
  - Bekenstein bound $\mathcal{E} \gtrsim \frac{J}{\mathcal{R}}$.
  - Black holes $\mathcal{E} \geq \sqrt{J}$ by Dain.

- Oblate objects
  - $\mathcal{R}_\mathcal{C}^2 \geq \frac{4J^2}{5\mathcal{R}_A m_{ADM}}$

- No equations of state for matter are assumed.
  - If $\mu = const. \quad \Rightarrow \quad m_\Omega = \mu\frac{4\pi}{3}\mathcal{R}_A^3$

- Maxwell fields
  - $m_{ADM} \geq m_\Omega + \frac{Q^2}{2\mathcal{R}_A} + \frac{4J^2}{5\mathcal{R}_A\mathcal{R}_\mathcal{C}^2}$

Next

- Convexity condition of surfaces along the flow.
- Improve bound. Changing the size measure?

- BH formation criteria.
- Penrose inequality

Thank you very much.