Limits set by causality on neutron-star deformability and on the tidally induced change in inspiral waveform

Eric Van Oeveren
John Friedman

Leonard E. Parker Center for Gravitation, Cosmology, and Astrophysics
Univeristy of Wisconsin-Milwaukee

Tidal Effects During Inspiral

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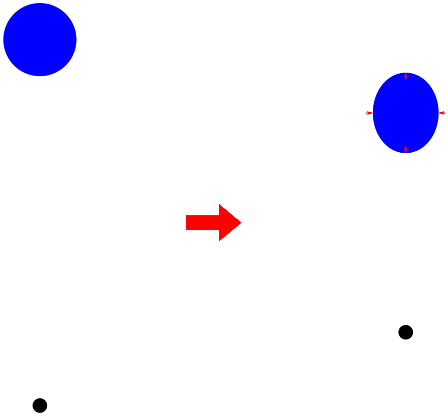
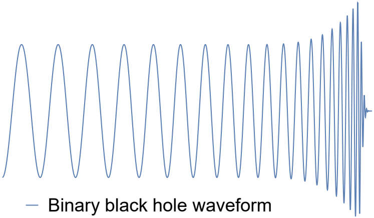
- Neutron star gets tidally deformed
- Deformation requires energy; taken from gravitational binding energy



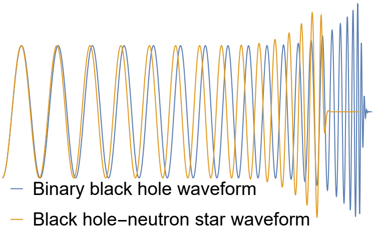
Tidal Effects During Inspiral

- Result: GWs from binaries including neutron stars accumulate phase more quickly than those from binary black hole systems



Tidal Effects During Inspiral

- Result: GWs from binaries including neutron stars accumulate phase more quickly than those from binary black hole systems
- GW observations of can constrain neutron star tidal deformability and EOS



Previous work constraining NS EOS from GWs:

- Kochanek (1992)
- Lai, Rasio, and Shapiro (1994)
- Mora and Will (2004)
- Berti, Iyer, and Will (2008)
- Hinderer, Lackey, Lang, and Read (2010)
- Flanagan and Hinderer (2008)
- Read, Markakis, Shibata, Uryū, Creighton, and Friedman (2009)
- Bauswein, Janka, Hebeler, and Schwenk (2012)
- Foucart, Duez, Kidder, Scheel, Szilagyi, and Teukolsky (2012)
- Many more

Goal of this Work

- Constrain the tidal deformability using causality
- Similar method to:
  - Rhoades and Ruffini (1972)
  - Brecher and Caporaso (1976)
  - Sabbadini and Hartle (1977)
  - Lattimer (2013)

Defining Tidal Deformability:

When a star is perturbed by a tidal field $\mathcal{E}_{ij}$, its quadrupole moment Q_{ij} can be written to linear order in $\mathcal{E}_{ij}$. Tidal deformability is then the constant of proportionality between tidal field and induced quadrupole moment:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

Constraining Tidal Deformability

- Larger stars are more easily deformed
- Constraint on size $\rightarrow$ constraint on λ

Steps to Estimate Tidal Effects on Waveform

1. Choose an equation of state $p(\epsilon)$
2. Construct spherical stars from TOV equation
3. Tidally perturb spherical stars and compute λ
4. Estimate effect of λ on waveform using numerical results

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$$c_s^2 = \frac{dp}{d\epsilon} \leq c^2$$

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Chosen EOS: Candidate EOS (MS1, Müller and Serot 1995) at low density, stiffest causal EOS above nuclear density

$$p = \begin{cases} p_{MS1}(\epsilon), & \epsilon < \epsilon_{nuc} \\ (\epsilon - \epsilon_S)c^2, & \epsilon \geq \epsilon_{nuc} \end{cases}$$

2. Construct Spherical Stars

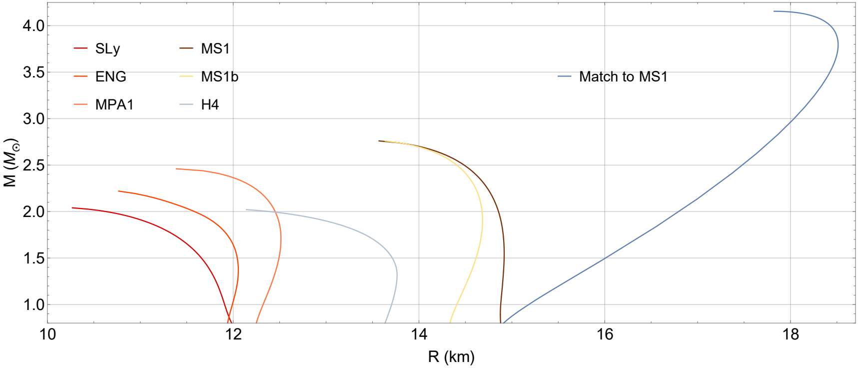
Tolman-Oppenheimer-Volkoff equation:

$$\frac{dp}{dr} = -\frac{G}{r^2}\left(\epsilon + \frac{p}{c^2}\right)\left(m + 4\pi r^3 \frac{p}{c^2}\right)\left(1 - \frac{2Gm}{c^2 r}\right)^{-1}$$

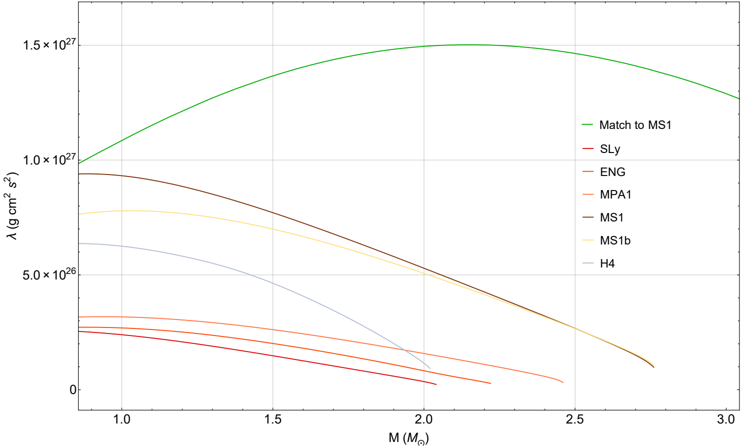
$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

With EOS $p(\epsilon)$, gives a differential equation for $m(r)$.

M-R relation

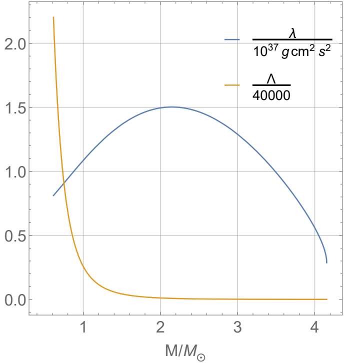

SLy
ENG
MPA1
MS1
MS1b
H4
Match to MS1
M ($M_\odot$)
R (km)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
10
12
14
16
18

Comparing resulting λ-values with realistic equations of state



4. Estimate Effect on Waveform

- Waveform depends on $\Lambda = \frac{c^{10}}{G^4} \frac{\lambda}{M^5}$
- Ways to estimate $\Delta\Phi = \Phi_{NS} - \Phi_{PP}$:
  - PN expansion
  - EOB
  - $\boxed{\text{Numerical Results}}$
- We will estimate $\Delta\Phi(f) < 0$



Fit to Numerical Data

Lackey et al. (2013) give fit to numerical data for amplitude and phase shift of the waveform from black hole-neutron star binaries:

$$A = \begin{cases} A_{PN}, & \frac{GM_{tot}f}{c^3} \leq .01 \\ A_{PN} e^{-\eta \Lambda b(\Lambda, \eta, \chi_{BH}) (\frac{GM_{tot}f}{c^3} - .01)^3}, & \frac{GM_{tot}f}{c^3} > .01 \end{cases}$$

$$\Delta\Phi = \begin{cases} \Delta\Phi_{PN}, & \frac{GM_{tot}f}{c^3} \leq .02 \\ -\eta\Lambda c(\eta, \chi_{BH}) (\frac{GM_{tot}f}{c^3} - .02)^{5/3} + & \\ \quad \Delta\Phi_{PN}(.02) + (\frac{GM_{tot}f}{c^3} - .02)\Delta\Phi'(.02), & \frac{GM_{tot}f}{c^3} > .02 \end{cases}$$

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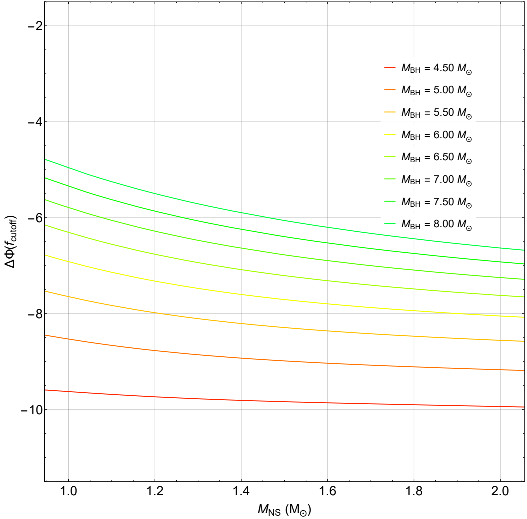
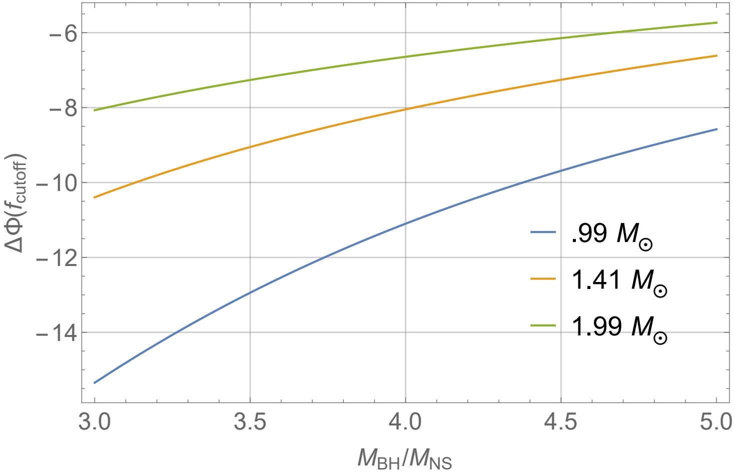
We choose f_{cutoff} to be f-value where A is damped by factor of e.

Upper limit on $|\Delta\Phi|$ as a function of neutron star mass



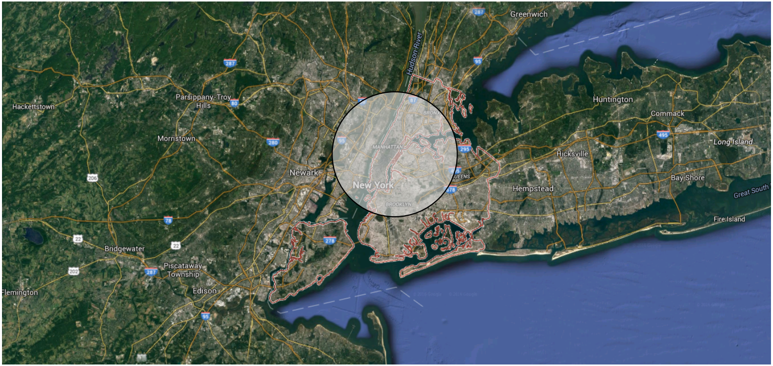
Upper limit on $|\Delta\Phi|$ as a function of mass ratio



Conclusion

- The tidal deformability of neutron stars has been constrained by causality and a match at nuclear density to a realistic equation of state
- The resulting effect on the BHNS inspiral and merger waveform has been estimated
- Future work: numerical simulations to get exact constraints of $\Delta\Phi$ for BHNS mergers and for BNS mergers
- Special thanks: Benjamin Lackey and Lee Lindblom

How big is a neutron star?



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