Graviton creation by an oscillating scale factor

in an expanding universe

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(arXiv:1602.08416v1)

21^{st} INTERNATIONAL CONFERENCE ON GENERAL RELATIVITY AND GRAVITATION

July 12, 2016

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Introduction

- The cosmological creation of gravitational waves was postulated by Grishchuk (Grishchuk, 1974) and, after that, this process has been studied specially in the context of inflation.
- We focus on a graviton creation by a sinusoidally oscillating scale factor in a spatially flat FRW background in two different models:
  - Standard general relativity (GR) plus a minimally coupled scalar field.
  - Modified gravity with a term proportional to the square of the Ricci scalar in the Einstein-Hilbert action.

Oscillating scale factor

- Standard GR plus a minimally coupled scalar field with a harmonic potential $(\omega\psi)^2/2$.
- When $\omega \gg H$, the ψ field rapidly oscillates.
- The scale factor $a(t)$ oscillates as

$$\bar{a}(t)\left[1 - C\left(\frac{\bar{a}(t_i)}{\bar{a}(t)}\right)^3 \cos(2\omega t)\right]$$

- The gravitational wave equation is (the tensor metric perturbation is given by $2H_T(\eta)Y_{ij}(\mathbf{x})$ where Y_{ij} is a symmetric, trace-free and transverse harmonic function, and η is the conformal time):

$$H_T(\eta)'' + \left[k^2 - a(\eta)''/a(\eta)\right] H_T(\eta) = 0$$

- $f(R)$-gravity with $f(R) = R + \frac{a_2}{2}R^2$.
- Apply a conformal transformation and associate a scalar field ϕ with a function of R. Then, ϕ can also show an oscillatory behavior.
- The scale factor $a(t)$ oscillates as

$$\bar{a}(t)\left[1 - D\left(\frac{\bar{a}(t_i)}{\bar{a}(t)}\right)^{3/2} \cos(\omega t)\right]$$

- The production of gravitons is ruled by a modified gravitational wave equation (here $A(\eta) = a(\eta)\sqrt{1 + a_2 R}$) (Hwang & Noh, 1996):

$$H_T(\eta)'' + \left[k^2 - A(\eta)''/A(\eta)\right] H_T(\eta) = 0$$

Number density and energy density for gravitons

- Both scenarios show an oscillating scale factor and quantum production of gravitons.
- On short time scales, both scale factors can be expressed in flat spacetime as

$$a(\eta) = 1 + A_0 \cos(\omega_0 \eta)$$

- Here A_0 and ω_0 take different values for each case. For GR plus a minimally coupled scalar field $\omega_0 = 2\omega$ and for $f(R)$-gravity $\omega_0 = \omega$.
- We use a perturbative method (Birrel & Davies, 1980) to solve both gravitational wave equations using this scale factor and to calculate the number density (n_g) and energy density (ρ_g) of gravitons.
- We take the asymptotic behavior of n_g and ρ_g for times scales long compared to the period of oscillation.

- For the number density and energy density of gravitons in both cases we have

$$\left.\frac{dn_g}{dt}\right|_{GR} \sim \frac{C^2\omega_0^4}{16\pi} \quad \text{and} \quad \left.\frac{dn_g}{dt}\right|_{f(R)} \sim \frac{9D^4\omega_0^4}{16\pi}$$

$$\left.\frac{d\rho_g}{dt}\right|_{GR} \sim \frac{C^2\omega_0^5}{32\pi} \quad \text{and} \quad \left.\frac{d\rho_g}{dt}\right|_{f(R)} \sim \frac{9D^4\omega_0^5}{16\pi}$$

- In GR with a minimally coupled scalar field, the number and energy creation rate are proportional to the square of the metric oscillations.
- By contrast, in $f(R)$-gravity the number and energy creation rate are proportional to the fourth power of the metric oscillations.
- Basically, the graviton energy density for each case can be extended to an expanding universe in a similar way.

- We have to consider two main phenomena in an expanding universe:
  - Damping of the metric oscillations
  - Redshifting and dilution of created gravitons
- So long as the expansion rate of the background is slow compared to the oscillation rate, $\frac{1}{\bar{a}}\frac{d\bar{a}}{dt} \ll \omega$, we may treat the background spacetime as approximately flat.
- Then, we can use the previous results in flat spacetime to include the damping of the metric oscillations as:
  - For GR plus the scalar field: $C \to C_{eff} = C_i \left[\frac{\bar{a}(t_i)}{\bar{a}(t)}\right]^3$
  - For $f(R)$-gravity: $D \to D_{eff} = D_i \left[\frac{\bar{a}(t_i)}{\bar{a}(t)}\right]^{3/2}$
- After creation, the expansion causes redshift and dilution of the created gravitons:
  - $\rho_g(t)$ in both cases scales as $1/\bar{a}^4(t)$

- Putting all together, we have the same main result from both cases:

$$\frac{d\rho_g(t')}{dt} \propto \omega_0^5 \left[\frac{\bar{a}(t_i)}{\bar{a}(t)}\right]^6 \left[\frac{\bar{a}(t)}{\bar{a}(t')}\right]^4$$

- The unique difference between both cases rises from the nature and the value of the constant of proportionality and the angular frequency of oscillations.
- This expression tells us that the present contribution of earlier graviton production is suppressed by a factor of $(1+\bar{z})^{-4}$ due to redshifting and increased by a factor proportional to $(1+\bar{z})^6$ due to greater oscillation amplitude at earlier times.
- Assuming that oscillations start at time t_i, the present graviton energy density in both cases will be (here we take t_0 to be the present time):

$$\rho_g(t_0) \propto \omega_0^5 \; \bar{a}(t_i)^6 \int_{t_i}^{t_0} \bar{a}^{-2}(t)dt$$

Conclusion

- Small oscillations of a scale factor in a FRW-background lead to quantum creation of gravitons.
- Such oscillations can rise either in GR plus a coherent oscillating scalar field or in $f(R)$-gravity.
- Both models show a similar expression for the graviton energy density in a FRW-background with differences in the constant of proportionality and the angular frequency of oscillations.

- GR plus a scalar field
- The fraction of oscillations over the background spacetime damps as $1/\bar{a}^3$
- The graviton number and energy creation rate in flat spacetime are proportional to the square of the metric oscillations.

- $f(R) = R + \frac{a_2}{2} R^2$
- The fraction of oscillations over the background spacetime damps as $1/\bar{a}^{3/2}$
- The graviton number and energy creation rate in flat spacetime are proportional to the fourth power of the metric oscillations.

Further discussion

- The scale factor in each case is the following:
  - Gr: $a(t) = \bar{a}(t)\left[1 - \frac{\psi(t_i)^2}{16Mpl^2}\left(\frac{\bar{a}(t_i)}{\bar{a}(t)}\right)^3 \cos(2\omega t)\right]$
  - $f(R)$: $a(t) = \bar{a}(t)\left[1 - \frac{\phi(t_i)}{\sqrt{6}Mpl}\left(\frac{\bar{a}(t_i)}{\bar{a}(t)}\right)^{3/2} \cos(\omega t)\right]$
- The energy density creation rate for each case is the following:
  - $\left.\frac{d\rho}{dt}(t')\right|_{GR} = \frac{\psi(t_i)^4\omega^5}{256\pi M_{pl}^4}\left[\frac{\bar{a}(t_i)}{\bar{a}(t)}\right]^6\left[\frac{\bar{a}(t)}{\bar{a}(t')}\right]^4$
  - $\left.\frac{d\rho}{dt}(t')\right|_{f(R)} = 4 \times \frac{\phi(t_i)^4\omega^5}{256\pi M_{pl}^4}\left[\frac{\bar{a}(t_i)}{\bar{a}(t)}\right]^6\left[\frac{\bar{a}(t)}{\bar{a}(t')}\right]^4$
- In both cases we focus on a scenario in which the energy density for matter fields, $\rho_M(t)$, is greater than the scalar energy density at any time. Then, we have (here χ refers to either ψ or ϕ scalar field) in a rough approximation

$$3\left(\frac{1}{\bar{a}}\frac{d\bar{a}}{dt}\right)^2 M_{pl}^2 \approx \rho_M(\bar{a}) + \rho_\chi(\bar{a}) \approx \rho_M(\bar{a})$$

- Consider a simple model of universe which is spatially flat and contains radiation, non-relativistic matter, and a cosmological constant associated with the dark energy. The model is first radiation dominated, then non-relativistic matter dominated, and now is entering into its dark energy dominated phase.

- In this model, the condition $\rho_M(t) \gtrsim \rho_\chi(t)$ is equivalent to require

$$\frac{\chi}{M_{pl}} \lesssim 7 \times 10^{-12} \left(\frac{2\times 10^{-3} ev}{\omega}\right) \left(\frac{Ti}{1GeV}\right)^{3/2}$$

- This condition constraints strongly the present graviton energy density today, which obeys in both cases the expression

$$\rho_g(t_0) \propto \omega^5 \bar{a}(t_i)^6 \int_{\bar{a}(t_i)}^{1} \left(\frac{\bar{a}^{-1}}{H_0\sqrt{\Omega_{r,0}+\Omega_{m,0}\bar{a}+\Omega_{\Lambda,0}\bar{a}^4}}\right) d\bar{a}$$