Towards a charged Myers-Perry black hole

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Introduction

1986 MP: higher dimensional gen of Kerr.

5D, two independent rotations

$$
\begin{aligned}
ds^2 &= -dt^2 + \rho^2\left(\frac{r^2\,dr^2}{\Delta(r)} + d\theta^2\right) \\
&\quad + \left(r^2 + a^2\right)\sin^2\theta\,d\phi^2 + \left(r^2 + b^2\right)\cos^2\theta\,d\psi^2 \\
&\quad + \frac{\mu}{\rho^2}\left[dt + a\sin^2\theta\,d\phi + b\cos^2\theta\,d\psi\right]^2 \\
\rho^2 &= r^2 + a^2\cos^2\theta + b^2\sin^2\theta \\
\Delta(r) &= \left(r^2 + a^2\right)\left(r^2 + b^2\right) - \mu\,r^2
\end{aligned}
$$

t and r, “zenith” $0 \le \theta < \pi/2$, two “azimuthal” $0 \le \phi, \psi < 2\pi$.

Ergoregion, event horizon, and asymptotically flat.
Causal structure outside EH essentially same as Kerr.

Adding electric charge

With a 5D version of Kerr, what about 5D version of Kerr-Newman?

Myers and Perry solved equations within Kerr-Schild approach. Argued such would not work for Einstein-Maxwell so presented only uncharged BH solution.

Numerical solutions with charge have been constructed by Kunz et al in special cases or with additional fields.

Perturbative approach by Aliev considers small rotation in which the charge can be included. Suggests that charge and rotation in 5D can coexist.

Another numerical try

We have some background in numerical solutions for rotating, NS in 4D.

Use "self-consistent field approach" of Hachisu et al.
Useful for matter spacetimes. Used less for vacuum or electrovac.

Exploit symmetries, reducing spacetime to an effective 2D space with extra fields living on reduced manifold.

Using coords similar to one MP expressed in, take Killing vectors $\phi^a = \delta^a_\phi, \psi^a = \delta^a_\psi$ and $t^a = \delta^a_t$.
Start with ϕ^a, perform successive rescalings and projections.

End up with what amounts to an orthonormal frame.

Another numerical try

Take metric as

$$g_{ab} = -T_a T_b + U_a U_b + V_a V_b + \sigma_{ab}$$

Reducing to this form leaves 2D gravity with 6 scalar fields. These related to norms and inner products of the Killing vectors.

Einstein eqns become six elliptic equations for six scalars; nice from numerical perspective.

Remaining function on 2 metric solved by integrating two first order equations, analogous to Kerr.

Elliptic equations

Equations take the form

$$\nabla^2_{(p,q)} f = S(f, \partial f, \cdots)$$

where source can be nonlinear in the function, its derivative as well as other metric functions. $p, q \in \{0, 1, 2\}$.

In chosen coordinates, the elliptic operator takes the form

$$\nabla^2_{(p,q)} = \partial_r \partial_r + \frac{3+p+q}{r} \partial_r + \frac{1}{r^2} \partial_\theta \partial_\theta + \frac{1}{r^2} \left[(p+1) \cot\theta - (q+1) \tan\theta \right] \partial_\theta$$

The eigenfunctions of the (angular part of the) elliptic operator are Jacobi polynomials. Green's function easily constructed.

Boundary conditions

Impose asymptotic flatness at spatial infinity ($r = \infty$) and regularity on the horizon, $\Delta(r_H) = 0$.

r can be numerical challenge, use compactified coordinates

$$r = \frac{r_H}{1 - s}$$

so (radial) computational domain becomes $s \in [0, 1]$.

Most metric components satisfy Neumann BCs at horizon ($s = 0$) and vanishing Dirichlet BCs at spatial infinity ($s = 1$). Exceptions satisfy Dirichlet everywhere.

Allows exact imposition of BCs.

Electromagnetism

So far, just Einstein. Must decompose Maxwell, too.

On doing so, three new scalar fields (components of gauge potential along Killing directions).

Satisfy same elliptic equations as previously with mixed or Dirichlet BCs.

Same formalism carries over to Einstein-Maxwell.

Parameter values

How do BH parameters come in?

We use an iterative approach. Need "starting" values, including on EH.

These are not BCs, but amplitudes for certain quantities; not independent.

KN: $\Omega|_{r_H}$, $\Phi|_{r_H}$, $\Omega|_{r_H}\Phi|_{r_H}$

MP: $\Omega_\phi|_{r_H}$, $\Omega_\psi|_{r_H}$, $\Omega_\phi|_{r_H}\Omega_\psi|_{r_H}$,

Rigidity: constants on horizon; related to ang mom(s) and charge.

Assume
cMP: $\Omega_\phi|_{r_H}$, $\Omega_\psi|_{r_H}$, $\Phi|_{r_H}$, $\Omega_\phi|_{r_H}\Omega_\psi|_{r_H}$, $\Phi|_{r_H}\Omega_\phi|_{r_H}$, $\Phi|_{r_H}\Omega_\psi|_{r_H}$

Numerical approach

Usual Green's function:

$$\begin{aligned}\nabla^2_{(p,q)} f &= S_f \\ f &= \int G_{(p,q)}\, S_f\, dV\end{aligned}$$

Iterate such that

$$f_{i+1} = \int G_{(p,q)} S_f(f_i, \cdots)\, dV$$

until $|f_{i+1} - f_i| < \mathrm{tol}.$
Numerical integration: Simpson and Gaussian quadrature.
Accommodate nonsmoothness of $G_{(p,q)}$
Use filters to handle Gibbs.
Use finite number of Jacobi polys in expansion of $G_{(p,q)}$
Use successive overrelaxation to aid convergence.

Test cases: 4D

Tested approach in 4D: reproduce Schwarzschild, Reissner-Nordstrom, Kerr and Kerr-Newman.

Self-consistent field method (iterative Green's method):

- converges to a solution iteratively
- converges to the exact solution iteratively
- converges to the exact solution with resolution.

Gives confidence in the approach as applied to black holes.

Test cases: 5D

Tested approach in 5D: reproduces Schwarzschild-Tangherlini, charged Tangherlini and single rotation Myers-Perry.

Results similar to 4D: converges iteratively to exact solution and converges with resolution.

However ... double rotation MP fails to converge with resolution.

Fails with respect to a single metric function; very close, i.e. the error is small, but it does not converge away.

Suggests a mistake/bug/problem in equation for which convergence is not achieved.

Still looking.

Charged Myers-Perry

Despite this, can solve for charged Myers-Perry with two rotations.

Iterations converge so that a solution is found to tolerance for range of angular momentums and charge, but solution does not converge with resolution.

Good evidence that a solution exists.

Outlook and possible directions

Settle convergence (with resolution) issues.

Explore solution properties.

Large charge, extremal limits

Add cosmological constant

Provide insight into an analytic solution?